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Voting and Power in the Small Firm: Alternatives to the One-Share, One-Vote Rule

Robert Goon and John L. Teall

The one-share, one-vote rule applicable to the governance of most business firms provides for proportional voting power which differs substantially from proportional shareholdings of investors. This problem is particularly acute in small firms where several (or many) shareholders may hold significant proportions of shares. This paper reviews well-known game theoretic algorithms (weighting or vote assignment schemes) for the alignment of power with proportional shareholdings. It also provides a simple measure of the "misalignment of power from proportional shareholdings" and discusses its application in determining more equitable vote reassignment schemes.

I. INTRODUCTION AND LITERATURE REVIEW

The one-share, one-vote system of corporate governance is intended to provide a fair distribution of power among shareholders with diverse interests and expectations. However, it can be shown rather easily that the one-share, one-vote system provides a distribution of power that is significantly out of proportion to the distribution of votes among shareholders (Dubey & Shapley, 1978; Shapiro & Shapley, 1978; and Shapley & Shubik, 1954). This is particularly true for many smaller companies where each of the individual shareholders or partners may hold significant numbers of shares relative to the total number outstanding. The distribution of power among investors is particularly important in smaller companies for a number of reasons:

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1. Investors in small companies tend to maintain less diversified portfolios. With more significant proportions of their wealth at stake in a particular firm, control and risk management is of greater importance to these investors.
2. Small firms subject themselves to significant shifts in power due to their need to raise capital as they grow. Prospective shareholders in the firm will be sensitive to the possibility of being exploited by controlling shareholders. This potential for abuse may inhibit the small firm's ability to raise capital and grow. The reassignment of voting rights may be an excellent means to deal with this problem.
3. Given that smaller firms are likely to have several shareholders holding significant proportions of the firm's stock, the power level of each shareholder is likely to be of greater consequence.
4. Shareholders of small firms frequently form readily identifiable coalitions affecting the power structure of the firm.
5. Small firms differ from larger firms in that their securities tend to be less marketable and more closely held, rendering the distribution of control and minority discounts difficult to evaluate by owners and prospective purchasers.[1] Frequently, valuations are mandated and determined by court systems and tax authorities. Dant (1975) discusses the increased willingness of the court system to recognize the value of control when establishing minority discounts.

Small firms are particularly suitable for various schemes to deviate from one-share, one-vote rules. In addition to the importance of the distribution of power to small firms discussed above, it is often easy to determine how many shares are owned by an investor at a given point in time when shares are transferred and which investors are most likely to vote as blocks.

The game theory literature provides substantial information on the measurement of power (e.g, Milnor & Shapley, 1978; Owen, 1972; von Neumann & Morgenstern, 1944). These and other works have provided a foundation for the measurement and valuation of control in the business and finance literature (Rydqvist, 1987; Robinson & White, 1989). The Shapley value and its "oceanic" variations (for large firms) have been used most extensively in the financial literature (Rydqvist, 1986, 1987; Robinson & White, 1989) and there have been occasional references to the Banzhaf index (Rydqvist, 1986). Each of these papers note the discrepancy between investor shareholdings proportions and relative voting power levels. Ratner (1970) argues that the one-share, one-vote rule gives excessive power to holders of large blocks, resulting in significant misallocations of resources and redistributions of wealth. Meeker and Joy (1980) and Meeker, Joy, and Cogger (1983) in their

studies of closely held banks demonstrate the importance of voting and control in the smaller, closely held firm.

The firm is regarded here as a set of contracts (the corporate charter, bylaws, bond indentures, managerial contracts, etc.) characterizing the joint activities of and the payoffs to the contracting parties (see Alchian & Demsetz 1972; Fama, 1980; Jensen & Meckling, 1976). This contractual structure specifies a wide range of the firm's activities. However, Easterbrook and Fischel (1983) argue that it is impractical, costly, or impossible for this set of contracts to fully prespecify all of the activities which may be necessary in reaction to unknown future conditions. Thus, the importance of the voting mechanism is that it is intended to provide for "fair" reactions to varying conditions on a timely basis. Presumably, the number of votes an investor holds is a function of the value of his investment in the firm; therefore, his voting power is based on the importance of the election to him. Nonetheless, the one-share, one vote rule provides for voting power which is not proportional to shareholdings (For example, consider the obvious case of two voters, where one has 49 percent of the votes). Voting reassignment schemes such as those discussed in the next section have been applied in the political arena (e.g., New York State, certain county supervisorial boards) and may be applied to the corporate arena. The small firm, with its readily identifiable controlling factions represents an excellent arena for applications of voting reassignment schemes.

II. MEASUREMENT OF POWER

Consider the following example where a firm with 99 outstanding shares has five shareholders (i) whose shareholdings (w_i) are given as follows:

i	w_i
A	35
B	30
C	20
D	13
E	1
Total	99

Assume that a simple majority vote will determine the outcome of a simple corporate election ($\alpha = 50$) with only two possible outcomes (yes/no). What are the relative power levels of each participant in this corporate election?

Although the answer to this is a function of exactly how power is measured, it will be clear that power is not proportional to shareholdings. An investor is said to have power if his vote may be pivotal in a corporate election or if he has the potential to "swing" the result of the election. We might measure the power of a participant in an election by determining the likelihood that an investor will be pivotal or swing election results. The reader may notice immediately that shareholders A, B, and C are capable of influencing election results; shareholders D and E are not. Thus, despite their investments in the firm, shareholders (or partners) D and E have no voting power in the elections of the firm. Clearly, power is not proportional to shareholdings.

One of the earliest and simplest power indices is discussed in Shapley and Shubik (1954) and Shapiro and Shapley (1978). The oceanic variation of the Shapley Index or Value discussed later has been by far the most influential in the financial and economics literature. This index is based on an election where n voters queue to vote in any one of $n!$ equiprobable orders or permutations. The Shapley Power Index (S_i) for a particular voter determines the probability that his block of votes will be pivotal assuming that prior votes in the queue are cast unanimously, in sequence, and that his position in that sequence is random. Thus, this index determines the average marginal contribution of voter Y to any voting coalition to which he might belong. A coalition is defined here to be a subset of voters who cast identical votes. The Shapley Index for voter Y is determined as follows:

$$S_Y = \frac{1}{n} \sum_{q=1}^{n} \frac{1}{c(q)} \sum_{Y \in Q} [v(Q) - v(Q - \{Y\})] \tag{1}$$

where $n =$ the number of participants in the election;

$q =$ the number of participants in coalition Q;

$v(Q) =$ the characteristic function or maximum potential worth of coalition Q; 1 if the coalition wins and 0 if it loses

$c(Q) = \binom{n-1}{q-1}$

The maximum worth or characteristic function $v(Q)$ of a coalition Q (combination of voters voting identically) might be interpreted as the total of its members' benefits of belonging to the coalition. If $v(Q)$ or $v(Q - \{Y\})$ are limited to values of either zero or one for the purpose of measuring power, the normalized Shapley Value $S_Y/\Sigma_i S_i$ is regarded as the probability that voter Y is pivotal. In this case, a coalition has a maximum worth $v(Q)$ of one if it wins the election or zero if it loses. Using equation (1) (Results and computations are summarized in Tables 1 and 2), we find that Shapley Values for shareholders A, B and C are 1.733 and zero for shareholders D and E.

Table 1
Pivotal Voters in 120 Potential Election Outcomes

A<u>B</u>CDE	B<u>A</u>CDE	A<u>C</u>BDE	A<u>C</u>ADE	C<u>A</u>BDE	C<u>B</u>ADE
A<u>B</u>DCE	B<u>A</u>DCE	A<u>C</u>DBE	B<u>C</u>DAE	C<u>A</u>DBE	C<u>B</u>DAE
AD<u>B</u>CE	BD<u>A</u>CE	AD<u>C</u>BE	BD<u>C</u>AE	CD<u>A</u>BE	CD<u>B</u>AE
DA<u>B</u>CE	DB<u>A</u>CE	DA<u>C</u>BE	DB<u>C</u>AE	DC<u>A</u>BE	DC<u>B</u>AE
A<u>B</u>CED	B<u>A</u>CED	A<u>C</u>BED	B<u>C</u>AED	C<u>A</u>BED	C<u>B</u>AED
A<u>B</u>DEC	B<u>A</u>DEC	A<u>C</u>DEB	B<u>C</u>DEA	C<u>A</u>DEB	C<u>B</u>DEA
AD<u>B</u>EC	BD<u>A</u>EC	AD<u>C</u>EB	BD<u>C</u>EA	CD<u>A</u>EB	CD<u>B</u>EA
DA<u>B</u>EC	DB<u>A</u>EC	DA<u>C</u>EB	DB<u>C</u>EA	DC<u>A</u>EB	DC<u>B</u>EA
A<u>B</u>ECD	B<u>A</u>ECD	A<u>C</u>EBD	B<u>C</u>EAD	C<u>A</u>EBD	C<u>B</u>EAD
A<u>B</u>EDC	B<u>A</u>EDC	A<u>C</u>EDB	B<u>C</u>EDA	C<u>A</u>EDB	C<u>B</u>EDA
ADE<u>B</u>C	BDE<u>A</u>C	ADE<u>C</u>B	BDE<u>C</u>A	CDE<u>A</u>B	CDE<u>B</u>A
DAE<u>B</u>C	DBE<u>A</u>C	DAE<u>C</u>B	DBE<u>C</u>A	DCE<u>A</u>B	DCE<u>B</u>A
AE<u>B</u>CD	BE<u>A</u>CD	AE<u>C</u>BD	BE<u>C</u>AD	CE<u>A</u>BD	CE<u>B</u>AD
AE<u>B</u>DC	BE<u>A</u>DC	AE<u>C</u>DB	BE<u>C</u>DA	CE<u>A</u>DB	CE<u>B</u>DA
AED<u>B</u>C	BED<u>A</u>C	AED<u>C</u>B	BED<u>C</u>A	CED<u>A</u>B	CED<u>B</u>A
DEA<u>B</u>C	DEB<u>A</u>C	DEA<u>C</u>B	DEB<u>C</u>A	DEC<u>A</u>B	DEC<u>B</u>A
EA<u>B</u>CD	EB<u>A</u>CD	EA<u>C</u>BD	EB<u>C</u>AD	EC<u>A</u>BD	EC<u>B</u>AD
EA<u>B</u>DC	EB<u>A</u>DC	EA<u>C</u>DB	EB<u>C</u>DA	EC<u>A</u>DB	EC<u>B</u>DA
EAD<u>B</u>C	EBD<u>A</u>C	EAD<u>C</u>B	EBD<u>C</u>A	ECD<u>A</u>B	ECD<u>B</u>A
EDA<u>B</u>C	EDB<u>A</u>C	EDA<u>C</u>B	EDB<u>C</u>A	EDC<u>A</u>B	EDC<u>B</u>A

Note: The underlined shareholder is the pivotal voter for that particular permutation of voters.

Table 2
Computing Shapley Values

$S_A = 1/5 \cdot \{(1/1) \cdot 0 + (1/4) \cdot 12 + (1/6) \cdot 16 + (1/4) \cdot 12 + (1/1) \cdot 0\} = 1.733$

$S_B = 1/5 \cdot \{(1/1) \cdot 0 + (1/4) \cdot 12 + (1/6) \cdot 16 + (1/4) \cdot 12 + (1/1) \cdot 0\} = 1.733$

$S_C = 1/5 \cdot \{(1/1) \cdot 0 + (1/4) \cdot 12 + (1/6) \cdot 16 + (1/4) \cdot 12 + (1/1) \cdot 0\} = 1.733$

$S_D = 1/5 \cdot \{(1/1) \cdot 0 + (1/4) \cdot 0 + (1/6) \cdot 0 + (1/4) \cdot 0 + (1/1) \cdot 0\} = 0$

$S_E = 1/5 \cdot \{(1/1) \cdot 0 + (1/4) \cdot 0 + (1/6) \cdot 0 + (1/4) \cdot 0 + (1/1) \cdot 0\} = 0$

Notes: There are 5!=120 permutations of five voters from the example. Thus, there are (5–1)!=24 permutations where a given shareholder votes in a given slot one through five. This table computes the number of times a particular voter will be pivotal given his position in the voting queue. For example, if Voter A is third to cast his votes, he will be pivotal in 16 out of 24 potential election outcomes. He is pivotal only if either shareholder B or C (but not both) are ahead of him in slots one or two in the queue. Fifty votes out of 99 are required for a favorable majority.

Normalized values are 1/3 for each shareholder A, B and C; normalized Shapley Values are zero for Shareholders D and E. Shareholders A, B and C have equal power; shareholders D and E have no power.

Banzhaf (1965) develops a second power index based on the probability that the particular voter is a "swinger". A voter is a "swinger" if he could change the election result by changing his vote. The Banzhaf Index has the advantage in the corporate setting over the Shapley Indices in that it is based on equiprobable voting coalitions or combinations (perhaps generating many "swingers", each of whom are capable of influencing the election result) rather than equiprobable voting permutations (orderings of voters where only one voter in the "queue" can be pivotal). Thus, it is possible in a given election for more than one voter to swing election results.

In a given election, n voters may form 2^{n+1} coalitions (including the null set), half of which, or 2^n are winning coalitions (assuming $0.5n$ votes are required for a majority). The number of swings for a particular voter j, $\mu_j(v)$, equals the number of coalitions which require his participation to win. To determine a voter's relative power, one may compute the normalized Banzhaf Power Index as follows:

$$B'_Y(v) = \mu_Y(v) / \sum_{j \in N} \mu_j(v) \tag{2}$$

where $\mu_Y(v)$ = the number of coalitions that require Voter Y to win
$\mu_j(v)$ = the number of coalitions that require Voter j to win
N = the set of all voters

The Banzhaf Index permits multiple swingers in any given election outcome. If an election outcome generates multiple swingers, increments to their power indices are equally distributed. Dubey and Shapley (1978) suggest that Banzhaf indices may be revised to reflect probabilities of a given voter being a swinger:

$$B_Y(v) = \mu_Y(v) / 2^{n+1}, \tag{3}$$

where n is the number of voters and 2^{n+1} is the number of potential coalitions which may be formed. Table 3 provides an example of applying the Banzhaf Index in the small firm.

Each of the indices discussed above has the advantage, particularly in the regulatory and judicial arenas, of being "sociologically neutral" in that they do not require assumptions regarding the election preferences of any of the contestants in the election. Each voting permutation or combination is regarded as being equally likely to be realized. However, this sociological neutrality may present some disadvantages in the applied corporate setting.

Table 3
Swingers in 32 Potential Election Outcomes

Voters for	*Voters against*	*Voters for*	*Voters against*	*Voters for*	*Voters against*	*Voters for*	*Voters against*
A	BCDE	A E	BCD	AB D	C E	CDE	AB
B	A CDE	BC	A DE	AB E	CD	*ABCD	E
C	AB DE	B D	A C E	A CD	B E	*ABC E	D
D	*ABC E	B E	A CD	A C E	B D	AB DE	C
E	*ABCD	CD	AB E	A DE	BC	A CDE	B
AB	CDE	C E	AB E	BCD	A E	BCDE	A
A CD	B DE	DE	*ABC	BC E	A D	*ABCDE	
A D	BC E	*ABC	DE	B DE	A C		*ABCDE

Notes: 1. From the example.
2. Underlined voters are swingers in that potential outcome.
3. Shareholder A is a swinger in 16 of 32 potential outcomes.
4. Shareholders B and C are each swingers in 16 potential outcomes.
5. Shareholders D and E are never swingers.
6. Raw Owen power indices are simply $O_A = O_B = O_C = 2/3$, and $O_D = O_E = 0$.
7. The sum of power indices is two.
8. Normalized power indices are simply raw values divided by two.
9. Asterisks denote potential winning coalitions without swingers.
10. Note that the total number of coalitions = 64 = 26 = 2^{n+1}.

In many instances the manager, raider or other contestant for control may have known specific preferences regarding the outcome of an election. Furthermore, one or more of his competitors for control may also have indicated preferences or seem likely to form certain coalitions. These stated or implied preferences may change the corporation's balance of power significantly. Hence, the corporate charter may provide for the application of a power index which reflects contestants' preferences or likelihoods of joining particular coalitions or voting in a certain manner.

Owen (1972) develops a power index which accounts for contestants' preferences by assigning probabilities p_i to each voter i of voting for the proposal. Let N be the set of all voters in a corporate election and T be a subset of voters who might form a coalition. The characteristic function (maximum worth which is one for winning coalitions and zero for losing coalitions) for coalition T is $v(T)$. The maximum worth or characteristic function $v(T)$ of a coalition might be interpreted as the total of its members' benefits of belonging to the coalition. Owen's power index for voter Y is

simply the sum of his contributions to all coalitions, each weighted for its probability of being formed:

$$O_Y = \sum_{T \in N \setminus Y} \left\{ \prod_{i \in T} p_i \right\} \cdot \left\{ \prod_{i \notin T} [1 - p_i] [v(T \cup Y) - v(T)] \right\} \tag{4}$$

where $N \setminus Y$ is the set of all voters, excluding voter Y;
$v(T \cup Y) - v(T)$ is voter Y's contribution to coalition T;
p_i is the probability that voter i joins coalition T

If the characteristic function results in a value of one for a winning coalition and zero for a losing coalition, then O_Y might be interpreted as the probability that voter Y will be a "swinger" on a winning coalition.

If shareholders are equally likely to form any coalition ($p_i = 0.5$), Owen power indices will be the same as Banzhaf indices. The relative strength of the Owen index in measuring power in the small firm is that it allows for varying uncertainties with regard to formation of coalitions. One may predict in many firms that certain coalitions are more likely than others to form (for example, among family members). Such prespecified coalitions significantly affect the distribution of power in the firm. Thus, one uses the Owen Index when participants vary in their probabilities (p_i) in joining a given coalition of voters.

In the example given above, Shapley, Banzhaf and Owen Power Indices all have the same values. Voter shareholdings (w_i), fractional holdings (f_i) power indices (P_i; each index is the same for a given voter) and deviations

Table 4
Shareholdings, Power Indices and Squared Differences

i	w_i	f_i	P_i	s^2
A	35	0.354	0.33	0.000576
B	30	0.303	0.33	0.000729
C	20	0.202	0.33	0.016384
D	13	0.131	0	0.017161
E	1	0.010	0	0.000100
Total	99	1.00	1.00	0.034953

$\sigma^2 = S_T^2/5 = 0.0069906.$

$\sigma = 0.0836098$

Note: S_T^2 = Total of s^2.

squared $s^2 = (f_i - P_i)^2$ are given for each voter in Table 4. We compute the sum of squared errors of differences between power index values and actual shareholdings: $\sigma^2 = 0.0069906$. The square root of this value, 0.0836, measures the "error" or "misalignment of power from proportional shareholdings" in using the one-share, one-vote rule rather than a power index to assign votes. Our objective will be to determine how to minimize this error, so as to minimize the discrepancy between a voter's shareholdings and his power.

III. ESTABLISHING A MORE EQUITABLE DISTRIBUTION OF POWER

Shapley, Banzhaf, and Owen Power Indices all indicated that shareholders A, B and C share equally the power in the firm (assuming either that all permutations or combinations of shareholders are equiprobable), though their shareholdings are far from equal. Shareholder D has no power, even though his holdings are significantly larger than those of shareholder E and 65 percent as large of those of Shareholder C. In addition to being regarded as unfair, inequitable distributions of corporate power may lead to inequitable distributions of corporate cash flows and suboptimal investment and financing policy. A variety of measures may be employed to distribute power more equitably among shareholders. Included among these might be to:

1. Assign voting power among shareholders such that the power index of each is proportional to the number of shares that he holds.
2. Combine issues to be voted such that shareholders may spread their votes among issues based on their relative importance. This provides for what is typically termed a cumulative voting process.
3. Establish super-majority voting requirements for certain issues.

Cumulative voting processes are already well established in corporate charters in most states. However, this process is useful only if there is more than one issue of relatively equal importance to be determined in one election. The selection of a series of board members is an obvious example of such a situation. Many corporate issues such as proposed mergers and proposed amendments to corporate charters are not likely to be combined with other issues of comparable importance on a given election slate. Cumulative voting differs from simple majority voting only when more than one issue is to be determined by the corporate election.

Super-majority voting rules have the rather undesirable characteristics of being arbitrarily determined (e.g., why are 67 percent and 75 percent the most common super-majority thresholds?) and may confer upon minority shareholders unduly large levels of power. Furthermore, is it any more reasonable to require a 67 percent majority to pursue a given activity (e.g., to settle a given lawsuit out of court) than it is to require a 67 percent majority to not pursue that activity? This issue may be of particular importance to the small firm which is likely to make a larger number of decisions based on votes among its shareholders.

We suggest here that voter power will be more closely aligned with shareholdings in the elections of many small firms by employing a weighted voting scheme, using either Shapley, Banzhaf, or Owen Indices to weight or reassign shareholder votes. To use the Shapley value scheme, we first note that there are 120 = 5! voting permutations and pivots. We then rearrange the shareholders' 99 votes such that their proportions of the 120 pivots are as close as possible to their proportional shareholdings; that is, we reassign votes so as to minimize σ.[2] Such a vote weighting scheme is defined here to be optimal. Under most circumstances, we will not be able to eliminate all voting power discrepancies (or reduce σ to zero) because each vote reassignment affects the ability of each coalition or combination to win. However, the accuracy of our reassignments would be expected to improve if we had a larger number of voters and potential pivots to rearrange. In our example, the minimum σ is determined with a simulation based on equation (1), with numerous possible combinations of votes held among investors. However, each solution held in common the same winning combinations (coalitions

Table 5
Votes, Shapley Values (S_i) and Squared Differences

i	w_i	f_i	S_i	s^2
A	35	0.354	0.3666	0.000159
B	30	0.303	0.2833	0.000388
C	20	0.202	0.2020	0.000004
D	13	0.131	0.1166	0.000207
E	1	0.010	0.0333	0.000542
Total	99	1.00	0.9998	0.001301

$\sigma^2 = S_T^2/5 = 0.0002595.$

$\sigma = 0.016109.$

Note: S_T^2 = Total of s^2.

AB, AC, AD, BC and BDE) and the same number of pivots out of 120 for each shareholder (A: 44, B: 34, C: 24, D: 14, E: 4). One of the optimal vote weighting schemes is given in Table 5 such that shareholder A has 40.5 votes, B has 40 votes, C has 9.1 votes, D has 8.9 votes and E has 0.5 votes. If fractional votes are not possible, vote assignments can be scaled. All of the other optimal weighting schemes resulted in similar reassignments. The σ resulting from this assignment scheme is 0.0161, representing a significant improvement over the σ of 0.0836 for the one-share one-vote rule.

Unlike the number of "Shapley pivots", the number of "Banzhaf swings" is a function of the number of votes outstanding as well as the number of shareholders. Whereas the original vote assignment resulted in 32 swings, the optimal Banzhaf weighting scheme results in a total of 25 swings. Shareholder A will have nine, B will have seven, C will have five, D will have three and E will have one. The number of votes in one of these optimal schemes are given in Table 6. Again, other optimal Banzhaf weighting schemes result in similar vote reassignment levels. The σ value is determined to be 0.0178, a significant improvement over the one-share, one vote rule and approximately the same as the Shapley weighting scheme. If we assume that all shareholders are equally likely to join all coalitions, the Owen Index would result in vote reassignments identical to those of the Banzhaf weighting schemes. However, the Owen Index weighting scheme would permit the flexibility to reassign votes based on varying likelihoods of different coalition formations.

Table 6
Votes, Banzhaf Values (B_i) and Squared Differences

i	w_i	f_i	B_i	s^2
A	35	0.354	0.36	0.000036
B	30	0.303	0.28	0.000529
C	20	0.202	0.20	0.000004
D	13	0.131	0.12	0.000121
E	1	0.010	0.04	0.000900
Total	99	1.00	1.00	0.001590

$\sigma^2 = s_T^2/5 = 0.000318.$

$\sigma = 0.0178325.$

Note: S_T^2 = Total of s^2.

IV. CONCLUSIONS

It is not clear which of the three weighting schemes here is best, though minimum σ values provide some information. No voting assignment scheme may be regarded as being perfect. The Banzhaf Index has the advantage in the corporate setting over the Shapley Indices in that it is based on equiprobable voting coalitions or combinations (perhaps generating many "swingers" in a single winning coalition) rather than equiprobable voting permutations (orderings which will generate only one pivotal vote). This feature of the Banzhaf index may seem more intuitively reasonable since it is quite possible that more than one shareholder can influence results in a given election. However, Banzhaf values may behave rather oddly or bear little relationship to desirable characteristics of a power weighting scheme when the number of voters is large (see Dubey & Shapley, 1978). Nonetheless, it does seem clear from reductions in σ that reassignment of shareholder voting rights by either the Shapley, Banzhaf, or Owen weighting schemes of voting rights may provide for a better alignment of shareholdings and relative power.

NOTES

1. A minority discount is a deduction from the proportional net asset value intended to reflect the shareholder's less than proportional control in the firm. Such discounts are frequently permitted by tax authorities in determining share values in estate sales, for gift taxes, etc.
2. Of course, we could even vary the number of votes outstanding in the 99 share firm such that the we can establish a reassignment scheme such that the σ value can be set equal to zero.

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