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Profitability Ratios in the Early Stages of a Startup

Cover Page Footnote

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Profitability ratios in the early stages of a startup

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ABSTRACT

This study develops a mathematical framework to analyze the time series of profitability ratios in the early stages of a startup. It is assumed that the expenditure of the startup grows at a steady rate and generates a proportionally identical flow of revenue in each period. The profitability in terms of the internal rate of return (IRR) and the lag structure of revenue flows are assumed constant over time in describing the adjustment process towards the steady state. The startup is assumed to expense in each period a constant part of periodic expenditure and beginning-of-the-period assets. The adjustment processes of three kinds of profitability ratios are investigated: return on investment ratio, profit margin (as percent of net sales), and (traditional) cash-flow margin (as percent of net sales). It is shown that IRR, growth, expense rate, and lag structure strongly affect the early time-series behavior of profitability ratios. Thus, in the early years, due to unstable adjustment processes, profitability ratios are unable to reflect profitability (IRR) properly and can give distorted signals of the performance of a startup. These findings are supported by numerical analyses with the parameters estimated for a sample of 2608 Finnish startups classified into five clusters.

Keywords: Startup, profitability ratios, early time-series, steady state, Finnish firms

JEL: M13, L26, C22, D21, D22, G33, M41

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1. Introduction

The importance of startups is remarkable for any economy investing on innovations and growth. Thus, the early stages of startups are dominant themes of business research, literature, and government policy debate (Davila, Foster, He & Shimizu, 2015). The death and survival of startups are often associated with Schumpeter's creative destruction process as startups with new ideas enter and replace old and stagnant firms (Huynh, Petrunia & Voia, 2012). Huynh et al. (2012) emphasize that a potential hindrance to this process is that startups have insufficient financial resources to carry out their plans. Startups typically find raising equity capital difficult and rely heavily on internal finance and borrowing to finance initial operations. Thus, Zingales (1998) suggests internal finance as the main source of financial capital for these firms. Sufficiency of internal finance is directly related to the early-stage growth and profitability of startups. Therefore, it is essential to the investors, financiers and entrepreneurs that the early development of growth and profitability of startups is well understood.

Profitability is in practice measured by reported financial ratios, which have obvious pitfalls (Murphy, Trailer & Hill, 1996; Losbichler, Hofer, Eisl & Zauner, 2012). Therefore, understanding of early profitability development of startups is connected with the characteristics of financial reporting. This is of importance, since one of the objectives of financial reporting is to make managers accountable to investors so that there is efficient allocation of capital. The most efficient firms should receive financing and have higher valuations than worse firms (Ak, Dechow, Sun & Wang, 2013). The efficiency of firms is measured by financial statement analysis where the general approach is to calculate ratios that represent key underlying constructs, such as profitability. The user can then analyze time-series and cross-sectional trends in the ratios. However, in order to get a reliable view the user must understand the content and the signals of the ratios. If a firm lives in a steady state, financial ratios generate predictable profitability and investors can generally agree on its value (Ak et al., 2013). However, in the early stages of development startups typically suffer from a strong nonstationary adjustment process towards a potential steady state, which makes prediction of future profitability and valuation extremely challenging.

The survival rate of startups in the first five years is only about 45-55% in U.S. and Europe (U.S. Bureau of Labor Statistics, 2016; Eurostat, 2016). Therefore, it is of importance to understand the signals of potential failure given by financial ratios in the early stages of startups. Failure prediction methods based on financial ratios may give reliable signals also for startups (Laitinen, 1992). However, the reliability of these models typically suffers from the occurrence of extreme ratio values and nonstationary failure processes (Balcaen & Ooghe, 2006; Moses & Liao, 1987). These models also suffer from the low quality of financial information that is typical for small firms and especially for startups (Balcaen & Ooghe, 2006). Therefore, understanding of the nonstationary nature of the development of startups and the quality of financial information plays a central role in applying failure prediction models to startups.

The academic research of the stationarity of the time series of financial ratios is wide (McLeay & Stevenson, 2009; Ioannides, Peel & Peel, 2003; Gallizo & Salvador, 2003). Typically, this research is concentrated on testing stationarity assumption for large and older firms, often using different adjustment models. The results are mixed. Whittington and Tippett (1999) found that the components of financial ratios may exhibit nonstationarity whereas Ioannides et al. (2003) concluded that ratios are globally stationary, but that the behaviour close to equilibrium may result from a nonlinear partial adjustment process. There is however also a large number of startup studies on the stationarity of general development in the early years (Garnsey, Stam & Heffernan, 2006; Reid, 2003; Coad, Frankish, Roberts & Storey, 2013; Davila et al., 2015). Typically, these studies do not pay attention to financial ratios but are general indicating that the development of startups is non-linear and prone to interruptions and setbacks, which are stochastic and quite difficult to explain using different variables and processes (Garnsey et al., 2006). None of the time series or development studies concentrates on financial ratios and their intrinsic properties in the early years of startups from the accounting point of view.

The puzzle related to the profitability ratios of startups is clearly empirically observable. Table 1 presents exemplary time series of profitability ratios for a set of randomly selected Finnish startups (source: the ORBIS database of Bureau Van Dijk). In the three panels of the table, the three first cases describe time series of surviving startups and the two last columns those of failed ones. These time series are in line of early startup development as described above by Garnsey et al. (2006). In some cases, profitability ratios are almost constant but generally, they are nonstationary increasing or decreasing rapidly as a result of an adjustment process. For a financial analyst it is difficult or even impossible to identify what kinds of signals these time series are giving about the future and the value of a startup or even what kind of profitability these time series are reflecting upon. These time series are a result of startup growth process and financial reporting. Therefore, the endogenous properties of these time series can only be explained by a conceptual model based on growth and accounting concepts. This puzzle forms the starting point for the present study.

Table 1. Examples of profitability ratios of startups in early years.

Panel 1. Cash flow to net sales ratio (%)

Status of the startup after ten years:

Period	Active	Active	Active	Bankruptcy	Bankruptcy
1	-3,13	5,00	-51,00	4,13	23,17
2	1,30	4,79	-11,58	3,45	16,53
3	3,20	4,40	12,72	0,87	3,25
4	5,14	6,59	13,19	-3,40	-17,00
5	5,78	4,37	16,04	5,32	24,91

Panel 2. Profit margin to net sales ratio (%)

Status of the startup after ten years:

Period	Active	Active	Active	Bankruptcy	Bankruptcy
1	2,00	-52,77	-22,70	21,44	4,75
2	1,75	-12,62	-3,71	18,21	3,72
3	2,82	12,51	-3,39	3,11	0,36
4	2,09	17,23	3,33	3,35	-4,01
5	0,12	21,27	3,12	3,82	4,89

Panel 3. Return on investment ratio (%)

Status of the startup after ten years:

Period	Active	Active	Active	Bankruptcy	Bankruptcy
1	25,68	-91,61	-10,20	-32,26	76,51
2	24,23	-91,50	5,40	-85,96	35,68
3	24,78	82,10	13,03	33,59	9,23
4	27,10	65,48	9,61	-8,43	8,12
5	23,69	67,23	6,65	-0,72	7,13

This study attempts to fulfill the gap in startup research outlined above. Thus, the objective is to develop a mathematical growth model of a startup based on accounting concepts in order to analyze the time series of profitability ratios in the early stages of startup. This objective is important since the study of new

firm development suffers from an absence of conceptual models (Garnsey et al., 2006). The development of a startup is a very complicated process. Therefore, it is strongly simplified in this framework assuming that the expenditure of the startup is growing at a steady rate and generating a proportionally identical flow of revenue in each period. The profitability in terms of the internal rate of return (IRR) and the lag structure of revenue flows generated by periodic expenditure are assumed constant.

Furthermore, the startup is assumed to expense in each period a constant part of periodic expenditure and beginning-of-the-period assets. In spite of the steady assumptions, profitability ratios in the first years may suffer from nonstationary and follow a strong adjustment process towards a steady state. It is shown that IRR, growth rate, expense rate, and revenue lag structure strongly affect this adjustment process. Thus, it is argued that in the early years profitability ratios are unable to reflect profitability (IRR) properly and can give wrong signals of the performance of a startup. The interpretation of mathematical results is empirically supported by analyzing numerical adjustment processes for steady state estimates of a large sample of Finnish startups. In all, the parameters of the steady model are estimated for nine-year time series from financial statements of 2608 startups.

The content of the paper is organized as follows. Firstly, the introductory section presented the motivation, objective, and contribution of the study. Secondly, the analytical model of the time series of three profitability ratios (cash flow ratio, profit margin ratio, and return on investment ratio) is drawn up and analyzed in the second section in detail. The main point is to explain how the model variables endogenously affect the adjustment process of the profitability ratios towards a steady state. The theoretical interpretation of the results is supported by illustrative numerical examples. Thirdly, the mathematical results are demonstrated by empirical time series data from a sample of Finnish startups in the third section. The purpose is to show how different combinations of model variable estimates in practice can lead to different adjustment processes generating different signals of the profitability of a startup. Finally, the last section discusses and concludes the main findings of the study.

2. Theoretical analysis of profitability ratios

2.1. Growth and profitability

The early development of time series for a startup is a complicated and stochastic process (Garnsey et al., 2006). Therefore, Reid (2003) applied a dynamic theory to predict trajectories for key financial variables of a startup whereas Coad et al. (2013) used Gambler's Ruin framework by arguing that startup performance is best modelled as a random walk process. However, the present framework is based on a set of simplified assumptions about behavior of profitability ratios under certainty. These kinds of accounting-oriented frameworks have earlier been applied to analyzing the steady state association between the accounting rate of return (ARR, ROI) and the internal rate of return (IRR) (Feenstra & Fang, 2000; Laitinen, 2006; 2012; Brief, 2013). In these frameworks, growth plays an important role. For a startup, growth is critical for survival in the early stages, since new firms that do not grow are more likely to fail or close (Garnsey et al., 2006; Coad et al., 2013). In the present framework, it is assumed that the entrepreneur periodically invests on the startup an expenditure growing at a steady rate in order to strengthen the early development of the firm. Thus, the time series of periodic expenditure can be described as follows:

$$M_n = \sum_{i=0}^n M_0 (1+g)^i \quad (1)$$

where M_t refers to expenditure spent in period t and g is the steady rate of growth. For simplicity, random elements (Coad et al., 2013) are neglected in this framework.

It is important for the survival of the startup that the business starts to generate revenue as quickly as possible after foundation. In this way, the startup strengthens internal finance and ensures a successful entry to the market. Gilbert, McDougall & Audretsch (2006) review 48 empirical studies on new firm growth concluding that growth of sales revenue is one of the most important measures of growth. The present model

assumes that the business process of the startup is proportionally fixed and repetitive so that periodic expenditure generates a similar but steadily growing flow of revenue already beginning in the investment period. It is assumed that the lagged flow of revenue generated by periodic expenditure follows a geometric distribution, which leads to the following expression:

$$R_n = KM_0 \sum_{i=0}^n (1+g)^i q^{n-i} = KM_n q^n \sum_{i=0}^n (1+g)^i q^{-i} \quad (2)$$

$$= KM_n \left[\frac{(1+g)^{n+1} - q^{n+1}}{(1+g)^n (1+g-q)} \right]$$

where K is the level parameter of the lagged revenue distribution whereas q is the lag parameter describing the geometric lag structure.

Equation (2) shows that the time series of total revenue (generated by the past and present expenditure) is a nonstationary process where the growth path is largely determined by the difference between g and q . In the adjustment process, the growth rate of revenue is converging towards g faster, the lower is q . When n approaches the infinity, the growth rates of expenditure and revenue are equal and the startup lives in a steady state. When it is assumed that the lag distribution is infinite, IRR or r as the principal measure of startup profitability can be incorporated in the model in the following way:

$$M_n = M_n K \sum_{i=0}^{\infty} q^i (1+r)^{-i} \Rightarrow K = \frac{1+r-q}{1+r} \quad (3)$$

which leads to the following steady relation between periodic expenditure and revenue

$$R_n = M_n \left[\frac{(1+r-q)(1+g)}{(1+r)(1+g-q)} \right] \quad n \rightarrow \infty \quad (4)$$

Equation (4) indicates that in a steady state the revenue-expenditure ratio is symmetric with respect to g and r . It is also dependent on the lag parameter q . This lag parameter tells how quickly invested expenditure generates revenue to the startup. The revenue lag is increasing in q with the average lag defined as $q/(1-q) =: L$.

2.2. Expenses and assets

When the startup is founded, it must periodically prepare the income statement and the balance sheet according to the accounting conventions and doctrine. The going concern convention is based on continuity of activity assuming that the business will be operating indefinitely. The doctrine of consistency requires that financial statements for different accounting periods are based on the same accounting principles making financial results comparable. These general rules justify us to assume that certain accounting parameters are fixed over time. Therefore, it is assumed that the startup periodically expends a fixed proportion C of expenditure and non-expensed expenditure in the balance sheet. This systematic accounting procedure leads to the following time series of expense:

$$D_n = CM_0 \sum_{i=0}^n (1+g)^i (1-C)^{n-i} = C(1-C)^n M_0 \sum_{i=0}^n (1+g)^i (1-C)^{-i} \quad (5)$$

$$= CM_n \left[\frac{(1+g)^{n+1} - (1-C)^{n+1}}{(1+g)^n (g+C)} \right]$$

which converges towards the following steady state:

$$D_n = CM_n \left[\frac{1+g}{g+C} \right] n \rightarrow \infty \quad (6)$$

Equation (5) shows that the relation of periodic expense to periodic expenditure depends on g and C . This relation in the steady state (6) is decreasing in g whereas the marginal effect of C is positive for $g > 0$ and negative for $g < 0$. In (5) and (6), C is not specified. It is possible to consider different expense methods specifying C in different ways. For example, it can be assumed that the expiration of expenditure as expenses in time follows the lag structure of revenue so that $C = 1-q$. This kind of expense method is consistent with accounting standards (such as IASB Framework: paragraph 94). Appendix 1 presents the time series and the steady state of expense for $C = 1-q$.

The assets of the startup directly follow from the definition of the expense concept, since they are drawn up from the unexpired expenditure. This systematic procedure leads to the following expression for the time series of assets:

$$\begin{aligned} A_n &= (1-C)M_0 \sum_{i=0}^n (1+g)^i (1-C)^{n-i} = (1-C)^{n+1} M_0 \sum_{i=0}^n (1+g)^i (1-C)^{-i} \\ &= (1-C)M_n \left[\frac{(1+g)^{n+1} - (1-C)^{n+1}}{(1+g)^n (g+C)} \right] \end{aligned} \quad (7)$$

which gives the following steady state solution:

$$\begin{aligned} A_n &= (1-C)M_0 \sum_{i=0}^{\infty} (1+g)^i (1-C)^{n-i} \\ &= (1-C)M_n \left[\frac{1+g}{g+C} \right] n \rightarrow \infty \end{aligned} \quad (8)$$

The comparison of equation (7) with (5) shows that the relation between the time series of expense D_n and assets A_n is simply $C/(1-C)$. Appendix 2 shows the time series and steady state for A_n in a special case of expense method when $C = 1-q$.

2.3. Profitability ratios

2.3.1. Cash flow ratio (CFR)

The cash flow refers to the sufficiency of internal finance being therefore an important indicator of a startup facing financial constraints. Startups with strong cash flows continue to operate and grow, while startups with weak cash flows close and eventually die (Huanh et al., 2012). Usually it is suggested that highly cash-flow sensitive firms are those facing the least constraints (Almeida, Campello & Weisbach, 2004). Some researchers further argue that cash flows reveal information about investment quality (Alti, 2003). In this framework, the cash flow ratio (CFR) is defined with the aid of the difference between periodic revenue and expenditure as follows:

$$CFR_n = \frac{R_n - M_n}{R_n} = 1 - \frac{(1+r)}{(1+r-q)} \left[\frac{(1+g)^n (1+g-q)}{(1+g)^{n+1} - q^{n+1}} \right]$$

$$= 1 - \frac{(1+r)}{(1+r-q)} \left[\frac{1+g-q}{1+g-(q/(1+g))^n q} \right] \quad (9)$$

based on equations (2) and (3). Equation (9) shows that the level of CFR is determined by r , q , and g . However, the adjustment process towards a steady state is based on the difference between $1+g$ and q . The speed of adjustment is determined by $q/(1+g) =: S$. The initial level of CFR is reduced to $CFR_0 = 1-1/K$ that is independent of g but increasing in r and decreasing in q .

Equation (9) indicates that the speed of convergence towards the steady state is higher, the lower is q and the higher is g . The steady state of CFR can be presented in the following way:

$$CFR_n = \frac{R_n - M_n}{R_n} = \frac{q(r-g)}{(1+g)(1+r-q)} \quad n \rightarrow \infty \quad (10)$$

which shows that in the steady state CFR is positive if $r > g$ and zero if $r = g$. The level of steady CFR is decreasing in g but influenced positively by q for $r > g$ and negatively for $g > r$.

Table 2 presents numerical examples of CFR for different values of r , g , q , and C . The examples are classified in three groups according to the relationship between r and g ($r > g$, $r = g$, and $r < g$). In each group, one of the examples is drawn for the special case that $C = 1-q$. The height of C not arbitrary in practice but regulated by accounting regulations and conventions. The range between minimum C and maximum C can be called the expense management flexibility limits (Choy, 2012). However, CFR is based on the cash flow and is thus invariant to C . This is also indicated by the figures in Table 2 where cases that differ only with respect to C are identical. Table 2 also indicates that CFR is sensitive to g . When $g > r$, the adjustment process of CFR is in the first years similar than for $g < r$ but CFR converges towards a negative steady state. When $g = r$, the steady value is 0. The effect of q on CFR is strong: the higher q , the more negative is CFP in the first years of a startup, *ceteris paribus*.

2.3.2. Profit margin ratio (PMR)

Cash flow is not based on the profit concept and is therefore independent of the expense method used by a startup. However, for the survival of a startup, profit is of importance (Davidsson, Steffens & Fitzsimmons, 2009). Profitability is usually theoretically associated with IRR (Feenstra & Wang, 2000). IRR is also a widely used method in capital budgeting assessing investment projects (Graham & Harvey, 2001). However, in practice the profitability at the level of the firm is typically measured by financial ratios, mainly by the profit margin ratio and the return on investment ratio (Murphy et al., 1996; Losbichler et al., 2012). These measures are the main types of profitability ratios that can aid an in-depth understanding of the business of a startup. In this framework profit margin ratio (PMR) is calculated as the ratio of profit to revenue. Using equations (2) and (5), PMR can be presented in the following form:

$$PMR_n = \frac{R_n - D_n}{R_n} = 1 - \frac{C(1+r)}{(1+r-q)} \left[\frac{((1+g)^{n+1} - (1-C)^{n+1})(1+g-q)}{(g+C)((1+g)^{n+1} - q^{n+1})} \right] \quad (11)$$

$$= 1 - \frac{C(1+r)}{(1+r-q)} \left[\frac{((1 - ((1-C)/(1+g))^{n+1})(1+g-q))}{(g+C)((1 - (q/(1+g))^{n+1}))} \right]$$

Equation (11) shows that the initial level of PMR is largely determined by r , C , and q but that the adjustment process towards a steady state is complicated.

Table 2. Examples of time series of the cash flow ratio (CFR).

	IRR > Growth rate				IRR = Growth rate (Golden path)				IRR < Growth rate			
Case	1	2	3	4	5	6	7	8	9	10	11	12
r	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10
g	0,05	0,05	0,05	0,05	0,10	0,10	0,10	0,10	0,15	0,15	0,15	0,15
q	0,35	0,35	0,35	0,55	0,35	0,35	0,35	0,55	0,35	0,35	0,35	0,55
C	0,55	0,65	0,75	0,55	0,55	0,65	0,75	0,55	0,55	0,65	0,75	0,55
Period		$C = 1-q$				$C = 1-q$				$C = 1-q$		
0	-0,4667	-0,4667	-0,4667	-1,0000	-0,4667	-0,4667	-0,4667	-1,0000	-0,4667	-0,4667	-0,4667	-1,0000
1	-0,1000	-0,1000	-0,1000	-0,3125	-0,1126	-0,1126	-0,1126	-0,3333	-0,1244	-0,1244	-0,1244	-0,3529
2	-0,0154	-0,0154	-0,0154	-0,1122	-0,0333	-0,0333	-0,0333	-0,1429	-0,0499	-0,0499	-0,0499	-0,1717
3	0,0100	0,0100	0,0100	-0,0299	-0,0104	-0,0104	-0,0104	-0,0667	-0,0291	-0,0291	-0,0291	-0,1011
4	0,0182	0,0182	0,0182	0,0085	-0,0033	-0,0033	-0,0033	-0,0323	-0,0230	-0,0230	-0,0230	-0,0703
5	0,0209	0,0209	0,0209	0,0275	-0,0010	-0,0010	-0,0010	-0,0159	-0,0211	-0,0211	-0,0211	-0,0561
6	0,0218	0,0218	0,0218	0,0372	-0,0003	-0,0003	-0,0003	-0,0079	-0,0205	-0,0205	-0,0205	-0,0495
7	0,0221	0,0221	0,0221	0,0422	-0,0001	-0,0001	-0,0001	-0,0039	-0,0204	-0,0204	-0,0204	-0,0463
8	0,0222	0,0222	0,0222	0,0448	0,0000	0,0000	0,0000	-0,0020	-0,0203	-0,0203	-0,0203	-0,0448
9	0,0222	0,0222	0,0222	0,0461	0,0000	0,0000	0,0000	-0,0010	-0,0203	-0,0203	-0,0203	-0,0441
10	0,0222	0,0222	0,0222	0,0468	0,0000	0,0000	0,0000	-0,0005	-0,0203	-0,0203	-0,0203	-0,0438
Steady	0,0222	0,0222	0,0222	0,0476	0,0000	0,0000	0,0000	0,0000	-0,0203	-0,0203	-0,0203	-0,0435

The steady state of the profit margin ratio (11) is as follows:

$$PMR_n = \frac{R_n - D_n}{R_n} = 1 - \frac{C(1+r)(1+g-q)}{(1+r-q)(g+C)} \quad n \rightarrow \infty \quad (12)$$

Equation (11) shows that the convergence of the ratio towards the steady state (12) depends on the relation between g , C , and q . However, when the startup makes use of the expense method to set C equal to $1-q$, the ratio (11) can be presented in the following form:

$$PMR_n = \frac{R_n - D_n}{R_n} = 1 - \frac{(1+r)(1-q)}{(1+r-q)} = \frac{rq}{1+r-q} \quad C = 1-q \quad (13)$$

which shows that PMR is initially constant (steady state) without any adjustment process. Thus, the use of an expense method strongly affects the stationarity of the time series of the ratio. The level of the constant ratio (13) is significantly affected by q in addition to r .

Table 3 presents examples of PMR for the same parameter values as used in the previous table (Table 2). For the special case $C = 1-q$ PMR is constant irrespective of g . In spite of positive r , PMR is in years 0 and 1 negative when q or C is high ($C > 1-q$). For the cases where $C < 1-q$ the time series of PMR is of a similar form independently of the relation between r and g . PMR is in the early years highly positive but converges towards the steady state that is increasing in g . This steady state is increasing in q when $r > g$ and decreasing when $r < g$. Thus, the influence of both C and q on the time-series behavior (adjustment process) of PMR is strong.

2.3.3. Return on investment ratio (ROI)

Profit margin ratio is only based on profit and revenue flows without paying attention to the assets of the startup. These assets are taken into account by the return on investment ratio (ROI) that is likely the most widely adopted measure of profitability (Losbichler et al., 2012). It is also often considered a proxy of IRR in theoretical analyses (Feenstra & Wang, 2000). Using equations (2), (5), and (7) this profitability ratio can be presented as follows

$$\begin{aligned} ROI_n &= \frac{R_n - D_n}{A_{n-1}} = \frac{(1+r-q)(q^{n+1} - (1+g)^{n+1})(g+C)}{(1+r)(1+g-q)(1-C)((1-C)^n - (1+g)^n)} \\ &\quad - \frac{C((1-C)^{n+1} - (1+g)^{n+1})}{(1-C)((1-C)^n - (1+g)^n)} \\ &= \frac{R_n - D_n}{A_{n-1}} = \frac{(1+r-q)((q/(1+g))^{n+1} - 1)(g+C)(1+g)}{(1+r)(1+g-q)(1-C)((1-C)/(1+g))^n - 1} \\ &\quad - \frac{C(((1-C)/(1+g))^{n+1} - 1)(1+g)}{(1-C)((1-C)/(1+g))^n - 1} \end{aligned} \quad (14)$$

Equation (14) indicates that the adjustment process of ROI is a complicated function of the four model parameters. Its speed of convergence towards the steady state depends on the relationship between C , q , and g .

Table 3. Examples of time series of the profit margin ratio (PMR).

Case	IRR > Growth rate				IRR = Growth rate (Golden path)				IRR < Growth rate			
	1	2	3	4	5	6	7	8	9	10	11	12
r	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10
g	0,05	0,05	0,05	0,05	0,10	0,10	0,10	0,10	0,15	0,15	0,15	0,15
q	0,35	0,35	0,35	0,55	0,35	0,35	0,35	0,55	0,35	0,35	0,35	0,55
C	0,55	0,65	0,75	0,55	0,55	0,65	0,75	0,55	0,55	0,65	0,75	0,55
Period		$C = 1-q$				$C = 1-q$				$C = 1-q$		
0	0,1933	0,0467	-0,1000	-0,1000	0,1933	0,0467	-0,1000	-0,1000	0,1933	0,0467	-0,1000	-0,1000
1	0,1357	0,0467	-0,0214	-0,0313	0,1377	0,0467	-0,0241	-0,0333	0,1396	0,0467	-0,0267	-0,0353
2	0,0996	0,0467	0,0140	0,0137	0,1041	0,0467	0,0089	0,0091	0,1082	0,0467	0,0042	0,0048
3	0,0793	0,0467	0,0286	0,0421	0,0859	0,0467	0,0220	0,0350	0,0919	0,0467	0,0160	0,0284
4	0,0687	0,0467	0,0343	0,0595	0,0769	0,0467	0,0268	0,0502	0,0842	0,0467	0,0201	0,0418
5	0,0634	0,0467	0,0364	0,0698	0,0726	0,0467	0,0285	0,0589	0,0807	0,0467	0,0215	0,0491
6	0,0610	0,0467	0,0371	0,0758	0,0707	0,0467	0,0291	0,0637	0,0792	0,0467	0,0220	0,0530
7	0,0598	0,0467	0,0374	0,0792	0,0699	0,0467	0,0293	0,0663	0,0785	0,0467	0,0222	0,0551
8	0,0593	0,0467	0,0375	0,0811	0,0695	0,0467	0,0294	0,0677	0,0783	0,0467	0,0222	0,0561
9	0,0591	0,0467	0,0375	0,0821	0,0693	0,0467	0,0294	0,0684	0,0782	0,0467	0,0222	0,0566
10	0,0590	0,0467	0,0375	0,0827	0,0693	0,0467	0,0294	0,0688	0,0781	0,0467	0,0222	0,0569
Steady	0,0589	0,0467	0,0375	0,0833	0,0692	0,0467	0,0294	0,0692	0,0781	0,0467	0,0222	0,0571

If the startup is using an expense method to make C equal to $1-q$, equation (14) can be simplified as follows:

$$ROI = \frac{R_n - D_n}{A_{n-1}} = \frac{(1+r-q)((q/(1+g))^{n+1} - 1)(1+g)}{(1+r)q(((q/(1+g))^n - 1))} \quad C = 1 - q \quad (15)$$

$$- \frac{(1-q)((q/(1+g))^{n+1} - 1)(1+g)}{q((q/(1+g))^n - 1)}$$

where the adjustment process of ROI towards its steady value is dependent on the relation between q and g . The complexity of equation (15) largely depends on the fact that periodic profit is in ROI divided by A_{n-1} (assets defined on the beginning-of-the-period basis). If ROI is calculated for A_n , a constant ratio is resulted as $ROI = r/(1+r)$ giving also a reasonable but slightly biased proxy of IRR. In practice, assets are often defined as average of the beginning- and end-of-the-period bases, which leads to a time-series of ROI with diminished non-stationarity in comparison to (15).

The steady state of ROI is the following:

$$ROI_n = \frac{R_n - D_n}{A_{n-1}} = \frac{(1+r-q)(1+g)(g+C)}{(1+r)(1+g-q)(1-C)} - \frac{C(1+g)}{1-C} \quad n \rightarrow \infty \quad (16)$$

which is strongly affected by r but also by g , q and C . If the startup systematically uses the expense method making C equal to $1-q$, the steady state ratio can be presented in the following form:

$$ROI_n = \frac{R_n - D_n}{A_{n-1}} = \frac{r(1+g)}{(1+r)} \quad n \rightarrow \infty \ \& \ C = 1 - q \quad (17)$$

which is increasing in r and g . Equations (16) and (17) indicate that ROI in the steady state correctly reflects IRR only when $r = g$ (golden path) which is a well-known result in theoretical analyses of IRR (Feenstra & Wang, 2000).

Table 4 presents exemplary figures of ROI for the previous sets of the model parameters. If $C < 1-q$, the values of ROI are in the first years very high clearly overestimating r . If $C = 1-q$, this overestimation is not strong and the time series converges quickly towards to the steady state. When $g = r$, this steady state ROI equals r according to the golden path rule. When q or C is high ($C > 1-q$), ROI is in the first year negative and increases after that towards the steady state. Thus, in the first years of a startup, ROI does not give a reliable signal of the true level of IRR due to the adjustment process.

Table 4. Examples of time series of the return on investment ratio (ROI).

Case	IRR > Growth rate				IRR = Growth rate (Golden path)				IRR < Growth rate			
	1	2	3	4	5	6	7	8	9	10	11	12
r	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10
g	0,05	0,05	0,05	0,05	0,10	0,10	0,10	0,10	0,15	0,15	0,15	0,15
q	0,35	0,35	0,35	0,55	0,35	0,35	0,35	0,55	0,35	0,35	0,35	0,55
C	0,55	0,65	0,75	0,55	0,55	0,65	0,75	0,55	0,55	0,65	0,75	0,55
Period		$C = 1-q$				$C = 1-q$				$C = 1-q$		
0												
1	0,2879	0,1273	-0,0818	-0,0556	0,3025	0,1318	-0,0955	-0,0611	0,3172	0,1364	-0,1091	-0,0667
2	0,1603	0,1034	0,0467	0,0202	0,1748	0,1077	0,0308	0,0138	0,1893	0,1120	0,0151	0,0075
3	0,1159	0,0979	0,0937	0,0592	0,1319	0,1023	0,0748	0,0509	0,1478	0,1067	0,0564	0,0427
4	0,0965	0,0963	0,1120	0,0828	0,1139	0,1007	0,0912	0,0723	0,1311	0,1052	0,0710	0,0622
5	0,0877	0,0957	0,1189	0,0971	0,1060	0,1002	0,0970	0,0847	0,1240	0,1047	0,0760	0,0731
6	0,0836	0,0955	0,1214	0,1056	0,1026	0,1001	0,0990	0,0917	0,1211	0,1046	0,0776	0,0789
7	0,0818	0,0955	0,1223	0,1105	0,1011	0,1000	0,0997	0,0956	0,1199	0,1046	0,0782	0,0820
8	0,0810	0,0955	0,1226	0,1133	0,1005	0,1000	0,0999	0,0977	0,1194	0,1046	0,0783	0,0836
9	0,0806	0,0955	0,1227	0,1148	0,1002	0,1000	0,1000	0,0988	0,1192	0,1045	0,0784	0,0844
10	0,0804	0,0955	0,1227	0,1157	0,1001	0,1000	0,1000	0,0994	0,1191	0,1045	0,0784	0,0848
Steady	0,0803	0,0955	0,1227	0,1167	0,1000	0,1000	0,1000	0,1000	0,1191	0,1045	0,0784	0,0852

3. Empirical analysis of adjustment processes

3.1. Sample of firms

The objective of empirical experiments is to illustrate the adjustment processes of early profitability ratios using parameter values estimated for a large sample of real Finnish (SME) startups. The data for the experiments are extracted from the ORBIS database of Bureau Van Dijk (BvD). Estimation of the steady parameters for a firm requires a long time-series of financial statements. The selection of the sample was therefore made under restriction that the selected startup must have successive financial statements available for at least 10 years. Because the parameters are intended to describe the adjustment process of ratios in early years, it was required that the startup has published financial statements beginning from 1-4 years after its foundation (registration) year. It was also required that the selected startup is founded after year 2000, is industrial, private limited company, and has less than 50 employees in its first data year. Therefore, this analysis excludes startups, which have failed during the first ten years of their active life. The sample also excludes large startups.

The search under these restrictions gave us financial statements from 4000 active Finnish firms, 2 firms in insolvency proceedings, 29 bankrupt firms, 160 dissolved firms, and 1 firm involved with merger or take-over, and 15 firms being in liquidation. For the present analyses, 4000 active and 29 bankrupt startups were selected due to their clear status. The financial status of the large group (160) of dissolved firms is not determined and therefore these firms were excluded from the sample. In Finland, limited liability companies must submit their financial statements to the Trade Register either with their tax return or directly to the Finnish Patent and Registration Office (PRH). Practically, ORBIS includes all Finnish firms, which have delivered their financial statements to PRH (The Finnish Trade Register) as required by Finnish Accounting Act. Therefore, the sample is considered statistically representative for all Finnish small startups (limited companies) surviving at least 10 years after foundation.

3.2. Estimation of parameters

For all startups in the sample (4029), time-series data of financial statements for 10 years were extracted from ORBIS. The estimation of the parameters of the distributed lag model is only based on the time series of total revenue R_t and total expenditure M_t . R_t is here measured as net sales and M_t as the sum of short-term (current) and long-term (fixed) expenditure. M_t is calculated from financial statements as the sum of total expenses (current expenses and depreciations) D_t and change in inventories and in fixed assets $A_t - A_{t-1}$. Therefore, finally we have nine-year time series of R_t and M_t available for statistical estimation. The estimation of the parameters g , q , and r was made in several stages. Firstly, the steady rate of growth g was estimated from the nine time-series observations applying the ordinary least squares (OLS) method to the logarithmic time series of both M_t and R_t as follows:

$$\begin{aligned} M_t &= M_0(1+g)^t \cdot e^\varepsilon \Rightarrow \ln M_t = \ln M_0 + t \cdot \ln(1+g) + \varepsilon \\ R_t &= R_0(1+g)^t \cdot e^\varepsilon \Rightarrow \ln R_t = \ln R_0 + t \cdot \ln(1+g) + \varepsilon \end{aligned} \tag{18}$$

where ε is a random residual. The final estimate of firm-level steady growth rate was calculated as the weighted average of the estimates of (18) using the sum of time-series of M_t and R_t over the nine observations as weights.

Secondly, q and r were estimated for the startups using a distributed lag model specification. However, a reliable estimation of these parameters is an exceptionally challenging task due to the sensitivity of estimates (Laitinen, 1997; compare with Hall, 2007) and to the time-series properties of startups (Garnsey et al., 2006). They were estimated using the Koyck transformation applied to the distributed revenue lag function. Furthermore, linear restrictions were incorporated in the estimation to make the estimates more stable

(Johnston, 1972: 155-159; Fomby, Hill & Johnson, 1984: 82-85). The Koyck transformation shows that the relation between R_t and M_t can be presented in the form of the following equation:

$$R_t = a + K \cdot M_t + q \cdot R_{t-1} + \varepsilon \quad (19)$$

where a is to be set equal to 0, $K = (1+r-q)/(1+r)$, and ε is a random residual.

The central linear restriction is based on the assumed steady relation (4) between revenue R and expenditure M . Because $R/M = K(1+g)/(1+g-q)$, the following steady relation holds:

$$(1+g) \cdot K + (R/M) \cdot q = (R/M) \cdot (1+g) \quad (20)$$

which leads to the following matrices of restrictions:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1+g & R/M \end{bmatrix} \quad (21)$$

$$\mathbf{h} = \begin{bmatrix} 0 \\ (R/M) \cdot (1+g) \end{bmatrix}$$

These restrictions can be presented in the matrix form as $\mathbf{h} = \mathbf{H} \cdot \mathbf{B}$ where \mathbf{B} is the 3 x 1 matrix of estimates. In these restrictions, the estimate of g and R/M were used to stabilize the estimates of q and r . R/M was estimated as the ratio of the cumulated sum of revenue and the cumulated sum of expenditure over the period of the nine time-series observations. Because of the sensitivity of estimates for startups, the procedure provided us with a number outliers (in the form of $q < 0$ or $q > 1$), which were excluded from the further analyses. Thus, the final sample only included 2599 (66.0%) active firms and 23 (79.3%) bankrupt firms.

Finally, the (constant) rate of expense C was estimated (as the weighted average) for the last nine years of startups as the ratio of cumulated expenses to the cumulated sum of expenditure M_t and assets $A_{t,1}$ in the beginning of the year. The three estimation stages gave us a set of estimates (for g , q , r , and C) based on a steady state assumption that rarely completely holds even for larger firms during their mature stages of life cycle. Therefore, the estimation results should be cautiously interpreted and mainly regarded as acting as demonstrative purposes only. It is expected that the reliability of the estimates is not very high for single startups. Thus, only statistical averages of estimates obtained for large sub-groups (types) of startups are analyzed in experiments to show the variations in adjustment processes of profitability ratios.

3.3. Cluster analysis

The purpose of further analyses is to find out representative types of startups and to demonstrate adjustment processes in these basic types. The search for the types was made by the TwoStep Cluster Analysis of SPSS. This analysis is an exploratory tool designed to reveal natural groupings (or clusters) within a dataset that would otherwise not be apparent. The algorithm employed by this procedure has several desirable features that differentiate it from traditional clustering techniques. It allows us to analyze large data files and it can automatically determine the optimal number of clusters. Here, the Bayesian Information Criterion (BIC) was specified as the criterion to determine the number of clusters. Bachler, Wenzig & Vogler (2004) regard the method as promising because it may solve some of the problems with traditional methods such as k -means and hierarchical clustering methods. They use simulation to show that the TwoStep Cluster Analysis performs well if the variables are continuous. In this analysis, the four continuous parameter estimates of the steady model (r , g , q , and C) were used as clustering variables.

The TwoStep Cluster Analysis method applied to the sample of active firms ($n = 2599$) provided us with five clusters. For technical reasons, the procedure dropped 14 startups (0.5%) from the final solution. The cluster quality (silhouette measure of cohesion and separation) was found fair (0.4 when 0.5-1.0 refer to good value). Table 5 presents the distribution of the startups between the five clusters. Each cluster includes a large number of firms. Clusters 3 ($n = 804$) and 1 ($n = 679$) are the largest sub-groups of startups making together 57.1% of the total sample. Table 6 presents statistics of total revenue (R) and expenditure (M) for the first and ninth data years of startups by cluster. For the first data year, F-statistics showed (not presented here) that the differences in size measured either by R (p -value 0.279) or M (0.522) are not statistically significant although startups in cluster 1 are relatively small. However, bankrupt startups are on average smaller than active startups. Until ninth data year the average size of startups in cluster 1 has increased very strongly whereas that of startups in cluster 4 has seriously diminished. In the ninth data year, all the differences in size between clusters are significant (p -value 0.0000).

Table 5. Distribution of active startups by cluster.

Cluster	Active startups	Percent	Valid percent
1	679	26,13	26,27
2	396	15,24	15,32
3	804	30,93	31,10
4	434	16,70	16,79
5	272	10,47	10,52
Total	2585	99,46	100,00
Missing	14	0,54	
Total	2599	100,00	

Table 6. Mean of total revenue and expenditure (TEUR) in active and bankrupt startups by cluster.

	Mean of revenue (TEUR):		Mean of expenditure (TEUR):	
	First data year	Ninth data year	First data year	Ninth data year
Active startups:				
Cluster				
1	768,5	3260,7	839,7	3043,7
2	1016,1	1305,1	1268,3	1232,3
3	1340,8	1989,4	1346,4	1886,6
4	1151,5	606,9	1094,0	561,0
5	1126,3	1945,5	1147,0	1983,1
Total	1086,4	1981,8	1138,0	1877,9
Bankrupt startups:				
All	698,0	658,8	741,6	711,0

Appendix 3 presents the industrial classification of startups by cluster. For the active firms, the contingency coefficient (not presented here) indicated that the distributions (industry & clusters) are highly statistically dependent ($p = 0.000$). The largest industry groups for active startups are wholesale and retail trade, repair of motor vehicles and motor cycles (20.1%), construction (16.7%), and professional, scientific and technical activities (14.0%). Cluster 2 has exceptionally large number of startups from human health and social work activities (14.1%) but includes only few construction startups (10.6%). Cluster 3 includes a large number

of transportation and storage startups (12.2%) while the number of professional, scientific, and technical startups is relatively small (9.0%). Bankrupt startups include a lot of wholesale and retail trade (30.4%), manufacturing (17.4%), and accommodation and food service (13.0%) startups.

3.4. Estimation results by cluster

Table 7 presents descriptive statistics of the estimation results by cluster. Panel A of the table shows the mean values of the final estimates for the four parameters of the steady model. These average values represent the centroids of the clusters and they are used to demonstrate different types of startups. F-statistics (not presented here) showed that the differences in the estimates between the clusters are all statistically significant (p -value 0.0000). The estimates of q are relatively high in clusters 1 (0.6146) and 2 (0.5866) but exceptionally low in cluster 5 (0.0831). The average estimate of r is very high in cluster 4 (0.4063) but extremely low in cluster 5 (-0.5165). The estimates of g show that startups in cluster 1 (0.1795) have grown very fast whereas startups in cluster 4 (-0.0847) report very negative rates of growth. In general, the differences in the average estimate of C are relatively small. However, startups in cluster 2 (0.4356) have a very low rate of expense. Bankrupt startups mainly differ from average active startups in that r is slightly negative (-0.0093) and C is quite low (0.7421). Except for low C , their profile is close to the centroid of startups in cluster 3.

Panel B of Table 7 presents statistics for the steady growth of startups. The means of golden path measure $(1+r)/(1+g)$ reflecting R/M show large variations between clusters. Cluster 5 (0.4387) that on average has a very low r has also an extremely low ratio while cluster 4 (1.5570) with a very high r reports an exceptionally high values for the ratio. The panel also shows that the coefficient of determination of equation (19) used to estimate r and q is generally high that is typical for distributed lag models. Thus, only for cluster 2 (0.5900) it is quite low. However, the coefficients for determination for logarithmic equations in (18) to estimate g for R and M are clearly lower. For cluster 1 (0.7360 & 0.6644) only these coefficients are relatively high. In general, the differences in growth estimate between R and M are quite large referring to violations against the steady state assumption. The special characteristic of bankrupt startups is that the growth estimate for R exceeds that for M notably by 10 per cent units.

3.5. Adjustment processes by cluster

Panel A of Table 7 shows that Cluster 1 (26.3% of startups) is characterized by a high growth rate (0.1795) and a high revenue lag parameter (0.6146). The startups in this cluster tend to use a high rate of expense (0.8170) although revenue lag is high being on average 1.6 periods (see Table 8). Panel B shows that in this cluster startups nicely follow the steady growth paths with high coefficients of determination. This coefficient is also relatively high for the revenue generation model. Thus, this cluster can be called “Rapidly growing steady startups with high revenue lag and high rate of expense”. Table 8 shows that in this cluster the inverse measure of adjustment speed (S) for CFR indicates slow adjustment. However, the differences between steady values of profitability ratios and their calculated values in early years are extremely large referring to a strong adjustment process. Thus, the time series of profitability ratios converge towards steady values very slowly (see Table 9 and Figure 1). These ratios refer to a very low negative profitability although average IRR is moderate (0.0792). The steady value of ROI (-0.3713) strongly underestimates profitability mainly due to the high rate of expense.

Table 7. Mean values of the estimated steady model parameters by cluster.

Cluster:							
Estimate	1	2	3	4	5	All	Bankrupt startups
Panel A. Parameters of the model							
q	0,6146	0,5866	0,2754	0,2411	0,0831	0,3862	0,3468
r	0,0792	0,0972	-0,0162	0,4063	-0,5165	0,0445	-0,0093
g	0,1795	-0,0037	0,0540	-0,0847	0,1112	0,0609	0,0573
C	0,8170	0,4356	0,8188	0,8232	0,8599	0,7647	0,7421
Nr. of firms	679	396	804	434	272	2585	23
Panel B. Measures of steady growth							
$(1+r)/(1+g)$	0,9284	1,1159	0,9398	1,5570	0,4387	1,0147	0,9775
R/M	0,9560	1,1245	0,9822	1,1466	0,8969	1,0157	0,9811
$R^2(q, r)$	0,8191	0,5900	0,7947	0,8298	0,8868	0,7853	0,8066
$g(R)$	0,2041	0,0179	0,0701	-0,0691	0,1345	0,0807	0,1091
$R^2(g(R))$	0,7360	0,4695	0,4579	0,4511	0,4152	0,5271	0,5249
$g(M)$	0,1556	-0,0300	0,0369	-0,1048	0,0859	0,0392	0,0055
$R^2(g(M))$	0,6644	0,3113	0,4138	0,4725	0,3959	0,4719	0,4941

Note:

$R^2(\cdot)$ = coefficient of determination of the equation used to estimate (\cdot)

$g(\cdot)$ = growth of (\cdot)

R = total revenue

M = total expenditure

R/M = estimate for the revenue-expenditure ratio

Table 8. Statistics of adjustment processes by cluster.

Cluster:						
	1	2	3	4	5	Bankrupt
L	1,5950	1,4190	0,3800	0,3178	0,0906	0,5309
S	0,5211	0,5888	0,2612	0,2635	0,0748	0,3280
r	0,0792	0,0972	-0,0162	0,4063	-0,5165	-0,0093
ROI	-0,3713	0,0970	-0,0676	0,4370	-0,6158	-0,0444
PMR	-0,0759	0,1121	-0,0144	0,0930	-0,0993	-0,0148
CFR	-0,1125	0,1164	-0,0259	0,1110	-0,1172	-0,0339
ROI-ROI ₁	1,4913	-0,0035	0,2310	0,1025	0,0045	0,2014
PMR-PMR ₀	0,8221	0,0481	0,1227	0,0866	-0,0610	0,1269
CFR-CFR ₀	1,2106	1,2653	0,3628	0,3180	0,0903	0,5046

Notes:

$L = q/(1-q)$ = average lag between expenditure and revenue

$S = q/(1+g)$ = inverse measure of adjustment speed for CFR

ROI = steady return on investment ratio

ROI₁ = return on investment ratio in period 1

PMR = steady profit margin ratio

PMR₀ = profit margin ratio in period 0

CFR = steady cash flow ratio

CFR₀ = cash flow ratio in period 0

Table 9. Theoretical time series of profitability ratios for the five clusters and bankrupt startups.

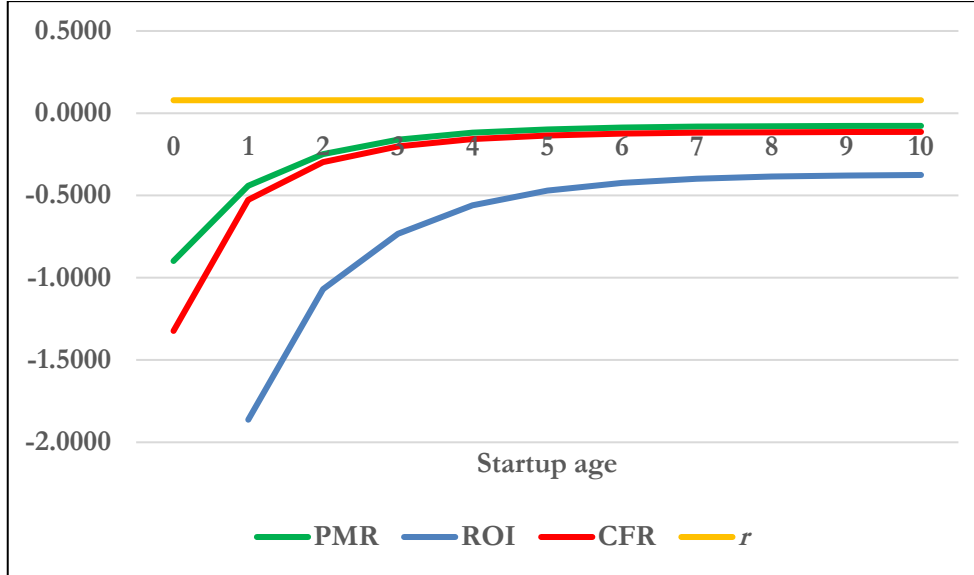
Panel 1. Clusters 1-3

Period	Cluster 1 ($r = 0,0792$)			Cluster 2 ($r = 0,0972$)			Cluster 3 ($r = -0,0162$)		
	PMR	ROI	CFR	PMR	ROI	CFR	PMR	ROI	CFR
0	-0,8980		-1,3231	0,0639		-1,1489	-0,1371		-0,3887
1	-0,4414	-1,8627	-0,5273	0,0771	0,1006	-0,3525	-0,0565	-0,2987	-0,1011
2	-0,2485	-1,0700	-0,2959	0,0872	0,0885	-0,1103	-0,0276	-0,1310	-0,0445
3	-0,1608	-0,7319	-0,2011	0,0947	0,0882	-0,0043	-0,0183	-0,0858	-0,0307
4	-0,1188	-0,5593	-0,1570	0,1002	0,0899	0,0491	-0,0155	-0,0727	-0,0272
5	-0,0978	-0,4693	-0,1353	0,1041	0,0917	0,0780	-0,0147	-0,0690	-0,0262
6	-0,0872	-0,4224	-0,1243	0,1068	0,0933	0,0942	-0,0145	-0,0680	-0,0260
7	-0,0818	-0,3980	-0,1186	0,1086	0,0945	0,1034	-0,0144	-0,0677	-0,0259
8	-0,0789	-0,3852	-0,1157	0,1098	0,0953	0,1088	-0,0144	-0,0677	-0,0259
9	-0,0775	-0,3786	-0,1142	0,1106	0,0959	0,1119	-0,0144	-0,0676	-0,0259
10	-0,0767	-0,3751	-0,1134	0,1112	0,0963	0,1138	-0,0144	-0,0676	-0,0259
Steady state	-0,0759	-0,3713	-0,1125	0,1121	0,0970	0,1164	-0,0144	-0,0676	-0,0259

Panel 2. Clusters 4-6 and bankrupt startups

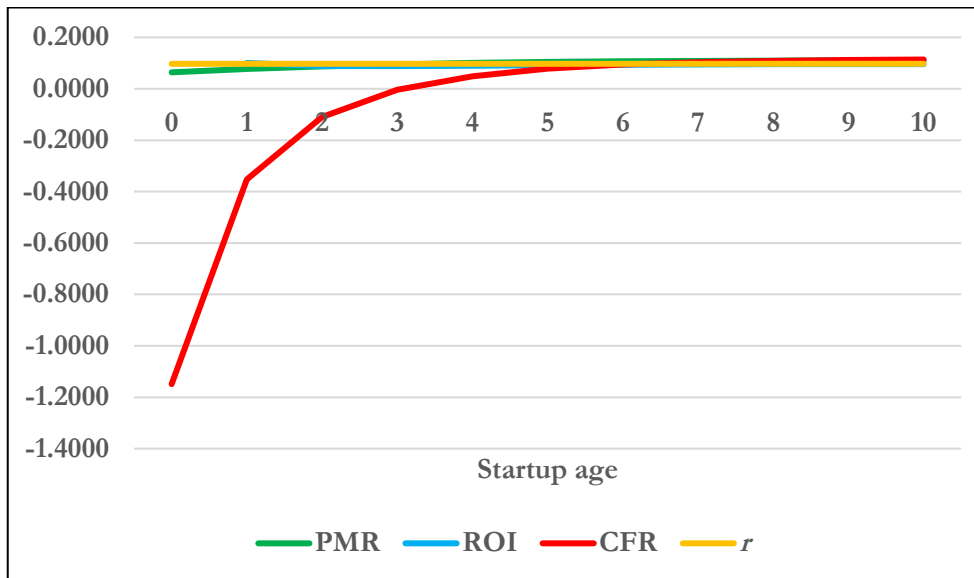
Period	Cluster 4 ($r = 0,4063$)			Cluster 5 ($r = -0,5165$)			Bankrupt startups ($r = -0,0093$)		
	PMR	ROI	CFR	PMR	ROI	CFR	PMR	ROI	CFR
0	0,0064		-0,2070	-0,0383		-0,2074	-0,1418		-0,5385
1	0,0617	0,3344	0,0447	-0,0879	-0,6203	-0,1235	-0,0695	-0,2459	-0,1586
2	0,0828	0,3966	0,0945	-0,0975	-0,6145	-0,1176	-0,0367	-0,1128	-0,0718
3	0,0899	0,4233	0,1067	-0,0990	-0,6155	-0,1172	-0,0231	-0,0694	-0,0461
4	0,0921	0,4328	0,1099	-0,0992	-0,6158	-0,1172	-0,0178	-0,0534	-0,0379
5	0,0927	0,4357	0,1107	-0,0992	-0,6158	-0,1172	-0,0159	-0,0476	-0,0352
6	0,0929	0,4366	0,1110	-0,0993	-0,6158	-0,1172	-0,0152	-0,0455	-0,0344
7	0,0930	0,4369	0,1110	-0,0993	-0,6158	-0,1172	-0,0149	-0,0448	-0,0341
8	0,0930	0,4369	0,1110	-0,0993	-0,6158	-0,1172	-0,0149	-0,0446	-0,0340
9	0,0930	0,4370	0,1110	-0,0993	-0,6158	-0,1172	-0,0148	-0,0445	-0,0340
10	0,0930	0,4370	0,1110	-0,0993	-0,6158	-0,1172	-0,0148	-0,0445	-0,0340
Steady state	0,0930	0,4370	0,1110	-0,0993	-0,6158	-0,1172	-0,0148	-0,0444	-0,0339

Figure 1. Development of profitability ratios in cluster 1 of active startups.



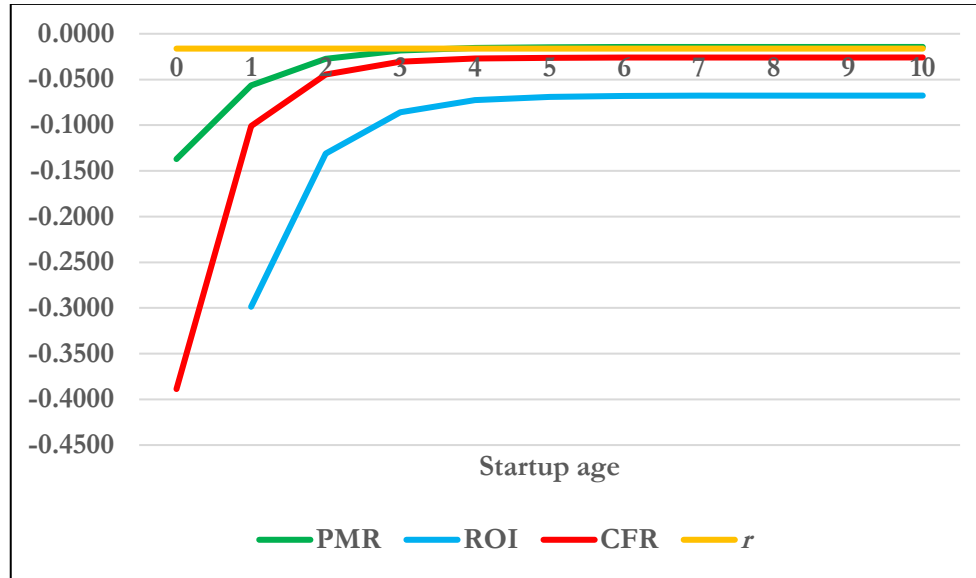
Cluster 2 (15.3% of startups) is also characterized by a high revenue lag parameter (0.5866) with a high average lag of 1.4 periods. However, its rate of expense is very low (0.4356) consistently with the high lag parameter. The coefficient of determination for the distributed lag model is low indicating a weak dependence between expenditure and revenue. In addition, the average estimates for growth rates in R and M are of different sign reflecting a non-steady state. Because the average rate of growth in this cluster is close to zero, this cluster can be entitled as “Zero-growth non-steady startups with high revenue lag and low rate of expense”. Table 8 shows that in this cluster the adjustment process (\mathcal{J}) is very slow. However, the differences between the steady values and values in early years for ROI and PMR are very small. Thus, the adjustment process for these ratios is very weak (see Table 9 and Figure 2). However, the steady value of CFR is far from its early values, which leads to a strong and long adjustment process. In this cluster, ROI gives a satisfactory estimate of r (0.0972) already in the early years.

Figure 2. Development of profitability ratios in cluster 2 of active startups.



Cluster 3 (31.1%) is the largest sub-group of active firms. It is characterized by a negative internal rate of return (-0.0162) and a low revenue lag parameter (0.2754) giving an average lag of 0.38 periods. Therefore, the cluster can be called as “Unprofitable startups with a low revenue lag”. The rate of growth (0.0540) in these startups is average. Table 8 shows that the adjustment process (S) due to the low revenue lag is quick. However, the differences between the steady values of profitability ratios and their early values are large. Therefore, the adjustment processes are strong although being quick (Table 9 and Figure 3). The three profitability ratios refer in the early years to a very unprofitable startup due to the adjustment process. In the steady state, ROI (-0.0676) still underestimates IRR.

Figure 3. Development of profitability ratios in cluster 3 of active startups.



Cluster 4 (16.8%) is characterized by a very high internal rate of return (0.4063), a negative growth rate (-0.0847) and a low revenue lag parameter (0.2411) with an average lag of 0.32 periods. Thus, this cluster can be entitled as “Very profitable shrinking startups with a low revenue lag”. Table 8 shows that the adjustment process (S) is as quick as in cluster 3. The differences between the steady state values and the early values of ROI and PRM are small leading to weak adjustment processes. However, for CFR the adjustment process is strong (Table 9 and Figure 4). In already early years, the three profitability ratios correctly refer to a profitable startup. The steady rate of ROI (0.4370) gives a good proxy of r .

Cluster 5 (10.5%) is the smallest sub-group of active startups. This cluster is characterized by a very low internal rate of return (-0.5165), a high growth rate (0.1112) and a very low revenue lag parameter (0.0831) leading to an average lag of 0.1 periods (one month). Consistently with the low revenue lag parameter, the rate of expense is high (0.8599). For this cluster, the difference between g and r is very large leading to a low steady revenue-expenditure ratio R/M (0.8969). Thus, this cluster can be called as “Very unprofitable rapidly growing startups with a low revenue lag”. Table 8 shows that the speed of adjustment process (S) is extremely quick. Because the steady values of the profitability ratios are quite close to their values in early years, the adjustment processes are very weak (Table 9 and Figure 5). These ratios correctly give an insight of very unprofitable startup already in the early years. Especially, the steady ROI (-0.6158) underestimates internal rate of return but anyway gives a reasonable proxy.

Figure 4. Development of profitability ratios in cluster 4 of active startups.

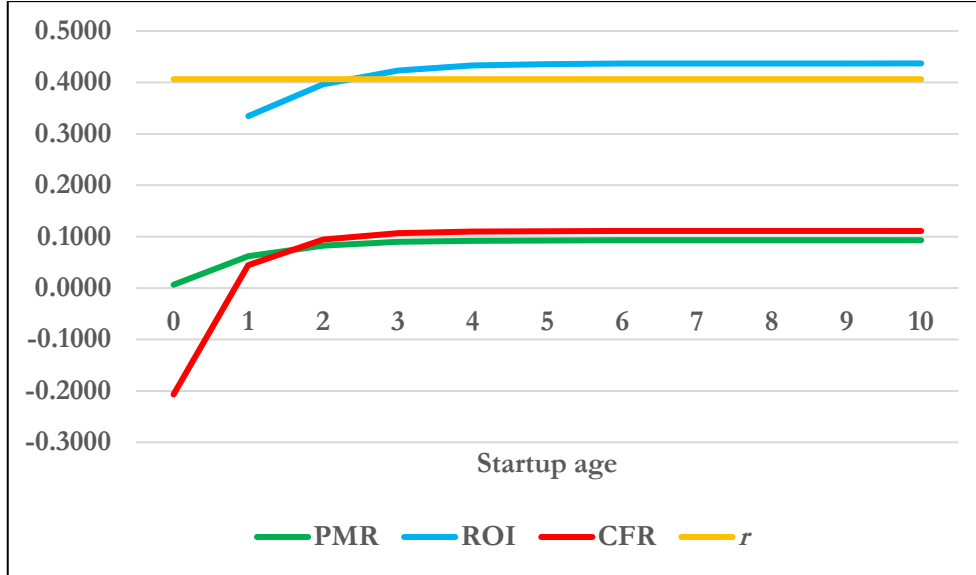
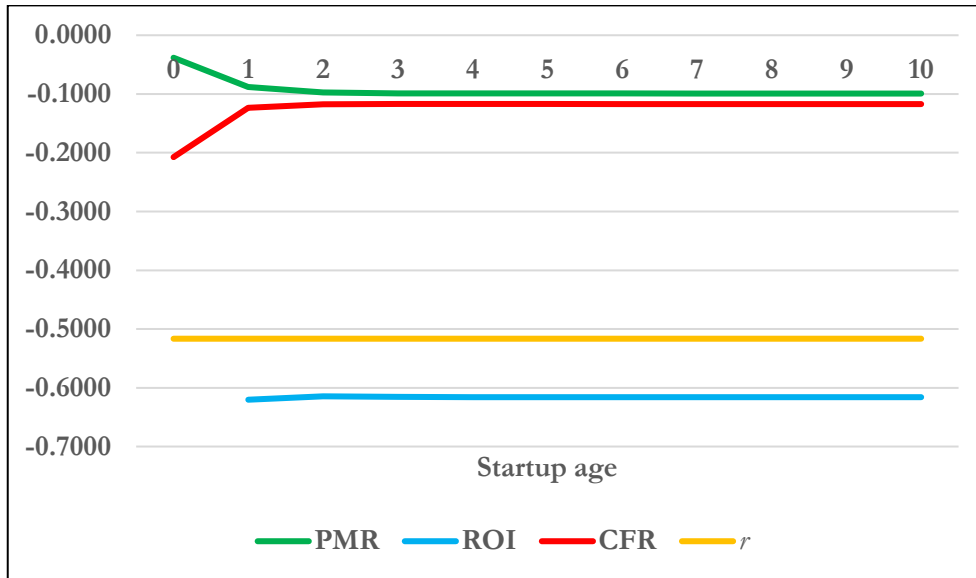
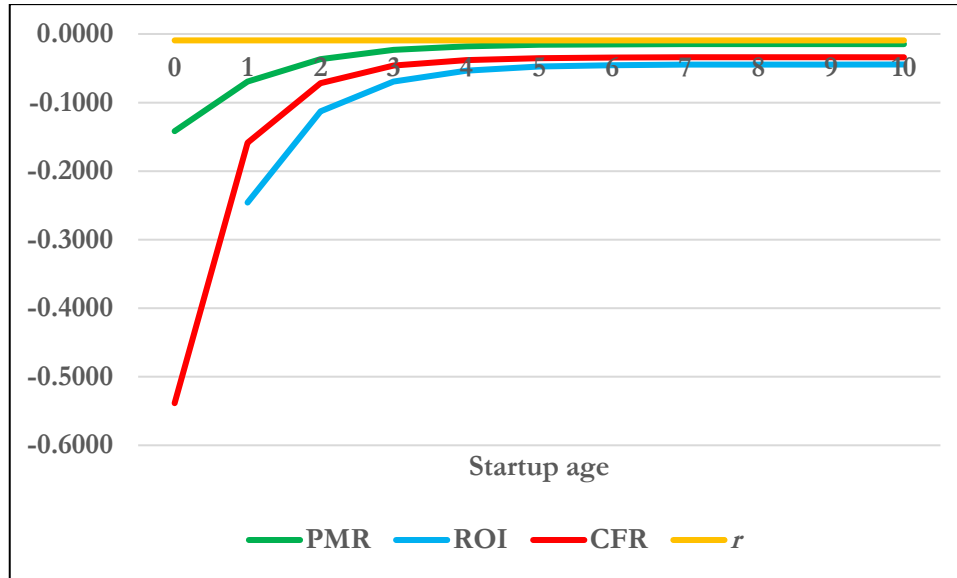


Figure 5. Development of profitability ratios in cluster 5 of active startups.



The bankrupt startups (23 firms) are characterized by average parameter values except for the internal rate of return (-0.0093) that is negative but very close to zero. For these startups, the estimate for the growth rate of R is high (0.1091) whereas that of M is close to zero (0.0055). The large difference in the growth estimates refers to a non-steady growth characterized by a structural change. Therefore, this sub-group of startups can be entitled as “Unprofitable non-steady bankrupt startups with a structural change”. Table 8 shows that the speed of adjustment processes (\mathcal{J}) is quite average. However, the differences between the steady values and the early values of the profitability ratios are considerable. Therefore, the adjustment processes are moderately strong referring to a very unprofitable startup in the early years (Table 9 and Figure 6). In the steady state, ROI (-0.0444) to some degree underestimates IRR giving an insight of an unprofitable startup.

Figure 6. Development of profitability ratios in bankrupt startups.



4. Concluding discussion

4.1. Research approach

The stationarity of the time series of financial ratios has been extensively studied with mixed results (McLeay & Stevenson, 2009). Some studies show that the components of financial ratios may exhibit non-stationarity, which is not eliminated by the ratio transformation (Whittington & Tippett, 1999). However, evidence has also showed that financial ratios may be globally stationary, but that behaviour close to equilibrium may result from a nonlinear partial adjustment process where the rate of adjustment towards the optimal value increases with deviation from the target (Ioannides et al., 2003). These studies are generally concentrated on long time-series of older and larger firms ignoring startups although the growth and contributions that startups make to the economy are important themes in research and business literature (Davila et al., 2015). The time series of startups and older firms may behave in a very different ways. The ratios of an older firm in steady state generate predictable time-series and investors can generally agree upon valuation (Ak et al., 2013). However, the ratios of startups may suffer from strong influence of adjustment process (towards a potential steady state) making a reliable ratio analysis difficult. The growth in these firms is often non-linear and prone to interruptions and setbacks overlooked in the literature (Garnsey, 2006).

Because of an obvious gap in startup research this study attempts to explain technically, the early time series behavior of startup profitability ratios. This behavior is not explained here by time-series of the components of ratios as in previous time series studies. It is explained endogenously by growth, IRR, and revenue lag, which all are important accounting concepts. Consequently, from the point of view of accounting, this study provides important findings about the time series of startup profitability ratios. These time series are usually considered non-stationary and therefore startups are often excluded from samples in accounting studies based on financial ratios (Balcaen & Ooghe, 2006). The present study supports this argument concluding that the signals given by the time series of startups may be largely distorted showing only a weak connection with IRR that is widely considered the principal measure of profitability.

However, the findings further indicate that profitability ratios in a steady state also suffer from disturbances, which may seriously impair the comparability of profitability between older firms. In startups, the profitability ratios in the early stages often tend to follow a strong adjustment process towards the steady

state. This adjustment process is typically non-stationary leading to the obvious problems as identified by the users of financial statement analysis. The signals given by the profitability ratios during the early stages of this adjustment process are generally vague and strongly distorted by several factors. Thus, the usefulness of these ratios in financial statement analysis can be seriously questioned.

The findings of this study are based on a framework following the strict assumptions of previous models (Feenstra & Fang, 2000, Laitinen, 2006; 2012; Brief, 2013). Therefore, the nature of this analysis is mainly technical in spite of empirical figures that are used in experiments. The present modelling is based on strong assumptions about the behavior of a startup, which do not exactly hold in reality. Therefore, the findings should be interpreted cautiously and only be regarded as a rough and simplified technical description of the potential behavior of profitability ratios in early years. The critical assumptions of the model are associated with a steady growth of expenditure and a constant lag structure of revenue. Empirical cases indicated that the steady growth assumption properly holds only for a part of startups. The constant lag structure assumption got more empirical support. However, even in the latter case it is difficult to assess how reliable the resulted estimates (for IRR and lag parameter) really are. Even for mature firms, estimation of a distributed lag model is not an easy task.

4.2. Profitability ratios in early stages

The findings indicate that under these simplified circumstances the cash flow ratio of a startup converges towards a steady state by a speed based on the relation between the revenue lag parameter and the growth rate: the longer lag and the lower growth rate, the lower is the speed of convergence. In the early stages, the sign of the cash flow ratio is negative but the sign of the steady ratio depends on the difference between IRR and the growth rate. The absolute value of the ratio is strongly increasing in the revenue lag. The findings thus indicate that the cash flow ratio in the early stages is very negative for startups with long revenue lag and that the ratio only slowly improves. The revenue lag is conceptually associated with the industry of a startup. Thus, the differences in the cash flow ratio between industries can be significant only due to differences in the lag (reflecting an industry effect). Since the cash flow ratio does not take account of expenses (match expenditures with revenue), its connection with profitability is even conceptually questioned.

The profit margin ratio is in the early stages of a startup determined by IRR, the rate of expense, and the revenue lag. Therefore, it is also exposed to the industry effect, which may be strong. The effect of the lag on the ratio is negative in early stages but later the sign of the effect depends on the difference between IRR and the growth rate being positive when IRR exceeds the growth rate. The speed of adjustment depends on the relation between the growth rate, the rate of expense and the revenue lag. The rate of expense plays in the early years a very important role. If the startup selects the rate of expense to follow the accumulation of lagged revenue, the rate immediately reaches the steady value. However, a selection of lower (higher) rate may seriously affect the ratio in early stages potentially making an unprofitable (profitable) startup look profitable (unprofitable). This is an important question since in early stages the management of a startup has only limited information about the generation of revenue making the selection of the expense rate at least somewhat ambiguous.

The return on investment ratio is the most widely used measure of profitability. However, the findings of this study indicate that for startups in early stages the link between the ratio and IRR is often very weak. The initial level of the ratios is largely determined by the rate of expense and the revenue lag, which also determine the speed of adjustment together with the growth rate. If the management selects the rate of expense to follow the accumulation of revenue, the resulted steady rate of ratio strongly depends on IRR but suffers also from a potential bias based on the rate of growth. If IRR equals the growth rate (golden path), the steady return on investment ratio correctly reflects IRR as is generally showed by previous studies. However, in the early stages of a startup the ratio is unstable reflecting a mix of several factors. Therefore, its link to IRR in these stages is often too weak to make the ratio a useful measure of profitability. The signal of profitability that it gives to a financial analyst may be badly distorted.

Empirical findings supported these theoretical results. However, the estimation of the distributed lag model was challenging due to the sensitivity of the estimates. For 35% of the startups, estimation was not technically successful. For active startups, cluster analysis revealed five different types of startup, which are summarized in Table 10. In general, profitability ratios in early years gave a pessimistic insight of startup profitability due to the adjustment processes. For clusters 1 and 3, the signal of profitability was distorted and misleading. These clusters made together 57.4% of startups. Cluster 1 was a serious case, since all profitability ratios gave an insight of a very low profitability although average IRR in reality was above average. Cluster 3 was not a serious case because ratios only underestimated low profitability due to the strong adjustment processes. Especially in cluster 1, the rate of expense was very high in comparison to the accumulation of revenue distorting the signal of profitability. Thus, empirically, the rate of expense plays an important role in profitability assessment. The accrual-based ratios gave in cluster 2 a reliable signal of profitability, since the rate of expense was consistent with the accumulation of revenue and, in addition, the rate of growth was close to zero. Therefore, according to (17), the steady ROI roughly equals $r/(1+r)$. For bankrupt startups, the profitability ratios underestimated (negative) profitability (IRR) due to strong adjustment processes.

Table 10. Description of the clusters.

Cluster	Description	Popularity	Adjustment processes	Profitability insight
1	Rapidly growing steady startups with high revenue lag and high rate of expense	26.3%	ROI, PRM and CFR very strong	Very misleading (pessimistic), steady ROI strongly underestimates
2	Zero-growth non-steady startups with high revenue lag and low rate of expense	15.3%	ROI and PRM weak, CFR strong	Correct except for CFR (pessimistic), steady ROI gives a good proxy (underestimation)
3	Unprofitable startups with a low revenue lag	31.1%	ROI, PRM and CFR quick and strong	Misleading (pessimistic), steady ROI underestimates
4	Very profitable shrinking startups with a low revenue lag	16.8%	ROI and PRM weak, CFR strong	Correct (pessimistic), steady ROI gives a good proxy (overestimation)
5	Very unprofitable rapidly growing startups with a low revenue lag	10.5%	ROI, PRM and CFR quick and very weak	Correct, steady ROI gives a reasonable proxy (underestimation)
Bankrupt startups	Unprofitable non-steady bankrupt startups with a structural change	23 startups	ROI, PRM and CFR moderately quick and strong	Misleading (pessimistic), steady ROI underestimates

4.3. Conclusion

In conclusion, the present study shows that the adjustment processes of profitability ratios in the early stages of startups are often non-stationary and strongly influenced by several factors in addition to IRR. The link to IRR is often empirically weak or potentially nonexistent and even misleading. Therefore, it is understandable that startups are often excluded from samples in studies based on financial ratios. The experiments for the present theoretical model based on estimates extracted from a large sample of Finnish startups. However, the findings are mainly theoretical in nature and should be better empirically assessed in further research. Thus, further research on the adjustment process of profitability ratios is called for.

Firstly, further research should relax the strong assumptions of the present model (especially the steady state growth) although this would probably lead to very complicated mathematical results. This theoretical work, however, would significantly support empirical analysis. Secondly, it is a challenging task to estimate of the parameters of a distributed lag model. New methods should be developed for estimating such parameters for non-stationary time series. Finally, the findings of this study showed that the rate of expense

plays an important role in the adjustment process of accrual-based profitability ratios. Therefore, it would be important to analyze empirically the selection of the rate in real-life startups and investigate its relation to the revenue lag.

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Appendices

Appendix 1. Time series of expense D_n for $C = 1-q$.

A. Time series

$$D_n = (1-q)M_n \left[\frac{(1+g)^{n+1} - q^{n+1}}{(1+g)^n (1+g-q)} \right]$$

B. The steady state

$$D_n = M_n \left[\frac{(1-q)(1+g)}{(1+g-q)} \right] n \rightarrow \infty$$

Appendix 2. Time series of assets A_n for $C = 1-q$.

A. Time series

$$A_n = qM_n \left[\frac{(1+g)^{n+1} - q^{n+1}}{(1+g)^n (1+g-q)} \right]$$

B. The steady state

$$A_n = M_n \left[\frac{q(1+g)}{(1+g-q)} \right] n \rightarrow \infty$$

Appendix 3. Industry of active and bankrupt startups by cluster.

Industry	Cluster number:					Total	%	Bankrupt	%
	1	2	3	4	5				
A AGRICULTURE, FORESTRY AND FISHING	15	23	6	3	4	51	2,0		
B MINING AND QUARRYING	2	2	3	1	1	9	0,3	1	4,3
C MANUFACTURING	43	34	62	31	17	187	7,2	4	17,4
D ELECTRICITY,GAS,STEAM AND AIR CONDITIONING SUPPL	1	12	3	1	0	17	0,7		
E WATER SUPPLY, SEWERAGE,WASTE MANAGEMENT AND REMEDIATION STATISTICS	2	5	1	0	0	8	0,3		
F CONSTRUCTION	77	42	157	93	62	431	16,7	1	4,3
G WHOLESALE AND RETAIL TRADE,REPAIR OF MOTOR VEHICLES AND MOTOR CYCLES	125	45	192	94	63	519	20,1	7	30,4
H TRANSPORTATION AND STORAGE	73	23	98	18	11	223	8,6	2	8,7
I ACCOMMODATION AND FOOD SERVICE ACTIVITIES	31	20	54	25	9	139	5,4	3	13,0
J INFORMATION AND COMMUNICATION	45	14	31	22	14	126	4,9	1	4,3
K FINANCIAL AND INSURANCE ACTIVITIES	8	9	6	6	1	30	1,2		
L REAL ESTATE ACTIVITIES	13	11	6	3	0	33	1,3		
M PROFESSIONAL, SCIENTIFIC AND TECHNICAL ACTIVITIES	113	54	72	69	53	361	14,0	3	13,0
N ADMINISTRATIVE AND SUPPORT SERVICE ACTIVITIES	32	21	38	17	24	132	5,1		
O PUBLIC ADMINISTRATION AND DEFENCE, COMPULSORY SOCIAL SECURITY	0	0	0	1	0	1	0,0		
P EDUCATION	8	3	9	8	1	29	1,1		
Q HUMAN HEALTH AND SOCIAL WORK ACTIVITIES	60	56	34	17	4	171	6,6	1	4,3
R ARTS, ENTERTAINMENT AND RECREATION	11	16	9	9	4	49	1,9		
S OTHER SERVICE ACTIVITIES	15	5	17	10	2	49	1,9		
X OTHER	5	1	6	6	2	20	0,8		
Total	679	396	804	434	272	2585	100,0	23	100,0
%	26,3	15,3	31,1	16,8	10,5	100,0			