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DRASTIC TIMES CALL FOR DRASTIC RISK MEASURES:
WHY VALUE-AT-RISK IS (STILL) A FLAWED PREVENTATIVE OF FINANCIAL CRISES AND WHAT REGULATORS CAN DO ABOUT IT

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ABSTRACT

Bank regulators recently proposed the most fundamental reforms to U.S. banking law in decades, yet the value-at-risk statistic—replete with known deficiencies—remains the basis of the capital adequacy requirement. Consequently, there exists an unresolved tension in the law: the purpose of the banking rules is to require riskier financial institutions to hold additional capital, yet the value-at-risk statistic used to make this assessment induces a perverse incentive to hold the riskiest securities. Overlaid on this framework is the wide latitude afforded to banks in designing their value-at-risk models.

This Article explores foreseeable issues with the regulatory reliance on value-at-risk. Moreover, it details specific problems associated with the design and implementation of this risk measure in the context of the capital adequacy requirement. The analysis draws on empirical data and uses advanced econometrics to engage these issues with the sophistication used at financial institutions. The Article introduces a supplemental risk measure that may mitigate the incentive for banks to hold the riskiest securities, and may allow regulators to introduce a system of capital surcharges accordingly.

"[O]perations for profit should be based not on optimism but on arithmetic."[1]

I. INTRODUCTION

In June 2012, U.S. banking agencies released three notices of proposed rulemaking and one final rule that collectively represent the most fundamental change in financial regulation since the introduction of risk-based capital requirements in 1989.[2] The new regulatory landscape will largely resemble the Basel III framework, despite some important differences.[3] The reforms will have profound implications for the future of banking practices in the U.S. and abroad, but the effectiveness of the banking rules as a safeguard against future financial crises remains an open question.[4]

At the center of the regulatory debate is the capital adequacy minimum: the appropriate type and quantity of capital that a bank must hold to be compliant with U.S. law.[5] The purpose of the capital adequacy minimum is to ensure that banks do not overextend themselves and become unacceptably vulnerable to underperforming loans and macroeconomic shocks.[6] In this way, the capital adequacy minimum is the amount of ballast necessary to ride out a financial storm.

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1 BENJAMIN GRAHAM, THE INTELLIGENT INVESTOR 523 (4th ed. 2003).

2 *See* 12 C.F.R. pt. 3 app. B (2012); Regulatory Capital Rules: Regulatory Capital, Implementation of Basel III, Minimum Regulatory Capital Ratios, Capital Adequacy, Transition Provisions, and Prompt Corrective Action, 77 Fed. Reg. 52,792, (Aug. 30, 2012) (to be codified at 12 C.F.R. pts. 324–25, 362) [hereinafter Proposed Rules].

3 *See* Proposed Rules, *supra* note 2, at 52,792. Among other reforms in connection with bank capital adequacy minimums, the Basel III framework involves modifying the definition of regulatory capital, altering the minimum capital ratio, and introducing the capital conservation buffer, the countercyclical capital buffer, and the Global Systemically Important Bank capital requirement.

4 *See generally id.*

5 *See* 12 C.F.R. pt. 3 app. B § 1(a).

6 *See generally* VIRAL V. ACHARYA ET AL., REGULATING WALL STREET 143–44 (2010).

At the same time, an excessively burdensome capital adequacy minimum will curtail lending operations and limit the funds available to corporations that require it for sustained growth.[7]

Since the publication of Basel I, U.S. regulators have embraced a risk-based approach to the capital adequacy minimum by requiring banks with riskier assets to maintain more capital in order to offset the higher statistical probability of loss.[8] In particular, existing regulations focus primarily on credit risk and market risk.[9] Credit risk is based on the possibility that the loan counterparty will default on its payment obligations,[10] and market risk is based on the possibility that trading securities will drop in value.[11]

Unfortunately, the proposed reforms leave the market risk rules materially unaltered with respect to the value-at-risk statistic—the basis of the market risk calculation—even though it systematically underestimates risk exposure and contributed to the financial crisis.[12] Even more troubling, present regulations allow banks to construct their own black box value-at-risk models with few meaningful restrictions on the underlying statistical assumptions. As a result, banks enjoy considerable discretion in the implementation choices that ultimately determine their capital adequacy minimums. Despite this latitude, there is no requirement to disclose the final algorithm used in these bespoke models. This lack of visibility seems at odds with Dodd Frank's call for "improv[ed] accountability and transparency in the financial system . . . ."[13]

This Article asks two important questions: First, how effective is value-at-risk as a capital adequacy measure for trading securities? Second, where are the points of discretion regarding the underlying statistical assumptions, and how great is their impact? This Article takes a very hands-on approach to answering these questions by using empirical data from domestic equity markets, reports from the Securities and Exchange Commission (SEC), and sophisticated econometrics where appropriate.

The organization of the Article follows the form of the questions. Part I introduces the conceptual design of value-at-risk and explains its perverse incentive for banks to overinvest in tail risk. Part I goes on to use empirical data to demonstrate the discretion that banks enjoy in making favorable statistical assumptions that influence their capital requirements. Part II spotlights the specific deficiencies of current regulations in curbing the problematic incentives that arise when banks construct their own undisclosed value-at-risk models.

Part III of the Article proposes "risktesting" as a regulatory tool intended to complement existing regulations and mitigate the unresolved incentive for banks to

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[7] *See* DOUGLAS J. ELLIOT, A PRIMER ON BANK CAPITAL 22 (2010).

[8] *See* BASLE COMM. ON BANKING SUPERVISION, INTERNATIONAL CONVERGENCE OF CAPITAL MEASUREMENT AND CAPITAL STANDARDS 8 (1988) [hereinafter BASEL I].

[9] *See* 12 C.F.R. pt. 3 app. A § 1(a) (2012); app. B § 1(a).

[10] *See* 12 C.F.R. pt. 3 app. A § 1(a)(1); BASEL I, *supra* note 8, at 9–10.

[11] 12 C.F.R. pt. 3 app. B § 2.

[12] Joseph Nocera, *Risk Mismanagement*, N.Y. TIMES MAG. (Jan. 2, 2009), http://www.nytimes.com/2009/01/04/magazine/04risk-t.html?pagewanted=all.

[13] *See* Dodd-Frank Wall Street Reform and Consumer Protection (Dodd-Frank) Act, Pub. L. No. 111–203, 124 Stat 1376 (2010).

overinvest in the riskiest securities. The analysis explains why risktesting is not vulnerable to the shortcomings of alternative approaches and then develops the various strengths of risktesting through a worked experiment that simulates the type of dataset that regulators receive from banks. The introduction of risktesting as a potential regulatory reform to address the unresolved incentive for banks to assume excessive risk concentrations is the primary contribution of the Article to the existing academic literature.

II. THE CHALLENGE OF RISK MEASUREMENT: A STATISTICAL PERSPECTIVE

During the financial crisis, many banks with substantial trading books sustained large mark-to-market losses that inched them closer to insolvency because they underestimated the market risk of the assets in their trading book.[14] As a result, prominent banks collapsed,[15] others required emergency federal funding to remain afloat,[16] and credit markets came to a grinding halt.[17] Failures in the financial sector spread to the real economy resulting in double-digit unemployment and negative economic growth. Many critics blamed value-at-risk as an inadequate risk management tool and the cause of a regulatory failure.[18]

The crisis emphasized the need for market risk banks to hold additional capital in order to protect themselves from the impact of another loss. Bank regulations require market risk banks—national banks with trading activity on their call report[19] greater than or equal to $1 billion or 10% of total assets on a consolidated worldwide basis—to hold extra capital because of the potential loss from trading activity.[20] Therefore, market risk banks have a higher capital adequacy minimum that reflects the value-at-risk of their trading securities.[21]

The concept of risk is foundational to the risk-reward tradeoff of modern portfolio theory, but it is a difficult notion to quantify. In fact, banks used a number of different statistics before value-at-risk came to predominate.[22] One measure was volatility, defined to be the standard deviation of asset returns.[23] Volatility is a sensible risk measure because it captures the dispersion of asset price movements and agrees with the intuition that a smoother, more reliable

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[14] *See, e.g.*, Mark Landler, *Losses at Deutsche Bank Reflect Depth of Credit Crisis*, N.Y. TIMES (Apr. 29, 2008), http://www.nytimes.com/2008/04/29/business/worldbusiness/29iht-bank.4.12440904.html; Louise Story, *Morgan Stanley Posts $2.36 Billion Loss*, N.Y. TIMES (Dec. 17, 2008), http://www.nytimes.com/2008/12/18/business/18morgan.html.

[15] FIN. CRISIS INQUIRY COMM'N, FINANCIAL CRISIS INQUIRY REPORT 289 (2011), *available at* http://*www.gpo.gov/fdsys/pkg/GPO-FCIC/pdf/GPO-FCIC.pdf*.

[16] *See id.* at 362–63.

[17] Markus K. Brunnermeier, Deciphering the Liquidity and Credit Crunch 2007–2008, J. ECON. PERSP. Winter 2009, at 77, 85–86, 90.

[18] *See, e.g.*, Nocera, *supra* note 12; *see also* JORGE MINA & JERRY YI XIAO, RETURN TO RISK-METRICS 73–74 (2001).

[19] The call report is the Report of Condition and Income. It is a quarterly report to the Federal Financial Institutions Examination Council, which distributes the reports to the regulatory agencies. The information includes balance sheet and income statement reports.

[20] 12 C.F.R. pt. 3 app. B § 1(b)(1)(i)–(ii) (2012).

[21] *See id.* § 4(a).

[22] *See* PHILIPPE JORION, VALUE AT RISK 113 (3d ed. 2006).

[23] *See id.* at 76.

stream of gains is less risky than a process of erratic returns. Still, volatility suffers from a directionality problem because the occurrence of one large gain has the same impact on the volatility computation as an equally large loss. In this way, volatility is not a downside risk measure.

Other common statistics are deficient at the portfolio level because they cannot aggregate across disparate asset classes.[24] These calculations use the sensitivity of an asset to a particular risk factor. These sensitivities, however, are not comparable because they are defined in different units. For instance, there is no meaningful comparison of interest rate risk of a bond with the beta risk of a stock because they measure different risk sensitivities.[25]

Value-at-risk resolves the directional issue by considering only trading losses.[26] Further, value-at-risk is capable of inputting trading securities across multiple asset classes and outputting a single measure for the entire portfolio because it computes risk in terms of dollars regardless of the financial instrument.[27] Still, there is no unique value-at-risk algorithm. Instead, value-at-risk is a statistical concept that leaves wide discretion to market risk banks in creating their models that in turn form the basis of their capital adequacy requirement.[28]

A. The Design

Market risk banks use the value-at-risk statistic because the gain or loss at the end of the trading day for a financial asset is probabilistic, meaning that the outcome may take any value along a continuum of potential returns.[29] Each potential return realization has its own probability. Even if it is impossible to determine in advance whether the asset will experience an uptick or downtick in price, it is still possible to estimate the probabilities associated with these potential returns.

In general, the observed probabilities for the potential returns form a bell-shaped curve because smaller gains and losses are more likely than larger ones.[30] Still, the particular shape of the bell curve differs depending on the risk level associated with the particular asset.[31] For example, a riskier asset may exhibit extreme fluctuations in value.[32] Therefore, the relative probability of extreme returns will be greater, the relative probability of modest returns will be lower, and the bell curve will have a flatter appearance.[33] By contrast, a more conservative

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[24] *See* CAROL ALEXANDER, VALUE-AT-RISK MODELS 5 (2008); *see also* JOHN C. HULL, OPTIONS, FUTURES, AND OTHER DERIVATIVES 435 (6th ed. 2006).

[25] *See* JORION, *supra* note 22, at 76; *see also* HULL, *supra* note 24, at 76.

[26] For this reason, value-at-risk is often called a downside risk metric. *See, e.g.*, ALEXANDER, *supra* note 24, at 9.

[27] *See* Thomas J. Linsmeier & Neil D. Pearson, *Value at Risk*, FIN. ANALYSTS J., Mar.–Apr. 2000, at 47, 48; MINA & XIAO, *supra* note 18, at 66.

[28] *See* 12 C.F.R. pt. 3, app. B § 4 (2012).

[29] *See* JORION, *supra* note 22, at 79–80.

[30] *See* JORION, *supra* note 22, at 88.

[31] *Id.*

[32] *Id.*

[33] *Id.*

asset will exhibit few extreme fluctuations and will therefore have a tighter, more peaked bell curve representing a greater probability of slight gains and losses.[34]

The value-at-risk statistic involves two steps: First, a model estimates the probability distribution for the returns of the financial portfolio by fitting an appropriate bell curve.[35] Second, the model identifies the particular return along the continuum that represents the specified percentage cutoff.[36] For example, if the bank wants to determine the 95% value-at-risk, then the model will identify the return on the continuum for which the sum of probabilities to the right of that return total 95%. If the model is accurate, then there is a 95% chance that the financial portfolio will beat the trading return that the model identified. It is equivalent to say that there is only a 5% chance that the bank will sustain trading losses worse than the 95% value-at-risk.

The design of value-at-risk hides potentially catastrophic losses because it is an ordinal statistic that simply ranks the universe of possible trading outcomes according to their magnitude and probability.[37] Ordinal statistics—such as the minimum, maximum, median, and percentiles—are useful as high-level descriptive summaries of a dataset, but are blind to outliers. Consequently, value-at-risk describes the order of potential returns, but ignores the spacing between successive returns thereby ignoring losses in the tail of the distribution.[38] For example, the 95% value-at-risk identifies the loss for which the sum of probabilities to the right of that return total 95%. The next potential return beyond this threshold could be an incrementally larger loss or a catastrophic loss, but value-at-risk is blind to the difference. For example, the value-at-risk of a portfolio could be a loss of $100 million, and the next worse potential return could be a loss of $101 million. Just as easily, however, the next worse potential return could be a loss of $200 million.

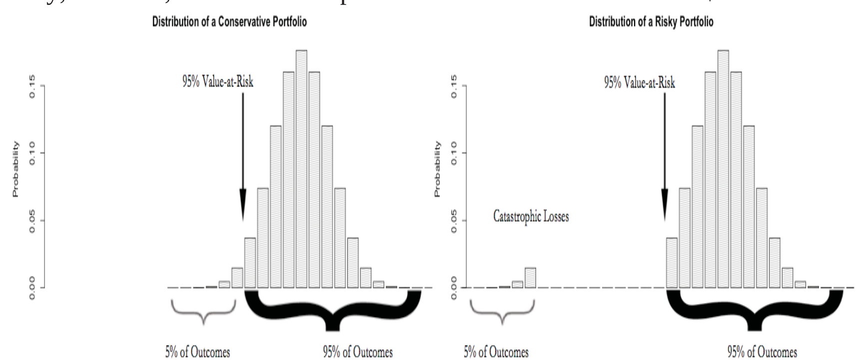

Figure 1. As an ordinal statistic, value-at-risk looks to the order–not the spacing–of potential asset returns in the probability distribution of a financial portfolio. As a result, a risky portfolio and a conservative portfolio may have the same value-at-risk if the catastrophic losses of the risky portfolio are hidden in the tail of the distribution.

In Figure 1, the 95% value-at-risk is identical for both distributions because the summation of the probability bars starting from the gains side moving leftward

[34] *Id.*

[35] *See* JORION, *supra* note 22, at 106–07.

[36] *Id.*

[37] Some academic literature uses the term quantile statistic, but the terms are interchangeable. *See generally id.* at 88–92.

[38] *See, e.g.*, HULL, *supra* note 24, at 436–37.

until reaching 95% identifies the same trading return for both distributions. The two distributions are identical up to the percentage cutoff, so the value-at-risk is the same. In this way, value-at-risk hides the catastrophic losses and fails to adjust the capital adequacy minimum of the risky portfolio.

Table 1. JPMorgan: Value-at-Risk

Primary Market Risk Category	95% Cutoff	99% Cutoff	Increase
Fixed Income	160	221	
Foreign Exchange	18	30	
Equities	47	75	
Commodities and Other	20	32	
Diversification	(91)	(131)	
Trading Value-at-Risk	154	227	47%

SEC Disclosures. Average One-Day VaR for Year Ended 12/31/09

The 2009 JPMorgan annual report demonstrates how quickly potential losses beyond the 95% threshold can mount because the value-at-risk for the 99% cutoff is almost 50% greater.[39] This comparison is useful because both value-at-risk calculations utilize the same estimation techniques, calibration period, and other model assumptions. An increase of 4% in the percentage cutoff corresponds to an almost 50% increase in potential trading losses.

Banks face a perverse incentive to overinvest in tail risk in order decrease the value-at-risk of the portfolio and lower their capital adequacy requirements.[40] If, for example, banks want to express a view of the market, they can lower the value-at-risk calculation by holding extremely risky assets because the potentially catastrophic losses are hidden deep in the tail of the distribution. Moreover, the lower reported value-at-risk makes the bank appear to possess a stronger balance sheet.

A state contingent last period problem occurs if the bank would be unable to sustain catastrophic losses, so there is no disincentive against holding additional risk related to that particular source of loss. A last period problem is state contingent if the losses arise only in a certain state of the world. In the value-at-risk context, the relevant state of the world is a loss in the particular trading security that the leads to the catastrophic losses. For example, a bank that writes deep out-of-the-money put options will face losses only if the underlying asset experiences very extreme, very unlikely losses. If a bank wrote so many options that it would become insolvent during a payout event, then there is no disincentive against writing additional options of the same kind. Therefore, banks holding the most extreme tail risk have the least disincentive not to increase that position.

B. The Implementation

In addition to a regulatory design that incentivizes suboptimal risk exposure, value-at-risk involves implementation choices about statistical assumptions that

[39] JPMorgan Chase & Co., Annual Report (Form 10-K), at 119, 121 (Feb. 24, 2010).
[40] 12 C.F.R. pt. 3 app. B, Section 5.

have important consequences for the overall capital adequacy computation.[41] Common assumptions introduce a downward bias in risk estimates, so many banks hold less capital than necessary because managers have a false sense of security in risk management practices at the bank.

The statistical challenge in quantifying market risk is complex because a properly specified model must capture the probability distribution of portfolio assets individually, collectively, and as they evolve over time. The marginal distribution of individual assets often does not match the symmetry and tails of theoretical distributions thereby introducing a downward bias in the probability estimate of extreme losses. Further, the co-movement structure of assets collectively is non-linear and therefore difficult to describe with intuitive models.[42] Finally, the marginal distribution and co-movement structure are time varying as the broader financial markets transition between periods of stability and distress.[43]

i. Three Value-at-Risk Methodologies

U.S. regulations require market risk banks to add a market risk adjustment to their capital adequacy requirements by using any of three value-at-risk methodologies: the historical, Monte Carlo, or variance-covariance.[44] Specifically, market risk banks must use a 99%, one-tailed, ten-day value-at-risk calculation based on a calibration period of at least one year.[45] Banks choose the value-at-risk methodology based on the asset classes they hold and the complexity of their hedging strategy.

The most intuitive approach for banks trading stocks, bonds, and other traditional securities is the historical methodology because it simply replicates the historical gains and losses that would have resulted from a buy-and-hold strategy of the present portfolio.[46] Moreover, the method makes no assumptions about the distribution of the assets other than assuming that past returns reflect future returns.[47] It is possible to replicate the historical returns of the portfolio by simply constructing a weighted average of the historical returns of the assets.[48] The returns of the replicated portfolio form a historical distribution that yields the value-at-risk estimate.[49]

This methodology is unsuitable for illiquid or bespoke securities such as

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[41] *See* Tanya Syblo Beder, *VAR: Seductive but Dangerous*, FIN. ANALYSTS J., Sept.–Oct. 1995, at 12, 12 (explaining that "[a] review of dozens of dealers' and end-users' VARs revealed radically different approaches to the calculation. . . . [E]ight common VAR methodologies were applied to three hypothetical portfolios . . . . [T]he magnitude of the discrepancy among these methods is shocking, with VAR results varying by more than 14 times for the same portfolio. These results illustrate the VAR's extreme dependence on parameters, data, assumptions, and methodology.").

[42] *See* JORION, *supra* note 22, at 207.

[43] *See id.* at 219.

[44] 12 C.F.R. pt. 3 app. B note 10.

[45] 12 C.F.R. pt. 3 app. B § 4 (d)(1)−(2).

[46] *See* Romain Berry, *An Overview of Value-at-Risk: Part II—Historical Simulations VaR*, J.P. MORGAN, http://www.jpmorgan.com/tss/General/Risk_Management/1159369485859 (last visited Mar. 18, 2013).

[47] *See* ALEXANDER, *supra* note 24, at 141.

[48] *See id.*

[49] *See id.*

structured products and over-the-counter derivatives because a long history of reliable prices is often unavailable, especially in thinly traded markets.[50] Many of these securities exist because of recent financial engineering and therefore lack the financial record necessary to estimate the distribution.[51] Many large banks trade these sophisticated instruments, so the historical methodology is typically appropriate only for small banks and traditional asset classes.

Banks trading highly complex financial derivatives often favor the Monte Carlo methodology because it can estimate value-at-risk even when the payoff structure of the derivative is too complicated for a direct mathematical computation.[52] The Monte Carlo methodology uses a random number generator to produce a dataset simulating the distribution of the underlying asset and then feeds those individual data points into the payoff structure of the derivative one-by-one.[53] The resulting values form a simulation of the distribution of the derivative, which is the basis for the value-at-risk calculation.[54]

The Monte Carlo methodology is practical for complex derivatives, but simulations are computationally intensive and generally less helpful for measuring risk exposure and developing hedging strategies.[55] By contrast, a methodology using equations can describe risk exposure more easily.[56] For example, the Black-Scholes equation values a call option as a function of the underlying stock price, interest rate, drift, volatility, and time to maturity.[57] Using calculus, it is possible to isolate the sensitivity of the derivative to adverse changes along any one of those risk dimensions and thereby allow banks to hedge more effectively.[58]

The last permitted approach is the variance-covariance methodology, which assumes that financial assets individually and collectively follow a particular bell-shaped distribution with certain algebraic properties that make the value-at-risk calculation straightforward.[59] The term "variance-covariance" refers to a mathematical matrix with statistical estimates for the co-movement of each asset with each other asset.[60] When the matrix is inputted in the mathematical equation, the result is a multivariate distribution describing the entire financial portfolio.[61] The equations themselves make critical assumptions regarding the distribution of the assets with important consequences for the value-at-risk estimates.

This Article examines the variance-covariance methodology because of its

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[50] *See id.*

[51] *See id.* at 145.

[52] *See* ALEXANDER, *supra* note 24, at 201; Romain Berry, *An Overview of Value-at-Risk: Part III—Monte Carlo Simulations VaR,* http://www.jpmorgan.com/tss/General/Risk_Management/1159380637650 (last visited Mar. 18, 2013); *see, e.g.*, JORION, supra note 22, at 308.

[53] *See* JORION, supra note 22, at 267.

[54] *See id.*

[55] *See id.*

[56] *See, e.g.*, HULL, *supra* note 24, at 353.

[57] *See id.* at 295.

[58] *See id.* at 241; *see also* JORION, *supra* note 22, at 255.

[59] *See* Romain Berry, *An Overview of Analytical VaR,* J.P.MORGAN, http://www.jpmorgan.com/tss/General/email/1159360877242 (last visited Mar. 18, 2013).

[60] *See* RICHARD A. JOHNSON & DEAN W. WICHERN, APPLIED MULTIVARIATE STATISTICAL ANALYSIS 68–70 (6th ed. 2007).

[61] *Id*. at 150.

prominence in the financial industry.[62]

ii. The Marginal Distribution

The market risk calculation requires assumptions regarding the marginal probability distribution, which is the movement of a single random variable such as the returns process of a financial asset.[63] This is the starting point for a more comprehensive value-at-risk model that describes an entire portfolio of multiple asset classes. The Gaussian probability distribution is the most common assumption for the movement of observed returns, but this assumption does not match the heavy tails and asymmetry of the empirical returns.[64] As a result, the Gaussian distribution underestimates market risk and biases the capital adequacy minimum to a lower value.

a. The Gaussian Distribution[65]

The variance-covariance methodology uses the Gaussian probability distribution—a mathematical equation describing a family of bell-shaped curves—to model the marginal distribution of financial assets returns.[66] The Gaussian distribution is a practical choice to describe the marginal distribution because it captures the majority of asset returns.[67] Moreover, the equation is algebraically tractable and effortlessly extends to higher mathematical dimensions, meaning that the variance-covariance methodology can describe a portfolio holding any number of financial assets.[68]

It is relatively easy to calculate value-at-risk because only two parameters, the mean and variance, are required to complete the Gaussian equation,[69] or are the parameters computationally intensive to estimate.[70] Perhaps most importantly, the natural interpretation of the Gaussian parameters comports with the intuition of traders about the average return and volatility of the asset. For instance, the mean

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[62] Nocera, *supra* note 12.

[63] RENÉ A. CARMONA, STATISTICAL ANALYSIS OF FINANCIAL DATA IN S-PLUS 63 (George Casella et al. eds., 2004).

[63] *Id.*

[64] *Id.* at 35.

[65] The Univariate Gaussian Distribution:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

[66] *See* MINA & XIAO, *supra* note 18, at 13.

[67] *See* MINA & XIAO, *supra* note 18, at 95–96.

[68] *See* JOHNSON & WICHERN, *supra* note 60, at 149.

[69] DENNIS D. WACKERLY, WILLIAM MENDENHALL III, & RICHARD L. SCHEAFFER, MATHEMATICAL STATISTICS WITH APPLICATIONS 170 (Carolyn Crockett et al. eds., 6th ed. 2002).

[70] *See, e.g.*, *id.* at 372.