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The author is indebted to an anonymous referee for helpful comments but remains responsible for any errors.

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ABSTRACT

The paper addresses the long standing asymmetry in the tax treatment of debt and equity costs through a direct comparison of two hypothetical regimes based exclusively on income taxation, broadly defined, and value added taxation. The model presented widens existing debate to encompass the choice between entrepreneurial and contractual use of inputs generally and including labour, as well as capital. Using representative functional forms and numerical illustrations the analysis explores the effect of the tax regimes on firm decisions concerning input selection, output level and vertical integration. The greater neutrality of value added taxation is shown to produce gains in terms of firm efficiency in production and concentration on competitive advantage.

Keywords: Residual income, income tax, value added tax, tax shield, neutrality, vertical integration

JEL Codes: H21,H25,H32

Value Added as the Tax Base for Enterprise Income

I. Introduction

The recent recession was characterised, in no small measure, by limitations in the availability of loans, especially to small businesses. Such limitations are discouraging for business investment. For loans, apart from helping to overcome shortages of equity finance, are also advantageous due to the deductibility of interest under the typical income tax regime, an advantage clearly foregone in the absence of a loan. The tax deductibility of wage costs however confers an equivalent tax advantage compared with the use of the entrepreneur's own labour. The favourable, in principle, effects of this on employment have attracted rather less attention.

Fundamental to the present discussion is the differentiation, in the Knightian tradition of entrepreneurship, between the *contractual* employment of either or both of loan capital and hired labour, and the *entrepreneurial* employment of either or both of equity capital and own labour. The entrepreneurial category is characterised by remuneration from *residual* enterprise income, after deduction of contractual costs. Taking together tax relief on both contractual inputs, it is clear that income taxation generally favours the use of these. Correspondingly it discriminates against the use of own (or externally obtained) equity capital but also against own (or other) labour engaged on entrepreneurial terms. This is a 'distortion' of the choice between entrepreneurial and contractual use of resources. In contrast, taxation of value added is neutral in this respect.

The asymmetric treatment by the UK tax system, among others, of debt and equity finance is pointed out by Meade (1978) who also highlights a great variety of ways in which the system has treated interest in different circumstances.[1] The idea of using value added as the main base of the '*Expenditure Tax'* mooted at that time was indeed considered as one option by Meade (1978). 30 years later, Auerbach et al (2008) reminded that the issue was still live.[2]

More recently public discussion has centred on international 'competition' to offer footloose business a favourable corporate tax regime, and well publicised cases of multinationals paying little or no corporation tax in some jurisdictions. We mention, indicatively, Schoemaker and Goodhart (2010)[3] and McCrae (2015),[4] as this aspect will not be pursued here.

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[1] See pp 55- 63 onwards.

[2] They write: "We point out that avoiding inconsistent treatment of debt and equity in the tax system has become an even more important issue since its discussion in the Meade Report, as the boundaries between the two forms of financial instrument have become increasingly blurred" (Executive Summary).

[3] "A removal of interest deductibility should be done in an international context. A first reason is to keep the playing field level. A second is to reduce the scope for circumventing the tax rules…The main objection to removing the

More interest currently, however, is centred on the United States. At the time of writing the Ben Cardin Progressive Consumption Tax Act (PCTA) would introduce a 10 percent value-added tax (VAT), while also cutting both individual and corporate income taxation. The PCT would require businesses to collect consumption tax imposed on the goods and services they sell or distribute, and claim a credit for the consumption tax they previously paid on inputs. Possible complaints that such a system is regressive would be dealt with through a PCT rebate, and important benefits would be retained in a much simpler income tax code. In addition, it is suggested that it would be possible to reduce the headline corporate rate by more than half, from 35 percent to 17 percent.

The interest 'tax shield' is treated in textbooks as part of an optimisation exercise. That is, leverage (gearing) is taken up to the point where the cost of the risk of distress overtakes the tax advantage. Distress risk increases as the equity 'buffer', and its ability to absorb losses, is weakened. But the tax disadvantage of the equity tends to be conflated with the premium that it would attract on account of is 'residual status' to result in an overall perception that equity is 'more expensive' than debt.

Much of the current discussion takes for granted the need to rely to a significant extent on some measure of income, alongside, or instead of, other tax bases. But ambiguities in the definition of 'profit' or 'net business income', notably as between the 'economic' and 'accounting' definitions, point towards a need to deemphasise the role of income generally as the tax base.[5]

Reform proposals generally centre on the abolition of the debt tax shield, while the possibility of extending tax relief to *dividends*, to put these on a par with debt interest, is sometimes mooted. Rare is any mention of the possibility of also exempting entrepreneurial *labour* on a par with contractual (wage) labour. The difference between the cost of dividends and the costs of the residual remuneration of entrepreneurial labour, from our perspective here, is the 'explicit' character of the former. While tax relief on dividends may be seen as a practical possibility, extension of relief to the 'implicit', or 'opportunity', costs of entrepreneurial labour (and indeed of entrepreneurial capital in so far as not fully reflected in the dividends) is generally not. One may surmise also that another reason for this is the need to preserve a sizeable base for income taxation.

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interest deductibility is that it would sharply raise the overall tax costs of corporates. This could, at least in principle, be addressed by allowing a lower tax deductibility both on interest and dividends at the same rate that would keep the net tax payment of corporates to government at the same level".

[4] He observes: "One obvious win to be to remove incentives for companies to add to debt rather than to equity capital. It is a bit ridiculous that owners are encouraged to load debt on to companies, thereby cutting corporation tax liability, while dividends on equity capital are highly taxed. But that would need international cooperation – not easy".

[5] The conventional 'economic' profit has been revived in the more recent development of 'Economic Value Added' (EVA) by Stern Stewart and related literature. Some accounting arbitrariness, or tax avoidance, may creep into the definitions of contractual and entrepreneurial employment to the extent that entrepreneurs may pay themselves fixed 'salaries' for part or even all of their income, thus making it tax deductible. See also Zafiris and Bayldon (2000). It should be noted that 'value added', as in EVA, is a very different concept from the base of value added taxation and should not be confused with it.

‘Economic profit’ (which would become the base after deduction of all costs, explicit, or implicit) would be a relatively small residual magnitude, normally positive but quite possibly negative. The tax would then be assessed on a very narrow base.

The case for a switch to value added taxation is made by Auerbach et al (2010). They suggest replacing corporation tax with a (destination basis) value added tax, albeit with deduction of labour costs. Yet that proposal also may suffer from the resulting narrowing of the tax base, a broader tax base would require *inclusion of labour costs*. But as pointed out by Crossley et al (2009),[6] very broad based taxes on expenditure would shade, in terms of their overall effects, into taxes on income. This would undermine the common, albeit erroneous, belief that value added taxation is neutral in terms of work incentives. This aspect, also discussed in Crossley et al (2009) is beyond the scope of the present work.

The choice between income/corporate and value added taxation choice is always present, implicitly, in tax policy, to the extent that reduced rates and increased exemptions on income taxes create pressures on the rates and exemptions regime of value added taxation, among other alternatives. Alongside the neutrality argument however, the case for a significant switch from income to value added is also made on the strength of the purported *verifiability* advantages of the latter, compared with the avoidance/evasion potential of the former.

A further aspect of value added taxation is generally unrecognised. While affecting uniformly all the factor incomes generated within the enterprise, such taxation still excludes the costs of materials and/or semi finished products. This would seem to encourage greater efficiency of specialisation by limiting activity within the firm to what the firm does best, i.e. focusing on its distinctive competitive advantages.

[6] They write: “Contrary to first appearances, VAT has the same economic impact as a suitably structured income tax. To see this, consider two very simple tax systems: one with a uniform rate of income tax of 20% and the other with a uniform VAT of 25%. For simplicity’s sake, assume that there is no borrowing or saving. An individual earning £10,000 would pay £2,000 in income tax under the income tax system, whilst his £10,000 expenditure would include £2,000 of VAT under the other system. In this instance, the uniform VAT and income tax are exactly equivalent – both allow the consumption of £8,000 of actual goods and services – and would therefore be expected to have the same behavioural impact.”

The rest of the paper is structured as follows. Section II presents a model to maximise a specific profit function in a regime where no tax applies. The exercise is repeated in section III for a proportional income tax regime and the respective optima are compared, allowing also for a change in gearing. Section IV undertakes the same analysis for the alternative regime based on an equalent rate value added tax and the results are compared with those of section III. In section V the choice of vertical integration is addressed relaxing earlier assumptions about intermediate input purchases. Section VI discusses a measure of residual income volatility The results are drawn together in the concluding section VII.

II. Definition of Profit and Maximisation in a no Tax Regime

The firm maximises 'economic' profit, i.e. revenue net not only of the explicit costs of contractual inputs but also net also of the implicit ones of own (entrepreneurial) resources.[7] Denoting such profit as π_e, and in the absence of taxation, the firm would work with a function of the basic form

$$\pi_e = P_f Q_f - iK_{cf} - wL_{cf} - rK_{ef} - hL_{ef} \qquad (1)$$

where K_{cf}, K_{ef}, L_{cf}, and L_{ef} are the amounts of capital and labour employed entrepreneurially and contractually in (final stage) production. Thus the total capital utilised is $K_f = K_{cf} + K_{ef}$ and total labour is $L_f = L_{cf} + L_{ef}$. Final output Q_f is a function of the inputs i.e. $Q_f = Q_f(K_f, L_f)$. Let P_f denote the exogenously determined price of the firm's final output (firm is a price taker). Let i and r be the unit costs of contractual and entrepreneurial capital respectively.

Specific functional forms have been selected for purposes of illustration. Final output is governed by a Cobb Douglas production function with a degree of homogeneity below 1.[8] Thus

$$Q_f = 10K_f^{0.25}L_f^{0.5} \qquad (2)$$

The unit capital costs i and r, representing Average Factor Costs (AFCs) although taken as externally determined, would also be subject to some differentiation as a result of the

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[7] Although (1) is the objective function that *should* be maximised in principle, it may just not be the behaviourally relevant one, to the extent that the firm's owners mistook the accounting calculation of the residual income as the function to maximise. This possibility will not be pursued further in what follows.

[8] The reason for this is to ensure that the optimisation problem has a finite solution for the output and the inputs. Linear homogeneity would make for indefinite expansion of output, so long as inputs increased at the same pace, unless expenditure on the inputs were constrained by a limited budget. Arguably that, too, is a plausible scenario for a firm faced with such a constraint at a particular decision time, possibly to be replaced by a larger or smaller, but still limited, budget at another decision point.

firm's own choice of *gearing* $g = K_{cf} / (K_{cf} + K_{ef})$ and hence $1 – g = K_{ef} / (K_{cf} + K_{ef})$.[9] Here both AFCs are increasing functions of gearing g of the form

$$i = 6 + 4g + 2g^2 \qquad (3)$$

and

$$r = 12 + 4g + 2g^2 \qquad (4)$$

Although both i and r rise by the same factor as g rises, it will be noted that (3) has a higher starting point, reflecting the cost of capital of an equity-only firm ($g = 0$). Eq (3) has a final value of 12 at $g = 0$, for a theoretical debt-only firm. The cost of the ever shrinking equity tends to a limit of 18 under eq. (4).[10] The optimisation of the gearing is an exercise 'nested' within the profit maximization problem and in principle should be needs to be tackled simultaneously with it. Viewing it separately for now, it means that the firm would aim to minimise the Weighted Average Cost of Capital (WACC), which occurs at $g = 0.5$. Profit maximisation however requires determination of the *marginal* costs (MFCs) of debt and equity or, strictly, the marginal aspect of WACC, a more complex concept, full discussion of which will not be undertaken here. For present purposes, and in view of the interdependence of the two forms of capital costs due to gearing, we shall follow Wang (1994) in defining WMCC, the *weighted average of the marginal costs*, to be the relevant MFC of both K_{cf} and K_{ef}. A property of WMCC is that it equals WACC at the minimum point of the latter.[11]

The relationship between the various aspects of unit capital costs, as defined here, is presented in Table 1. The expressions and valueswhich illustrate the examples appearing in the discussion are highlighted .

[9] Gearing in finance is defined in terms of debt and equity *values*, rather than factor quantities as here. This issue may be overcome, for simplicity's sake, if the market price of a unit of physical capital is taken to be 1, as at the time of decision making.

[10] The numbers selected are such as to allow possible interpretation as percentage rates of return.

[11] See Appendix for proof of the general marginal/average relationship. For a more accurate definition of WMCC and a full discussion see Zafiris (2016)

Table 1: Average and marginal factor costs

		g	a	b	c	di/dg =	$g - g^2$		$AFC_c = i$	$AFC_e = r$	WACC	MFC_c	MFC_e	WMCC
						= b + 2cg								
$i = a + bg + cg^2$		0	6	4	2	4	0		6			6		
	$r = a + bg + cg^2$	0	12	4	2	4	0			12	12		12	12
0.8i									4.8		12	4.8		12
$i = a + bg$		0.1	6	4	2	4.4	0.09		6.42			6.816		
	$r = a + bg + cg^2$	0.1	12	4	2	4.4	-0.09			12.42	11.82		12.024	11.5032
0.8i									5.136		11.6916	5.4528		11.36688
$i = a + bg$		0.2	6	4	2	4.8	0.16		6.88			7.648		
	$r = a + bg + cg^2$	0.2	12	4	2	4.8	-0.16			12.88	11.68		12.112	11.2192
0.8i									5.504		11.4048	6.1184		10.91328
$i = a + bg$		0.3	6	4	2	5.2	0.21		7.38			8.472		
	$r = a + bg + cg^2$	0.3	12	4	2	5.2	-0.21			13.38	11.58		12.288	11.1432
0.8i									5.904		11.1372	6.7776		10.63488
$i = a + bg$		0.4	6	4	2	5.6	0.24		7.92			9.264		
	$r = a + bg + cg^2$	0.4	12	4	2	6.6	-0.24			13.92	11.52		12.336	11.1072
0.8i									6.336		10.8864	7.4112		10.36608
$i = a + bg$		0.5	6	4	2	6	0.25		8.5			**10**		
	$r = a + bg + cg^2$	0.5	12	4	2	6	-0.25			**14.5**	**11.5**		**13**	**11.5**
0.8i									6.8		**10.65**	**8**		**10.50**
$i = a + bg$		0.6	6	4	2	6.4	0.24		9.12			10.656		
	$r = a + bg + cg^2$	0.6	12	4	2	6.4	-0.24			15.12	11.52		13.584	11.8272
0.8i									7.296		10.4256	8.5248		10.54848
$i = a + bg$		0.775	6	4	2	6.8	0.174375		10.30125			11.487		
	$r = a + bg + cg^2$	0.775	12	4	2	6.8	-0.174375			16.30125	11.65125		15.1155	12.30341
0.8i									8.241		10.0545563	9.1896		10.52293
$i = a + bg$		0.8	6	4	2	7.2	0.16		10.48			11.632		
	$r = a + bg + cg^2$	0.8	12	4	2	7.2	-0.16			16.48	11.68		15.328	12.3712
0.8i									8.384		10.0032	9.3056		10.51008
$i = a + bg$		0.9	6	4	2	7.6	0.09		11.22			**11.904**		
	$r = a + bg + cg^2$	0.9	12	4	2	7.6	-0.09			**17.22**	**11.82**		**16.536**	**12.3672**
0.8i									8.976		**9.8004**	**9.5232**		**10.2245**
$i = a + bg$		1	6	4	2	8	0		12			12		
	$r = a + bg + cg^2$	1	12	4	2	8	0			18	12		18	12
0.8i									9.6		9.6	9.6		9.6

Let also w and h be the opportunity unit costs of contractual and entrepreneurial labour L_c which are taken as fixed. Although a 'gearing' aspect may also be defined as between contractual and entrepreneurial labour, all the costs associated with the extra risk of the residual position are treated here as absorbed by the capital inputs.[12]

A final feature to be introduced here will allow for an *intermediate input* or semi finished product Q_b. Although not part of the firm's value added (VA) the input is necessary to achieve the final output. Given technology, Q_b will be needed on a one for one basis with Q_f . It is assumed available to be bought externally at a unit cost P_b. As only Q_f generates revenue, VA would be $P_fQ_f - P_bQ_b$ .The firm may however choose to produce the semi finished input internally (Q_s), if it can do so at a unit cost lower than P_b. At this stage in the discussion P_bQ_b is taken to be constant. But profit must be computed net also of the cost of the bought in semi finished input. Defining a 'net price of $P_n = P_f - P_b$, the profit function now becomes

$$\pi_e = P_n Q_f - iK_{cf} - wL_{cf} - rK_{ef} - hL_{ef} \qquad (5)$$

The decision variables of the problem of maximising (5) are the four inputs contributing to final production, namely K_{cf}, K_{ef} , L_{cf} and L_{ef} . The first order conditions (FOCs) are

$$\frac{\partial \pi_e}{\partial K_{cf}} = P_n \frac{\partial Q_f}{\partial K_f} - \frac{d(iK_{cf})}{dK_{cf}} = 0 \qquad (5.1)$$

$$\frac{\partial \pi_e}{\partial K_{ef}} = P_n \frac{\partial Q_f}{\partial K_f} - \frac{d(rK_{ef})}{dK_{ef}} = 0 \qquad (5.2)$$

$$\frac{\partial \pi_e}{\partial L_{cf}} = P_n \frac{\partial Q_f}{\partial L_f} - w = 0 \qquad (5.3)$$

$$\frac{\partial \pi_e}{\partial L_{ef}} = P_n \frac{\partial Q_f}{\partial L_f} - h = 0 \qquad (5.4)$$

It will be noted that, while the MFCs of capital and labour are differentiated as contractual or entrepreneurial, the respective Marginal Revenue Products (MRPs) are not, since the factors' physical productivity is not affected by the mode of their employment. The MRP of capital cannot of course be equated simultaneously with different MFCs, unless these happened to be equal by coincidence. To overcome this we will treat the WMCC, as defined earlier, as the relevant MFC for both K_c and K_e. The same dilemma would apply to the two categories of labour input unless their costs, too, happened to be equal. Again, it is necessary to apply weighted averaging to marginal labour costs by establishing a WMCL, analogously with the WMCC. It seems reasonable to adopt the same level of 'gearing' for both inputs. Once these adjustments are made the four equations reduce to only the following two.

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[12] Entrepreneurial labour emoluments may not always be higher than contractual, especially in adverse circumstances when residually remunerated workers may be more flexible in accepting a pay squeeze.

$$\frac{\partial \pi_e}{\partial K_f} = P_n \frac{\partial Q_f}{\partial K_f} - WMCC = 0 \qquad (5.5)$$

$$\frac{\partial \pi_e}{\partial L_f} = P_n \frac{\partial Q_f}{\partial L_f} - WMCL = 0 \qquad (5.6)$$

The earlier expressions of equations (2), (3) and (4) now have to be substituted into (5). Assumed numerical values of $P_f = 10$, $P_b = 1.25$, $w = 10$ and $h = 10$ are also inserted. The FOCs then take the following specific forms

$$\frac{\partial \pi_e}{\partial K_f} = 87.5(0.25)(K^{-0.75}L^{0.5}) = 11.5 \qquad (6.1)$$

$$\frac{\partial \pi_e}{\partial L_f} = 87.5(0.5)(K^{0.25}L^{-0.5}) = 10 \qquad (6.2)$$

Dividing both sides of (6.1) by (6.2) yields $L = 2.3K$, hence an optimal L/K ratio of *2.3.* Substituting $2.3K$ for L back into (6.1) yields an optimal $K = 69.25$, hence an optimal $L = 159.28$, approximately. The firm is assumed to be minimising WACC, thus selecting $g = 0.5$ and dividing the total K equally between K_{cf} and K_{ef}. Let us also hypothesise for now that the equal amounts of L_{cf} and L_{ef} would be selected to make up the total L.

The example is summarised in Table 2, the entries of which are meant to be self explanatory. The variables are listed in the first column on the left and defined, as necessary, in the second column, with assumed numerical values inserted and derivatives computed for the specific forms selected. The optimal calculation in the no tax regime is set out in the first column of calculations headed ‘No Tax’ , while the remaining columns will show the results under the tax regimes to be discussed in the next two sections. The marginal magnitudes which need to reach equality in each column are highlighted. Correctness of the calculations is confirmed through the equality of the MRP of capital *(= 11.5)* with the WMCC of capital and similarly equality of the MRP of labour *(= 10)* with the MFCs of L_{cf} and L_{ef}, each of which has been assumed to have fixed unit costs of *10.*[13]

<u>Table 2: The profit maximising calculation</u>

[13] An alternative, or auxiliary, procedure is to approximate the optimal values of the decision variables by trial and error around a plausible starting point and continuing until the relevant marginal products and costs reach equality. The procedure can be mildly entertaining for a while but becomes rather tedious eventually.

Variable	= .. (Definition/formula)	No Tax	Inc Tax	Inc Tax with gearing adjustmt	VA Tax
Q_f	$10(K_{cf} + K_{ef})^{0.25} (L_{cf} + L_{ef})^{0.5}$	**364.0048**	**252.0144**	**311.7770**	**231.2771**
P_f	10	10	10	10	10
K_{cf}		**34.6**	**21**	**48.024**	**18.9**
K_{ef}		**34.6**	**21**	**5.336**	**18.9**
K_f	$K_{cf} + K_{ef}$	**69.2**	**42**	**53.36**	**37.8**
g	$K_{cf}/(K_{cf}+K_{ef})$	0.5	0.5	0.9	0.5
i	$6 + 4g + 2g^2$	8.5	8.5	11.22	8.5
iK_{cf}		294.1	178.5	538.8293	160.65
r	$12 + 4g + 2g^2$	14.5	14.5	17.22	14.5
rK_{ef}		501.7	304.5	91.8859	274.05
L_{cf}		**79.64**	**49**	**119.76**	**43.5**
L_{ef}		**79.64**	**49**	**13.31**	**43.5**
L_f	$L_{cf} + L_{ef}$	**159.28**	**98**	**133.07**	**87**
w	10	10	10	10	10
wL_{cf}		796.4	490	1197.6	435
h	10	10	10	10	10
hL_{ef}		796.4	490	133.1	435
t	0 OR 0.2	0	0.2	0.2	0
$1 - t$		1	0.8	0.8	1
v	0 OR $t(P_fQ_f - iK_c - wL_c - P_bQ_b)/(P_fQ_f - P_bQ_b)$	0	0	0	0.1411
$1 - v$		1	1	1	0.8589
$P_n\partial Q_f/\partial K_f$	$P_n 2.5(K_{cf} + K_{ef})^{-0.75} (L_{cf} + L_{ef})^{0.5}$	**11.5067**	13.1258	12.7813	13.3841
$(1 - t)P_n\partial Q_f/\partial K_f$	$(1 - t)P_n 2.5(K_{cf} + K_{ef})^{-0.75} (L_{cf} + L_{ef})^{0.5}$	11.5067	**10.5006**	**10.2251**	13.3841
$(1 - v)P_n\partial Q_f/\partial K_f$	$(1 - v)P_n 2.5(K_{cf} + K_{ef})^{-0.75} (L_{cf} + L_{ef})^{0.5}$	11.5067	13.1258	12.7813	**11.4952**
$P_n\partial Q_f/\partial L_f$	$P_n)5(K_{cf} + K_{ef})^{0.25} (L_{cf} + L_{ef})^{-0.5}$	**9.9982**	11.2506	10.2504	11.6303
$(1 - t)P_n\partial Q_f/\partial L_f$	$(1 - t)P_n 5(K_{cf} + K_{ef})^{0.25} (L_{cf} + L_{ef})^{-0.5}$	9.9982	**9.0005**	**8.2003**	11.6303
$(1 - v)P_n\partial Q_f/\partial L_f$	$(1 - v)P_n 5(K_{cf} + K_{ef})^{0.25} (L_{cf} + L_{ef})^{-0.5}$	9.9982	11.2506	10.2504	**9.9889**
WACC	$gi + (1 - g)r$	11.5	11.5	11.82	11.5
WACC w inc tax relief	$g(1- t)i + (1 - g)r$	11.5	10.65	9.8004	11.5
$d(iK_{cf})/dK_{cf}$	$i + (4+ 4g)(g - g^2)$	10	10	11.904	10
$(1 - t)d(iK_{cf})/dK_{cf}$	$(1 - t)[i + (4+4g)(g - g^2)]$	10	8	9.5232	10
$d(rK_{ef})/dK_{ef}$	$r +(4 + 4g)[-(g - g^2)]$	13	13	16.536	13
WMCC	$[i + (4 + 4g)(g - g^2)]g + [r +(4+ 4g)[-(g - g^2)]](1 - g)$	**11.5**	11.5	12.3672	**11.5**
WMCC w inc tax relief	$(1 - t)[i + (4 + 4g)(g - g^2)]g + [r +(4 + 4g)[-(g - g^2)]](1 - g)$	11.5	**10.5**	**10.2245**	11.5
$d(wL_{cf})/dL_{cf}$	w	**10**	10	10	10
$(1 - t)d(wL_{cf})/dL_{cf}$	$(1 - t)w$	10	8	8	10
$d(hL_{ef})/dL_{ef}$	h	**10**	10	10	**10**
WMCL	$gw + (1 - g)h$	**10**	**10**	**10**	**10**
WMCL w inc tax relief	$g(1- t)w + 1 - g)h$	**10**	**9**	**8.2**	**10**
P_fQ_f		3640.0483	2520.1440	3117.7697	2312.7707
P_b	$0.125P_f$	1.25	1.25	1.25	1.25
P_n	$0.875P_f$	8.75	8.75	8.75	8.75
Q_b	Q_f	364.0048	252.0144	311.7770	231.2771
P_bQ_b		455.0060	315.0180	389.7212	289.0963
TC_f	$iK_{cf} + rK_{ef} + wL_{cf} + hL_{ef} + P_bQ_b$	2843.6060	1778.0180	2351.1364	1593.7963
Π_e	$P_nQ_f - iK_{cf} - rK_{ef} - wL_{cf} - hL_{ef}$	**796.4423**	**742.1260**	**766.6333**	**718.9744**
Π_e CHECK	$P_fQ_f - TC_f$	**796.4423**	**742.1260**	**766.6333**	**718.9744**
Π_a	$(P_nQ_f - iK_{cf}- wL_{cf})$	2094.5423	1536.6260	991.6192	1428.0244
T	$t(P_nQ_f - iK_{cf}- wL_{cf})$	0.0000	**307.3252**	**198.3238**	0.0000
Π_{te}	$(1 - t)(P_nQ_f - iK_{cf}- wL_{cf}) - rK_{ef} - hL_{ef}$	796.4423	**434.8008**	**568.3095**	718.9744
VA	P_nQ_f	3185.0423	2205.1260	2728.0485	2023.6744
VAT	vP_nQ_f	0.0000	0.0000	0.0000	**285.6049**
Π_{ve}	$(1 - v)(P_nQ_f) - iK_{cf} - wL_{cf} - rK_{ef} - hL_{ef}$	796.4423	742.1260	766.6333	**433.3695**

The last few entries in the first column calculate the profit achieved, in this case a healthy rate of over 20% of revenue. One reason for this is the relatively low cost P_b assumed for the intermediate input at *0.125* of the final product price P_f.

III. Firm Decisions under Income Taxation

At the risk of some oversimplification we shall subsume under 'income tax', for purposes of this discussion, the entire range of taxes assessed on the firm's income or capital, as opposed to taxes assessed on its sales. That is we lump together not only personal income tax on dividends and corporation tax but also (later) capital gains taxes on retained earnings and in principle also (eventual) inheritance taxes on bequeathed assets. E.g. there is some interchangeability between being taxed on 'profit' and transferring some income to a fixed 'base salary' which would be (tax deductible). Any net tax advantage would depend of course on the relative taxation of such 'salaries' and 'profits'. Keuschnigg and Nielsen (2004) discuss similarities of all taxes along that continuum in their effects on individual entrepreneurs, essentially adverse ones on effort, more on which later. Some modelling is on the assumption that the entire income may be received as capital gains. Simplification here is also dictated by the style of modelling attempted in this paper.

Let t now be the rate of income tax, assumed equal to *0.2.* Profit after income tax (π_{et}) to be maximised would now be

$$\pi_{et} = (1-t)[P_n Q_f - iK_{cf} - wL_{cf}] - rK_{ef} - hL_{ef} \qquad (7)$$

It should be recognised, for the avoidance of any confusion, that the tax is not assessed on the above expression of (7) but on the 'accounting' measure of the residual income, that is only on the square bracketed expression in (7). As already suggested, the 'economic' definition of (7) is a smaller magnitude than the 'accounting' one, to the extent of the deduction of the last two terms in (7).

Table 1 shows also the values of unit capital costs under the income tax relief (rows labelled 0.8i). It will be noted that while WACC has a minimum at $g = 0.5$ under no tax, once the tax relief is allowed for it does not reach a minimum until the implausible value *of* $g = 1$. WMCC had in the absence of tax has a starting and finishing value of *12*, dropping below that value in intermediate ranges of g. But it too, once the tax relief is included, it reaches a minimum of *9.6* at $g = 1$.

Under the postulated functional forms and values of exogenous variables, and after rearrangement of terms, the first order conditions (FOCs) of (7) are

$$\frac{\partial \pi_{et}}{\partial K_{cf}} = (1-t)[P_n \frac{\partial Q_f}{\partial K_f} - \frac{d(iK_{cf})}{dK_{cf}}] = 0 \qquad (7.1)$$

$$\frac{\partial \pi_{et}}{\partial K_{ef}} = (1-t)P_n \frac{\partial Q_f}{\partial K_f} - \frac{d(rK_{ef})}{dK_{ef}} = 0 \qquad (7.2)$$

$$\frac{\partial \pi_{et}}{\partial L_{cf}} = (1-t)[P_n \frac{\partial Q_f}{\partial L_f} - w] = 0 \qquad (7.3)$$

$$\frac{\partial \pi_{et}}{\partial L_{ef}} = (1-t)P_n \frac{\partial Q_f}{\partial L_f} - h = 0 \qquad (7.4)$$

Once again the FOCs essentially require equalisation of the net-of-tax marginal revenue product (MRP) of each input to its marginal cost (MFC), with the MRPs identical across the two modes of employment. But only the MFCs of the externally obtained capital and labour are now computed net of the tax. This introduces likely inconsistencies between conditions (7.1) and (7.2) and also between (7.3 and 7.4), as these require the MRPs of capital and labour to equal their marginal costs with, and at the same time without, tax relief. The dilemma was resolved in the previous section by adopting the WMCC as the relevant MFC for both K_c and K_e . As regards labour costs, the conditions can only be satisfied in the general case, if what we defined as the WMCL is also adopted as the MFC of both K_c and K_e .

We can then, again, compress conditions (7.1) to (7.4) into only two. Substituting the specific functional form and assumed parameter values, and with *g* still at *0.5*, the FOC are now

$$\frac{\partial \pi_{et}}{\partial K_f} = (0.8)(87.5)(0.25)(K^{-0.75}L^{0.5}) = 10.5 \qquad (8.1)$$

$$\frac{\partial \pi_{et}}{\partial L_f} = (0.8)(87.5)(0.5)(K^{0.25}L^{-0.5}) = 9 \qquad (8.2)$$

Proceeding as before we now find $L = 2.333K$ or an L/K ratio of *2.333*. That leads to new optimal values of, approximately, $K = 42$ and $L = 98$. Table 2 summarises the results in the column headed 'Income Tax' which shows that, compared with the no tax column, the firm would shrink to some 70% of the previous output level and experience an even greater, proportionately, reduction in profit.[14] We can thus state:
Proposition 1: A proportional income tax with relief on contractual inputs will reduce the employment of both contractual and entrepreneurial inputs, and hence the scale of final

[14] We have avoided, for illustration purposes, selecting numbers that would make profit turn negative post tax. The profits shown here are somewhat exaggerated as a consequence. Real world 'economic' profits are generally small, and may well be negative after tax, especially for firms which may have difficulty recognising opportunity costs fully. As suggested already, the typical size of 'economic' profit makes it unsuitable to serve as the tax base.

output and post tax profit, of a profit maximising firm which maintains the pre tax gearing levels.

As already seen however the optimisation of gearing is intertwined with decisions about scale. Under our specific assumptions here there is seems no reason why the firm should stick, under the tax, to the originally optimal *g* of *0.5*. As is clear from Table 1, the firm would tend towards the theoretical limit of $g = 1$. As that is unrealistic we shall assume in what follows a new optimal $g = 0.9$ and recompute capital (and labour) unit costs accordingly. The equations of (8.1) and (8.2) now become

$$\frac{\partial \pi_{et}}{\partial K_f} = (0.8)(87.5)(0.25)(K^{-0.75}L^{0.5}) = 10.225 \qquad (9.1)$$

$$\frac{\partial \pi_{et}}{\partial L_f} = (0.8)(87.5)(0.5)(K^{0.25}L^{-0.5}) = 8.2 \qquad (9.2)$$

Adopting the earlier procedure, we calculate $L = 2.494K$. That produces new values of, approximately, $K = 53.36$ and $L = 133.07$. Table 2 summarises the results in the column headed 'Income Tax w Gearing Adjustment'. It is evident that the gearing adjustment restores part of the firm's pre tax scale and profit, correspondingly reducing its tax liability.

Comparing the positions defined by (6.1) and (8.1) to (9.1), the employment of K_{cf} following the gearing adjustment goes to a *higher absolute level* (and *a fortiori* higher *relative* to K_{ef}) than in the absence of the tax. K_{ef} on the other hand is *reduced to a lower absolute level* than even under (8.2). On balance, the tax relief, although limited to the contractual resources, means a *larger* level of firm output Q_f than that which would have applied in the absence of *any* relief. Similar conclusions apply as regards the employment of L_{cf} and L_{ef}. We may therefore state:

Proposition 2: A proportional income tax with relief on contractual inputs will reduce to a minimum the employment of entrepreneurial inputs, but increase contractual inputs to a level higher than pre tax, if the profit maximising firm adjusts gearing post tax. On balance the gearing adjustment will partially restore the scale and profit towards the pre tax level.

Although the tax relief goes some way towards restoring the pre tax position, the increased use of contractual resources relative to entrepreneurial ones, (which may be an *absolute* increase under certain assumptions) remains a distortion occasioned by the income tax regime. Given the inevitability, however, of some form of tax on business a search for an alternative seems to be in order.

IV. Firm Decisions under Value Added Taxation

The foregoing discussion has assumed a proportional income tax and no VA tax (although in reality VA tax and/or other sales taxes will be additional to income taxes). Suppose nevertheless, that instead of a tax on enterprise income there were a *complete switch* to an *equal yield tax* on VA at a rate *v.* Such a rate can be derived from the formula

$$v = t\frac{P_f Q_f - iK_c - wL_c - P_b Q_b}{P_f Q_f - P_b Q_b} = t\frac{P_n Q_f - iK_c - wL_c}{P_f Q_f - P_b Q_b} \qquad (10)^{15}$$

In view of the larger base of the VA tax, it will generally be the case that $v < t$ or that $v/t < 1$. In effect the *v* rate is the *t* rate scaled down by entrepreneurial income as a proportion of VA. Alternatively, an income tax may be viewed as equivalent to one on VA, mitigated however by *rebates* on the cost of contractually employed factors and intermediate purchases. Applying this formula (see again Table 2) gives a *v* rate of approximately *0.14*.

Denoting profit after VA tax by π_{vt} the after tax function of (7) would now become

$$\begin{aligned}\pi_{vt} &= (1-v)[P_f Q_f - P_b Q_b] - iK_{cf} - wL_{cf} - rK_{ef} - hL_{ef} = \\ &= (1-v)P_n Q_f - iK_{cf} - wL_{cf} - rK_{ef} - hL_{ef}\end{aligned} \qquad (11)$$

where only intermediate purchase costs (still taken as constant) are tax deductible. We proceed as in the previous section to derive the FOCs, from (11)

$$\frac{\partial \pi_{ev}}{\partial K_c} = (1-v)P_n \frac{\partial Q_f}{\partial K} - \frac{d(iK_c)}{dK_c} = 0 \qquad (11.1)$$

[15] For comparability with the *t* rate the *v* rate is computed as a tax inclusive one. Value added tax is however usually levied at a tax exclusive rate.

$$\frac{\partial \pi_{ev}}{\partial K_e} = (1-v)P_n \frac{\partial Q_f}{\partial K} - \frac{d(rK_e)}{\partial K_e} = 0 \qquad (11.2)$$

$$\frac{\partial \pi_{ev}}{\partial L_c} = (1-v)P_n \frac{\partial Q_f}{\partial L} - w = 0 \qquad (11.3)$$

$$\frac{\partial \pi_{ev}}{\partial L_e} = (1-v)P_n \frac{\partial Q_f}{\partial L} - h = 0 \qquad (11.4)$$

The FOCs now require equalisation of the net of (VA) tax MRP of each input to its MFC. *None* of the factor costs are now computed net of tax, as no tax relief applies to any. Both factors in either mode of employment will be engaged up to the point of equality of the net of tax MRPs with the respective MCs, with no discrimination against the entrepreneurial inputs.

Reducing to two equations, as before, and substituting the specific functional form and parameter values we now have

$$\frac{\partial \pi_{vt}}{\partial K_f} = (0.86)(87.5)(0.25)(K^{-0.75}L^{0.5}) = 11.5 \qquad (12.1)$$

$$\frac{\partial \pi_{vt}}{\partial L_f} = (0.86)(87.5)(0.5)(K^{0.25}L^{-0.5}) = 10 \qquad (12.2)$$

Dividing the first equation by the second yields $L = 2.3K$ which, after substitution back into the first equation gives the optimal value of $K=37.8$, from which also $L = 87$.Table 2 sets out these results in the column headed ‘VA Tax’.

Compared with all the previous results (12.1) and (12.2) define *lower* levels of K and L and hence a lower scale of Q_f , lower even than under the income tax unadjusted for gearing in the second column. As VA tax does not favour either category of input no move to a better gearing ratio is possible here. Hence the amounts of K_c and K_e would also be the smallest, among the cases considered, save only for the amount of K_e which is reduced to a minimum in the third column, under the income tax relief. The reduction in output however enables the firm to achieve almost the same amount of post tax profit as under the income tax unadjusted for gearing (column 2)

The firm in the VA regime in fact realises a *larger* post tax profit *rate*. as a percentage of turnover. That is unfortunately not a generalizable result, as the amount of VA tax paid is also not quite as large as under the income tax. This highlights the fact that, although aiming for *equal yield* tax rates, the formula of (10) does not quite achieve equality of yields *ex post* the optimisation exercises. For that we would need a rate derived from the final tax yield of the calculation of column 2. Formula (10) will only produce ‘broadly equal’ yields from income and VA taxes and may be said to determine an ‘equivalent rate’. In summary we find:

Proposition 3: A proportional VA tax at a rate equivalent to a proportional income tax will lead to lower employment of both contractual and entrepreneurial inputs, and lower final output, tax yield and post tax profit, albeit possibly a higher profit rate. No adjustments in gearing are indicated.

What is the welfare significance of a smaller firm under VA taxation? Counterintuitively, perhaps, it can be argued that the smaller scale dictated under such taxation is the most appropriate. Given that the firm must be taxed somehow, the no tax regime does not establish a realistic benchmark. The long established principle then is that taxation should be as neutral as possible. Income taxation in its various forms is arguably disorientating in encouraging efforts to minimise liability, by varying or reclassifying the input mix, and diverting attention from the search for productivity and identification of competitive advantage.

V. The Choice of Degree of Integration

VA taxation has one discriminatory aspect in common with income taxation, in exempting bought in semi finished product, and taxing any internal intermediate (as opposed to final) production. Inputs in the form of semi-finished product, or subcontracted work are not part of VA and are thus tax deductible. *Ceteris paribus* the firm would be influenced in favour of buying in rather than producing internally. For this reason also the firm in the VA tax regime would tend to be smaller than it would be in the absence of this tax concession.

The discussion so far has assumed a single intermediate input linked to final output through a fixed technical coefficient ($=1$) and purchased externally at a fixed price $P_b = 0.125P_f$. But if that price where to rise, say to $0.15P_f$ it is reasonable to suppose that the firm would consider some substitution of internal production, to say $Q_s = 0.2Q_f$ and sourcing externally only $Q_b = 0.8Q_f$, in effect opting for a higher level of vertical integration. There is an implied demand curve for external sourcing of semifinished product, given in this instance by

$$Q_b = 2 - 8P_f \qquad (13)$$

which has a domain of $0.125 < P_b < 0.25$ and a range of $1 > Q_b > 0$.

Since $Q_s = Q_f - Q_b$, we also have

$$Q_s = -1 + 8P_b \qquad (14)$$

and Q_s now becomes a variable on a par with Q_f. The decision variables of the optimisation problem, the amounts of the two categories of each input K and L, would now need to be differentiated further depending on whether they were employed in final or intermediate production. We would then have $K = K_{cf} + K_{ef} + K_{cs} + K_{es}$ and similarly $L = L_{cf} + L_{ef} + L_{cs} + L_{es}$, the second subscript referring to final or semi finished production. Assuming that the same production function governed both final output Q_f and intermediate output Q_s, the firm would maximise the following, rather cumbersome, modified version of eq (11)

$$\pi_{vt} = (1-v)[P_f Q_f - P_b Q_b] - i(K_{cf} + K_{cs}) - w(L_{cf} + L_{cs}) - r(K_{ef} + K_{es}) - h(L_{ef} + L_{es}) \quad (14)$$

We would then proceed to optimise the profit function as in the previous sections. Although some of the simplification applied earlier is possible, a full formal statement is probably unnecessary. It will be noted however that any switch to internal production Q_s would be subject to VA tax. Not generating any sales revenue, Q_s does not appear explicitly in (14) but is there in the form of increased input costs, not alleviated by any tax relief inside the bracketed VA expression. In view the discouragement of internal production by the VA tax regime, the example of the previous section has illustrated the limiting case of $Q_s = 0$.

The choice of degree of integration is a strategic decision problem for the firm to make, alongside others, such as margin vs market share, the trade off between quality and price, and (horizontal) diversification. The last possibility of the (smaller) firm also becoming less *diversified* than otherwise is relevant to the present discussion. For such a restriction in the firm's *scope*, as well as scale, induced by a switch to value added taxation, would most probably also translate into greater specialisation in areas of core competence and competitive advantage. The switch would then represent a move in the direction of greater firm efficiency within the tax environment.

Realisation of such efficiency potential would clearly depend on (i) how far the firm's distinctive advantage lay in limited VA activities but also (ii) the removal of biases affecting the substitutability of entrepreneurial for contractual factor services. The literature on moral hazard in production is relevant here. We mention again, indicatively, Keuschnigg and Nielsen's (2004) analysis of the effects of the (income/capital) tax regime on entrepreneurial/venture capital effort incentives in the context of (possibly double-sided) moral hazard. These writers point to adverse tax effects on 'effort' between parties each of whom bears the whole of the cost of their increased effort but has to share with the other party the resulting revenue increase. Fairchild's (2011) discussion of empathy and trust between entrepreneurs and angels (as opposed to venture capitalists) is also of relevance in the same context.

Unlike however the distinction between ‘entrepreneur’ and ‘capitalist’ of contributions such as the above, we have identified here *all* such input suppliers as ‘entrepreneurial’, by virtue of their residual remuneration. We have indeed included residually remunerated ‘labour’ in the broader entrepreneurial category. It is the effort of all such participants, notwithstanding any tax (dis)incentives, that constitutes the main determinant of efficiency, both in production and in the selection of strategic alternatives. The relatively simple message that emerges here is that a tax regime which does not discriminate against entrepreneurial input engagement is more likely to favour pursuit of the effort rightly located in the wider domain of entrepreneurship. Qualities such as empathy and trust are also more likely to be developed among input suppliers contributing own ‘labour’.

Other things equal, the firm should now prefer entrepreneurial to contractual employment of inputs, of labour as well as capital, as entrepreneurial engagement is more likely to release expenditure of effort. Although not observable or verifiable, such effort and associated qualities are likely to be evidenced in efficient strategic choices, such as the avoidance of unnecessary intermediate production and inappropriate diversification. In view of widespread criticisms of ‘managerial’ behaviour in respect of such choices, emphasis on the entrepreneurial element in input selection seems to offer the best platform for efficient decisions. This, rather than the mechanics of the formal models, would be a reason for adjusting gearing in favour of towards the entrepreneurial inputs. We may state, if somewhat loosely,

Proposition 4: A regime based on VA would favour entrepreneurial input employment as the likelier location of effort to improve efficiency in production and strategic choices oriented towards distinctive firm capabailities.

VI. Measurement of Riskiness under Alternative Taxes

The potential effects of the asymmetry of (income) tax treatment on choices involving risk are, *prima facie*, ambiguous. There are mutually opposed influences at work. On the one hand, the greater than otherwise pre-emption of income to meet contractual commitments increases the riskiness of the residual accruing to the suppliers of entrepreneurial resources. That would encourage more risky choices to produce a larger residual. In the corporate world in particular “..the increased commitments to pay interest serve as an incentive to elicit greater efforts from entrenched managers. Thus, while a tax bias in favour of interest appears to encourage borrowing, it is harder to say whether it encourages *too much* borrowing” (Auerbach et al 2008, our emphasis). Equally, the increased riskiness of the residual may encourage attitudes of caution in the firm’s selection of products or projects. The perception of bankruptcy risk, possibly low at first, looms larger once a critical level of leverage is reached and that is indeed where the ‘traditional’ view in finance would locate the point of optimal leverage.

We employ below an objective, in principle, measure of variability or volatility of after tax profit. Variability, essentially represented by the variance of the residual, is traced to potential movements in the externally given parameters, once the values of the decision variables have been selected. Such selection is taken to have been made in response only to the *expected values* (EV*s*) of the external parameters.[16] That is, the *ex ante* optimisation exercises of earlier sections precede the calculation of variability. The selected values of the decision variables, and hence also of output Q_f, are treated as constants for purposes of the calculation. To note also that the variability measure does not allow for subsequent adjustments in the values of the choice variables based on any evidence of variability becoming available *ex post*.

Assume, as before, perfect correlation between and P_f and P_b. Of the ten possible pairs of random variables P_n, i, r, w and h we assume, for simplicity, positive correlation between i and r and also between w and h, the other eight correlations being taken to be zero.

The variance of the tax free profit of (5) is then

$$Var(\pi_e) = Q_f^2 Var(P_n) - K_{cf}^2 Var(i) - L_{cf}^2 Var(w) - K_{ef}^2 Var(r) - L_{ef}^2 Var(h) - \\ - 2K_{cf} K_{ef} Cov(i,r) - 2L_{cf} L_{ef} Cov(w,h) \qquad (15)$$

The variance of profit after income tax in (7) would be

$$Var(\pi_{et}) = (1-t)^2 [Q_f^2 Var(P_n) - K_{cf}^2 Var(i) - L_{cf}^2 Var(w) - 2K_{cf} K_{ef} Cov(i,r) - \\ - 2L_{cf} L_{ef} Cov(w,h)] - K_{ef}^2 Var(r) - L_{ef}^2 Var(h) \qquad (16)$$

Comparing (15) with (16) by inspection we find that the first five terms are preceded by $(1-t)^2$, the last two being identical. But the similarity of expressions conceals differences in the values taken by the variables in the respective optimal solutions. Referring back to Table 2, we find that the Q_f value of (16), corresponding to the 'Income Tax' column is some 70% of the Q_f of (15) i.e. the 'No Tax' column, while the values of the inputs of (16) are between 60% and 70% of those of (15). Each of the first five terms of (16) thus replicates a term in (15) preceded by a constant well below 1. While it seems that (16) defines a lower variance than (15), that has to be seen *relative* to the smaller scale of the of the post (income tax) profit of (16) unadjusted for gearing.[17] The comparison with the 'adjusted gearing' column, the variance of which is also represented by (16), is narrower still, as that solution involves output and input figures rather closer in total to those that corresponding to the no tax case of (15), and indeed

[16] In the interests of simplicity the EV*s* of parameters are not shown in EV notation.

[17] Thus the coefficient of variation (= standard deviation/expected value) would be a more appropriate measure.

exceeding those where the contractual inputs are concerned. Short of selecting arbitrary figures to illustrate the possible variation of price and input costs, we can unfortunately confirm nothing more here than the ambiguity that we started with.

Our real interest however is in comparing risk behaviour across the two alternative tax regimes. For that we need to compute the variance of (11) which is

$$\begin{aligned}Var(\pi_{vt}) &= (1-v)^2 Q_f^2 Var(P_n) - K_{cf}^2 Var(i) - L_{cf}^2 Var(w) - K_{ef}^2 Var(r) - L_{ef}^2 Var(h) - \\ &- 2K_{cf}K_{ef}Cov(i,r) - 2L_{cf}L_{ef}Cov(w,h)\end{aligned} \quad (17)$$

Comparison of that with (16), or indeed with (15), again does not seem to lead to an unambiguous result, especially when the probably lower absolute size of (17) is viewd in relation to the smaller size of the output and input variables of the solution headed ‘VA Tax’ in Table 2. The variance of the remaining equation (14), corresponding to post VA tax profit with possible own itermediate production, would clearly not produce anything more definite and is omitted for the sake of brevity.

Sad though it is to finish with a negative, we seem to have reached the limits of what the methodology employed in this section allows us to say about a firm’s risk bearing capacity under the alternative tax regimes hypothesised, and hence its likely behaviour in selecting risky projects.

VII. Conclusions

Difficulties of access to loan finance experienced by many firms have possibly reduced the advantages often associated with contractually engaged resources. That calls for a re-examination of the supposed tax advantages of debt and of the potential of value added taxation as non-discriminatory in respect of the choice between debt and equity finance but also, more generally, between any contractual and entrepreneurial resources.

Using the standard ‘economic’ profit approach as the firm’s objective function, a detailed comparison has been undertaken of the effects of tax regimes based wholly on business income or wholly on value added. Specific functional forms have been utilised to exemplify the predicted effects of the tax regimes on the firm’s choices regarding the employment of inputs in the entrepreneurial or contractual mode, and the firm’s size and degree of vertical integration. An attempt has been made also to compare residual income volatility under the alternative regimes.

The discussion supports the widely held expectation that tax neutrality in respect of choices between entrepreneurial and contractual input utilisation has certain desired efficiency effects. A switch towards value added taxation achieves such neutrality in principle and is likely to confer advantages through greater use of entrepreneurial inputs and more effort among the suppliers of these. It is recognised that value added taxation, favours intermediate input purchases over internal value generation, thus making for a smaller firm. But the associated incentives to concentrate on core competences and make better strategic choices can produce efficiency and welfare gains.

REFERENCES

Auerbach Alan J, Devereux Michael P and Simpson Helen (2008) "Taxing Corporate Income" University of California, Berkeley, Paper prepared for the Mirrlees Review "Reforming the tax System in the 21st Century".

Auerbach Alan J, Devereux Michael P and Simpson Helen (2010) in J Mirrlees et al (eds) *Dimensions of Tax Design: The Mirrless Review*, Oxford Uninversity Press for Institute of Fiscal Studies

Bronfenbrenner M (1960) "A Reformulation of Naïve Profit Theory" *Southern Economic Journal*, Vol 26, no 4, April, pp 300-309, reprinted in Breit W and Hochman H M eds (1968) *Readings in Microeconomics*, Holt, Rinehart and Winston.

Crossley Thomas F, Phillips David and Wakefield Mathew (2009), *The IFS Green Budget 2009*, Institute of Fiscal Studies, February, p 196

Fairchild, R (2011) "An entrepreneur's choice of venture capitalist or angel-financing: A behavioral game-theoretic approach" *Journal of Business Venturing*, 2011, vol. 26, issue 3, pages 359-374

Keuschnigg Christian, Soren Bo Nielsen (2004) "Start-ups, venture capitalists, and the capital gains tax", *Journal of Public Economics*, Issue 4, 88 pp 1011-1042

McCrea Hamish (2015) "There is no magic wand to tackle world debt woe but honesty would help" *Evening Standard*, Feb 9

Meade J E (1978) *The Structure and Reform of Direct Taxation*, Institute of Fiscal Studies, Committee Report

Mirrlees J et al (2011) "The Mirrlees Review: Conclusions and Recommendations for Reform" Fiscal Studies, vol. 32, no. 3, pp. 331–359

Schoenmaker Dirk and Goodhart Charles (2010) "Removing tax advantages of debt is vital" *Financial Times*, Dec 30

Wang Louie K (1994) "The weighted average cost of capital and sequential marginal costing: A clarification" *The Engineering Economist*, Vol 39, Issue 2, pp 187-192

Zafiris N and Bayldon R J (2000) 'Economic Value Added and Market Value Added: A Simple Version and Application'. *Journal of Applied Accounting Research*, Vol 3, No 1, pp 84-107

Zafiris N (2016) "The Weighted Average Cost of Capital as a Marginal Criterion" *Journal of Finance and Investment Analysis,* Vol. 5, No. 4, 2016, pp 1-27

APPENDIX

AVERAGE AND MARGINAL FACTOR COSTS

The general relationship used is $MC = AC + Q\frac{d(AC)}{dQ}$. Thus

$$\frac{di}{dK_c} = i + K_c(4+4g)\frac{di}{dg} = i + K_c(4+4g)\frac{K_e}{(K_c+K_e)^2}$$
$$= i + (4+4g)g(1-g) = i(4+4g)(g-g^2)$$

and

$$\frac{di}{dK_e} = r + K_e(4+4g)\frac{dr}{dg} = r + K_e(4+4g)(-\frac{K_c}{(K_c+K_e)^2})$$
$$= r + (4+4g)[-g(1-g)] = r + (4+4g)[-1(g-g^2)]$$

Whereas these MFC formulae do not quite capture the full interdependence of the costs of the two types of factor, they have the advantage of expressing the effects of changes in terms of values of *g* only, i.e. without reference to the actual quantities of K_c and K_e.