Network Evaluation of Influential Sensors: A Proposed Approach

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Motivation

With persistence in extreme weather conditions, maintaining an infrastructure for safety has grown in demand.

Despite modern instrumentation for acquiring data, interpreting the behavior for policy makers remains a challenge.

We propose a network-based evaluation procedure to identify which measured responses of an infrastructure are most critical for monitoring applications.

The Case Study

A magnitude 4.2 earthquake occurred roughly 5.5 miles south/southeast of the dam.

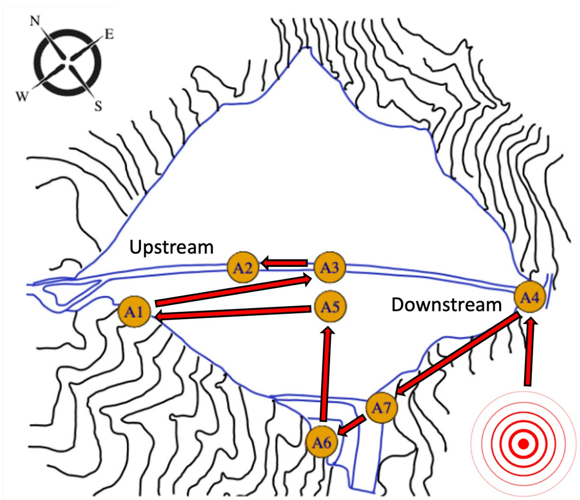
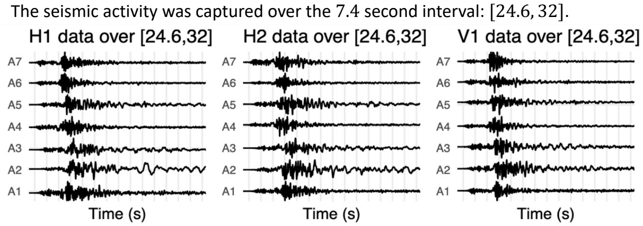


Figure: The progression of the earthquake through the dam.

Each sensor recorded activity for two minutes, at 200 samples per second.

- Horizontal ground motion (H1, H2)
- Vertical ground motion (V1)

The seismic activity was captured over the 7.4 second interval: $[24.6, 32]$.



Network Construction

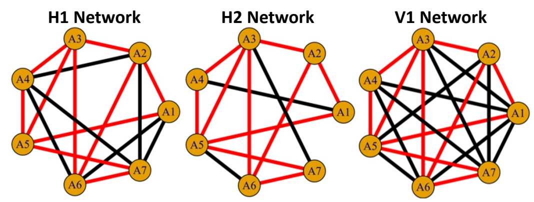
Networks, defined as an abstract graph, $G = (V, E, W)$, provide a natural framework to model complex systems.

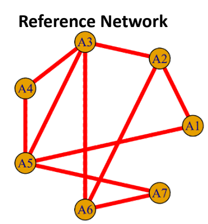
- **Vertices** (V): Sensors
- **Edges** (E): Statistically significant correlations

Functional connectivity[1] is any statistical relation between time series.

We tested the significance of the correlation, r_{x_i,x_j}, between two sensors' times series, x_i and x_j, over $[24.6, 32]$.

- **Null hypothesis** (H_0): $\rho_{x_i,x_j} = 0$
- **Alternative hypothesis** (H_A): $\rho_{x_i,x_j} \neq 0$
- **Significance level**: $\alpha = 0.05$
- **Test statistic**: $t_{x_i,x_j} = \dfrac{r_{x_i,x_j} \cdot \sqrt{n-2}}{\sqrt{1 - r^2_{x_i,x_j}}}$
- **Degrees of freedom**: $\mathrm{df} = n - 2 = 1479$





Assumptions:

1. Topology is fixed (i.e., Reference Network)
2. Information travels along the shortest path
3. Shortest paths change over time

By **Assumption 1**, the time series data was used to create three separately weighted networks, one for each orientation.

- **Weights** (W): $w_{A_i,A_j} = 1 - |r_{x_i,x_j}|$

By **Assumption 3**, the 7.4 second interval was partitioned into 149 sub-intervals of length 0.05 seconds.

$$[24.6, 32] = [24.6, 24.65] \cup [24.65, 24.7] \cup \cdots \cup [31.95, 32]$$

Betweenness Centrality

The betweenness centrality[2], C_B, measures the fraction of shortest paths that pass through sensor A_i.

$$C_B(A_i) = \sum_{A_i \neq A_j \neq A_k} \frac{\sigma_{A_j,A_k}(A_i)}{\sigma_{A_j,A_k}}$$

- σ_{A_j,A_k}: Number of shortest paths connecting sensors A_j and A_k
- $\sigma_{A_j,A_k}(A_i)$: Number of σ_{A_j,A_k} that pass through sensor A_i

By **Assumption 2**, the betweenness value of each sensor was calculated over each sub-interval across each orientation.

The mean betweenness value of each sensor across each orientation was calculated by averaging the betweenness values across the 149 sub-intervals.

H1 Orientation							
Sensor	A1	A2	A3	A4	A5	A6	A7
Normalized Mean C_B	0.19	0.40	0.87	0.03	1.00	0.33	0.13
H2 Orientation							
Sensor	A1	A2	A3	A4	A5	A6	A7
Normalized Mean C_B	0.20	0.53	0.89	0.06	1.00	0.43	0.16
V1 Orientation							
Sensor	A1	A2	A3	A4	A5	A6	A7
Normalized Mean C_B	0.18	0.51	0.84	0.10	1.00	0.41	0.21

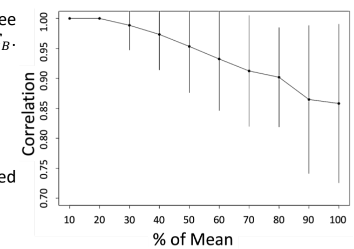
Findings

Averaging the normalized mean betweenness values across the orientations, the sensors that most influence the flow of information are A5 then A3.

Sensor	A1	A2	A3	A4	A5	A6	A7
Grand Normalized Mean C_B	0.19	0.48	0.87	0.06	1.00	0.39	0.17

The sensors were assigned one of three groups based upon their normalized C_B.

- **Group 1**: A3, A5
- **Group 2**: A2, A6
- **Group 3**: A1, A4, A7



Increased levels of noise were added to perturb the acceleration data; the correlation between the rankings remained very strong.

Acknowledgements

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[1] Friston, K. J. (2011). Functional and Effective Connectivity: A Review. *Brain Connectivity*, *1*(1), 13-36.

[2] Brandes, U. (2001). A Faster Algorithm for Betweenness Centrality. *Journal of Mathematical Sociology*, *25*(2), 163-177.