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I. INTRODUCTION

It is well known that risk-taking is one of the unique characteristics of entrepreneurs, especially those from the small business community. While the risk-taking behavior may result in innovations in new products or services that benefit society as a whole, it also significantly increases the financing costs of smaller companies. Knowing that the higher financing costs may discourage or even eliminate socially desirable risk-taking activities, government agencies usually offer loan guarantees to support these activities. In fact the practice became so common and popular that several years ago many politicians argued that more loan guarantees should be provided because the "ex-post" costs, such as loan losses, are very small in comparison to the jobs created. The most popular examples used to support this argument are the bail-out of the Chrysler Corporation in the early 1980s and the savings and loan associations' crisis in the mid 1980s. However, several years ago, the former House speaker, Mr. Newt Gingrich, and the former Senate majority leader, Mr. Robert Dole, both proposed to eliminate the Small Business Administration (SBA), especially the minority small business programs which were abused by some individuals. Given that the SBA programs in fact have provided significant contributions to assist small firms, this study argues that the issue should not be "to eliminate or not to eliminate", but how to modify or re-design the current program so that it will not be abused. Using some basic concepts in finance, this study shows that moral hazard or the agency problem indeed can be minimized or eliminated entirely if additional financial terms are added to the contract of the loan guarantee.

The plan of this study is as follows. In section II, the objectives of this paper will be briefly provided. The relationship between the agency problem and financing for smaller and larger firms is reviewed in section III. It is pointed out that larger corporations are able to reduce or eliminate the debt related agency costs but not smaller firms. The support from government agencies is therefore necessary. The impacts of these supports, especially the loan guarantee, on the market value of the firms, i.e. their stocks and bonds, are discussed in section IV. In section V, we show that accurate pricing of the loan guarantee is a necessary condition to control risk-taking and warrants can be used as monitoring devices. Finally, total risk, systematic risk, and project evaluation are reviewed in section VI and suggestions and comments are available in the last section.

II. OBJECTIVES

The objective of this study is twofold. First, we examine the relationship between the small business loan guarantee and the agency problem of small firms. Second, we recommend financial instruments or financial contracts that can minimize or eliminate the moral hazard problem.

According to the agency theory in finance, small business firms whose loans are guaranteed may have an incentive to take "excessive risk" if the loan guarantee is not properly priced, and/or if the investment activities of the companies are not properly monitored due to high transaction costs. Using a contingent claims approach, this study shows that the growth opportunity of smaller firms may be relatively larger than the assets-in-place. Unlike larger corporations, smaller firms do not have the financial strength or credit record to eliminate the debt related agency cost that is caused by the incentive to forgo positive net present value projects. Therefore the growth opportunity, even if it is really profitable, may not be undertaken without the credit support or other loan related programs from government agencies such as the Small Business Administration (SBA).

However, an accurate pricing of these supports, especially the loan guarantee, is very important to safeguard the financial interest of the SBA. Because the loan is guaranteed, the lenders, such as commercial banks who have expertise in processing information and assessing credit risk, do not have incentives to monitor the risk-taking behavior of the borrowers and because returns are determined by risk, the tendency for loan recipients to take excessive risk is very high. Applying the option pricing theory in finance, this study shows that a loan guarantee can be treated as a put option which can transform a risky loan into a riskless loan. Because a closed form solution for European call options or put options has been developed by Black and Scholes (1973), the price of a loan guarantee can be explicitly determined, at least conceptually.

In addition to the correct pricing of a loan guarantee, this study recommends that financial instruments, such as warrants, be included in the loan guarantee package. By doing so, the warrant holders, such as the SBA, will be able to share the "windfall" gains if excessive risk is indeed taken and excessive profits are realized. Given that extra returns have to be shared, windfall profits will be less for the risk takers and the motivation to take excessive risk will be reduced.

III. INVESTMENTS, FINANCING, AND AGENCY PROBLEM[1]

In order to distinguish small firms from large firms, let us assume that the market value of these firms is the sum of two components, assets-in-place and the growth opportunity. We define small firms as having a smaller proportion of assets-in-place than growth opportunity. Large firms, on the other hand, have a much larger proportion of assets-in-place and/or relatively smaller proportions of growth opportunity:

(1) $$V(F) = V(A) + V(G)$$

$$= V(A) + \int_0^{\infty} P(s) Z(s) \left[V(G,s) - I\right] ds$$

where $V(F)$ = the market value of the firm;

$V(A)$ = the market value of the assets currently held by the firm;

$V(G)$ = the value of an option for future investment. This is analogous to the value of the growth opportunity;

$P(s)$ = the present value of a dollar delivered next period if a particular state of the world, s, occurs;

$V(G,s)$ = the discounted value of the economic cash flows as of next period, given state s;

$Z(s)$ = the decision variables;

I = investment requirements.

Without an agency problem, the net present value decision rule is followed,

$$\begin{cases} Z(s) = 1 & \text{if } V(G,s) > I; \text{ or} \\ Z(s) = 0 & \text{if } V(G,s) < I. \end{cases}$$

With an agency problem, the principle of net present value may not be followed. In an efficient market, these agency costs will be assumed by the corporations in terms of higher cost of capital and therefore, these corporations will have a strong incentive to resolve the agency problem, as long as the marginal monitoring costs are not larger than the marginal agency costs. The relationship can be illustrated by the following numerical examples.

Assuming that a large corporation has the following assets-in-place and a growth opportunity[2]:

Time	0	1	2
Assets	-50	$100	$50
Growth		-75	100
Bonds		10	100

If the time value of money is zero, the net present value of the assets will be $100 and the net present value of the growth will be $25. Since both of them are greater than zero, the projects should be accepted when the agency problem does not exist or when the company is able to guarantee that both investments in asset and growth will be accepted. If this is the case the market value of the bonds and stock will be

Bonds = $(10 + 100) = $110

Stock = $(-50 + 15 + 50) = $ 15

and the total value of the company will be \$125. When an agency problem exists and when the company is not able to guarantee that both projects will be accepted, the value of the bonds will drop to \$60 and the stock value will be \$90:

$$\text{Bonds} = \$(10 + 50) = \$\ 60$$

$$\text{Stock} = \$(-50 + 90 + 0) = \$\ 40$$

The total value of the company will therefore become \$100. Notice that the market value of the bonds reduces from \$110 to \$60 and the stock value increases from \$15 to \$40. The shareholders are better off not to invest in the growth opportunity even though it has a positive net present value. The market value of the bonds changes from \$110 to \$ 60 because project A was accepted at time zero and thus the company has cash inflow of \$100 at time 1. After paying the bondholders \$10, the shareholders will have \$90. At time 2, due to the same investment made at time 0, the cash inflow will be \$50 and because the investment in growth opportunity was not made, no additional income is available. The company is therefore forced into bankruptcy and when the bondholders take over the firm, they receive \$50. In an efficient capital market, the market price of the bonds will be discounted to reflect the agency cost. Given that the bond price decreases by \$50 and the stock price increases by \$25, the net agency cost of debt is \$25.

In addition to providing a guarantee, the company can save the agency cost of debt by creating a sinking fund account or borrowing less. The sinking fund provision will prevent default. By borrowing less, the risk of default is reduced and the trust of the bondholders can be restored. For example, if the company borrows \$10 at time 1 and \$70 at time 2, the value of the stock and bonds will be

$$\text{Bond} = \$(10 + 70) = \$80$$

Stock= \$(-50 + 15 + 80) = \$45

Note that the total value of the firm is \$125 which is the same as the first case which assumes that a guarantee to invest in the positive net present value project is offered.

Although the guarantee and sinking fund provision may be feasible for large corporations which have sound financial backgrounds and credit histories and have significant assets-in-place that can serve as collateral for loans, these financial strengths are not available for small firms. A guarantee provided by a credit-worthy third party, such as a government agency is therefore necessary.

IV. LOAN GUARANTEE VALUATION, RISK-TAKING, AND SMALL FIRMS

Assume that a SBA loan guarantee is offered to a small firm. The impacts of the guarantee can be examined by comparing the market value of the stock, the bond, and the total value of the firm before and after the loan guarantee is offered.

A basic corporate finance textbook shows that "Whenever a firm borrows, the lender effectively acquires the company and the shareholders obtain the option to buy it back by paying off the debt. The stockholders have in effect purchased a call option from bondholders" (Brealey and Myers, p.430). The value of common stock can therefore be determined by using the Black-Scholes (1973) option pricing theory. At maturity, the market value of the common stock and bond can be expressed as

	$Vf > B$	$Vf \leq B$
Vs	$Vf - B$	0
Vb	B	Vf
Vf	Vf	Vf

where Vf = the market value of the firm;

Vs = the market value of the stock;

Vb = the market value of the bond;

B = the face value of the bond.

In other words, when the bond issued by the company expires, the shareholders will pay the bondholders the last interest payment and the face value of the bond if the total market value of the firm is greater than the total payments. On the other hand, if the firm is insolvent so that its value is less than the total bond payments, the shareholders are better off to declare bankruptcy and let the bondholders take over the firm. In the latter case, the shareholders are protected by the limited liability clause of the corporate charter and the bondholders suffer losses because Vf is less than B, the total promised payments. The bond is therefore very risky and risk averse lenders will either ask for a very high interest rate or refuse to finance small firms. If a loan guarantee is obtained from the SBA, the risky debt will become riskless for the lenders because loans in default can be recovered from the SBA:

	$Vf > B$	$Vf \leq B$
Vs	$Vf-B$	0
Vb	B	B
G	0	$Vf-B$
Vf	Vf	Vf

Notice that the debt is risk-free because the bondholders are able to receive B which is independent of the financial condition of the firm. The loan guarantee, G, on the other hand, is either zero or negative. When the company is solvent, the guarantee has no value. However, when the company is insolvent, the SBA has to pay the loan in full for the borrower. After considering the cash inflows from the firm, *Vf*, the SBA still suffers losses because *Vf* is always less than *B*. The losses can become a large financial burden for the government agency if *B* is much greater than *Vf*, i.e. small firms are allowed to borrow as much as possible and/or loan guarantees are under-priced. Given that the returns for the firms are associated with the risk they assume and given that the lenders do not have incentives to monitor the risk-taking behavior because their returns are guaranteed, the tendency for small firms to take excessive risk is very high. Several alternatives can be proposed to discourage or prevent excessive risk-taking; proper pricing of loan guarantees and the inclusion of inexpensive monitoring instruments, such as warrants.

V. PRICING OF LOAN GUARANTEE AND FINANCIAL INSTRUMENTS AS MONITORING DEVICES

According to Smith (1976) and Jones and Mason (1980), the value of the guarantee can be expressed as:

(2) $$g = l - l'$$

where g = value of the guarantee;

l = value of the guaranteed loan;

l' = loan value without guarantee.

Put-call parity and option pricing theory reveal that:

(3) $$l' = l - p$$

where p = put option.

Substituting (3) into (2) gives:

(4) $$g = p$$

In other words, the value of the loan guarantee is the put option value and therefore can be determined by

(5) $$p = x\, e^{-rt}\; N(-d2) - s\, N(-d1)$$

where $d1 = [ln\,(s/x) + (r + 0.5\sigma^2)t]/(\sigma\sqrt{t})$

$d2 = d1 - \sigma\sqrt{t}$

and x = the exercise price;

r = riskless interest rate;

t = time to maturity;

σ = standard deviation;

s = the underlying asset price

$d1$, $d2$ = standard normal cumulative probability density.

Since the put option has a positive value before maturity, the guarantee should therefore have a positive market value. Moreover, the government agent is exposed to risk because it has the obligation to repay the loan if the loan is in default. The risk and its expected losses are the "implicit" costs for the loan guarantee and should be charged to the loan recipients. Given that these costs have a positive relationship with risk, higher risk-taking firms will be required to pay a higher premium. Since a "free-lunch" is no longer available, excessive risk-taking can be prevented.

One major difficulty of the risk-sensitive approach is that it imposes another financial cost to the already-cash-strapped small firms. It is not likely that small firms have the financial capacity to pay the "up-front" premium. Moreover, in addition to other variables, the put premium requires an estimate of the risk level of the firm.

Instead of paying a "cash premium," the firms can be asked to issue warrants to the agents providing the loan guarantees. The warrants will not only reduce the financial burden of the firms but also provide a "profit-sharing" opportunity for the SBA. The financial instrument is not costly to introduce and implement and therefore should be recommended. In fact, it was a successful instrument used by the U. S. Treasury in the bailout of the Chrysler Corporation in the 1980s (Wall Street Journal, May 9, 1983). The use of the total risk estimate, however, is inconsistent with the systematic risk measurement recommended by modern portfolio theory. Some suggestions to link the total risk with the systematic risk are provided in the following section.

VI. TOTAL RISK, SYSTEMATIC RISK, AND PROJECT EVALUATION[3]

Using the capital asset pricing theory and the option pricing theory, Galai and Masulis (1976) show that the relationship between the beta of an option and the beta of the underlying asset is as follows:

(6) $$Bc = N(d1)\, A/C\, Ba$$

where Bc = the systematic risk of a call option;

A = the market value of the asset;

C = the market price of the call option;

Ba = the systematic risk of the asset.

Since common stock can be treated as a call option, equation (6) therefore shows that the systematic risk of the common stock is a function of $N(d1)$, total asset value, total common stock value, and the systematic risk of the common stock. After the systematic risk of common stock is determined, the security-market-line method can be used to calculate the required rate of required for the loan guarantee. Notice that in 1971, Lockheed paid the current Treasury bill rate plus a fee of roughly 2% to the government for the bailout it received [Reinhardt (1973)]. The fee essentially is the risk premium of the security market line. The premium, however, must be time varying because equation (6) reveals that even if the systematic risk of the underlying assets of the firm is constant, the common stock beta is likely to change over time when the time to maturity changes, altering the value of $d1$. For smaller firms, the time varying risk premiums will be more profound given the significant amount of growth opportunity.

Fewings (1975), Turnbull (1977), and Myers and Turnbull (1977) have all proposed models illustrating the effects of growth on a firm's operating beta. Turnbull and Myers and Turnbull, using very different models as starting points, show a firm's asset beta to be a decreasing function of growth; Fewings, though, shows it to be an increasing function of growth. Senbet and Thompson (1982) attempt to reconcile the models of Fewings and Myers and Turnbull by showing that the Myers and Turnbull model can be modified to be compatible with a firm's asset beta being an increasing function of growth, though they do not lay the controversy to rest.

The relationship between systematic risk and growth can also be examined by applying the multiperiod equilibrium model of Stapleton and Subrahmanyam (1978). The model overcomes the stationarity assumption made by Fama (1970) and the possible lack of normality of derived prices. Assuming exponential utility for terminal wealth, Stapleton and Subrahmanyam have shown that the two-period equilibrium prices at t=0 of cash flows received at t=1 and t=2, respectively are:

(7) $$\boldsymbol{P}_{0.1} = (E(\boldsymbol{X}_1) - \lambda_1 \boldsymbol{\Omega}_1 \mathbf{1})/r_1$$

(8) $$\boldsymbol{P}_{0.2} = (E(\boldsymbol{P}_{1.2}) - \lambda_1 \boldsymbol{\Omega}_1 \mathbf{1})/r_1$$

where $\boldsymbol{X}_1$ is the vector of cash flows at t=1 of all x_j firms (j=1, ..., m), $\boldsymbol{P}_{1.2}$ is the vector of firm prices at t=1 for cash flows received at t=2, and Ω_1 is a variance-covariance matrix of cash flows X_1 and $P_{1.2}$, i.e.

$$\Omega_1 = \begin{bmatrix} F & G \\ G' & H \end{bmatrix}$$

$$F = cov(X^j_1, X^k_1)$$

$$G = cov(X^j_1\, P^k_{1.2})$$

$$H = cov(P^j_{1.2}\, P^k_{1.2})$$

$P^j_{0.2}$ and $P^j_{0.1}$ are the values of cash flows X^j_2 and X^j_1 for firm j, assuming that the cash flows are paid out as dividends. The value of the market portfolio at t=1 is $X^m_1 + P^m_{1.2}$, where $P^m_{1.2}$ is the market portfolio (of the X^j_2) at t=1. The vector $\boldsymbol{P}_{1.2}$ contains the values of assets (with cash flows X^j_2) at t=1.

In the above multiperiod model, the systematic risk (β_{jt}) of project j is:

(9) $$\beta^j_t = cov(X^j_t + P^j_t, X^m_t + P^m_t \mid X_{t-1})\, P^m_{t-1} \,/\, var(X^m_t + P^m_t \mid X_{t-1})\, P^j_{t-1}$$

The "periodic" systematic risks of the single cash flow X^j_2 will be:

(10) $$\beta^j_2 = cov(X^j_2, X^m_2 \mid X_1)\, P^m_1 \,/\, var(X^m_1 + P^m_1)\, P^j_1$$

(11) $$\beta^j_1 = cov(P^j_1, X^m_1 + P^m_1)\, P^m_0 \,/\, var(X^m_1 + P^m_1)\, P^j_0$$

Since

$$\begin{aligned} cov(P^j_1, X^m_1 + P^m_1) &= cov(P^j_1, X^m_1) + cov(P^j_1, P^m_1) \\ &= cov(X^j_2, X^m_1)/r_2 + [cov(X^j_2, X^m_2) - cov(X^j_2, X^m_2 \mid X_1)]/r_2, \end{aligned}$$

the systematic risk β^j_1 and β^j_2 are dependent on the degree of risk, $cov(X^j_2, X^m_1)$ and $cov(X^j_2, X^m_2|X_1)$, being resolved over the period. The higher is $cov(X^j_2, X^m_1)$, the lower is the residual covariance $(X^j_2\ X^m_2|X_1)$ and the higher is β^j_1 relative to β^j_2. It is possible that the proportion of uncertainty over period 1 will be such that $\beta^j_1 = \beta^j_2$. However, this is only likely for mature firms, i.e. the Fortune 500 but not growth firms, i.e. the INC 100. The former group tends to be old and most uncertainty may have been resolved. The latter group tends to be young and the demand for their products is not stable. The uncertainty faced by this group will be higher because it cannot be resolved within a short time period. Note that Stapleton and Subrahmanyam have pointed out that the systematic risk of a firm over a particular period is a weighted average of the individual cash flows and, other things being equal, projects with predominantly later cash flows will have lower systematic risk, because a greater amount of uncertainty resolution takes place in a given period for early flows than for later flows.

One problem with the early work is that, with the exception of Turnbull (1977), growth and leverage are not considered simultaneously. However, in the Turnbull model the leverage factor and the operating beta are independent only if the value of the firm is independent of capital structure. Under such a condition, the asset beta is a decreasing function of the growth rate in earnings. Unfortunately, the relationship between the leverage factor and growth is not explored explicitly in the model.

In a later work, Myers (1984) takes a very different view of growth. He makes a distinction between real assets which have a value independent of a firm's future investments, and real options which are opportunities to purchase real assets at sometime in the future on favorable terms. Real options can loosely be identified with a firm's growth opportunities and real assets identified with a firm's assets-in-place. Myers does not specifically derive the relationship between the proportion of a firm's value in growth opportunities versus its assets-in-place and its asset beta. However, he does suggest that since options, in general, can be expected to be more risky than the assets on which they are written, firms with a large proportion of their value in assets-in-place will have lower asset betas than firms with a large proportion of their value in growth opportunities. This lower asset beta, though, does not necessarily translate into a lower equity beta since, as Myers also shows, assets-in-place should be more heavily financed by debt than should growth opportunities. This added leverage would lead to a higher equity beta for any given asset beta. Thus, while the net effect of having a large proportion of the value of the firm in growth opportunities is indeterminate, the model does predict that leverage and growth are likely to be related.

VII. CONCLUSIONS AND SUGGESTIONS

This study examines the inter-relationship among risk-taking, agency problems, and small business financing. It is argued that small firms have more intangible assets, such as growth opportunities, than tangible assets, such as real assets. Since the former assets are riskier than the latter and are heavily influenced by agency problems, it is more difficult for small firms to obtain conventional loans from financial intermediates. Small business loan guarantees are therefore necessary for most smaller firms.

The risk and the expected losses of the guarantee, however, can be very large if the guarantee is not properly priced and the risk-taking behavior of small firms are not properly monitored. This study recommends using the option pricing theory to determine the value of the guarantee. Financial instruments, such as warrants, can also be required if small firms cannot afford to pay for such premiums. This study also shows that the premium can be determined by the systematic risk of the firms.

Since this study only provides a conceptual framework, it will be useful for further study to complete the analyses with an equilibrium model. Empirical evidence will also be useful in testing whether the approach recommended is practical or not.

FOOTNOTES

1. See Beshouri and Nigro (1994) for an excellent discussion of the relationship between agency problem and the securitization of small business loans.

2. This example is adopted from the lecture notes provided by Galen Hite and Brealey and Myers (2000,chapter 18).

3. This section draws heavily on So et. al (1995).

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