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Evaluating Flexibility in Small Firm Financing*

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The choice of financing source is particularly difficult for a small firm due to the high uncertainty about future liquidity requirements. We show that the techniques of continuous time arbitrage and stochastic control theory may be used not only to value such firms but also to determine the optimal financing policies. In particular, we investigate the choice between liquid, but more expensive, forms of financing and restrictive, but cheaper, sources of capital. In addition to developing an optimal financing policy for a typical firm, we estimate the value of

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flexibility in a financing arrangement. This, in turn, provides a rational explanation for the otherwise surprisingly high levels of flexible financing used by small firms. Beyond small firm management, our findings have important implications for financial institutions and regulators.

Introduction

Small firms typically finance 50% or more of their cash requirements using flexible but very expensive lines of credit and bank overdrafts (Poutziouris et al 2000). The recent years have also seen an increase in very costly credit cards use to finance small business expenditure. For instance, the Bank of England (2001) finds that in 1999, 23.9% of businesses surveyed used credit cards in comparison to 17.1% in 1991. While convenient, the use of overdraft and credit line facilities comes at a high cost. Binks and Ennew (1997), for instance, find that the premium over base (or prime) rate charged on overdrafts and lines of credit is up to 30% higher then the premium charged on standard long-term bank loans. The premium charged on flexible instruments is higher even though such facilities tend to have substantially shorter maturities than conventional loans.

Since cheaper non-prepayable bank loans are available, such a high reliance on flexible instruments appears wasteful at a first glance. Explanations for these phenomena are rooted either in the financing availability to those firms (credit rationing and funding gaps), or in the agency frictions within them. While these are both important and relevant factors for most small firms, we take an entirely different approach. In particular, we use stochastic control theory to provide a rational explanation for the high presence of flexible financing. Our explanation does not preclude, nor is in conflict, with the financing availability or agency frictions explanations for the high use of flexible instruments.

Due to the lack of product and market diversification, small firms face a particularly high uncertainty about their future cash inflow levels and timing. Even a very successful firm may face temporary declines of sales, delayed or delinquent accounts receivable, or output price fluctuations. Again, due to lack of diversification and negotiation power, cash outflows are also highly uncertain and difficult to manage. Maintenance and repair of essential equipment, government intervention, employee turnover, or input price fluctuations, can all be particularly influential for a small firm. This heightened cash flow uncertainty leads to a substantial uncertainty about small firms’ future liquidity requirements.

The effects of liquidity uncertainty are magnified by the substantial financing constraints that small firms face. Even a sound small firm may have difficulty obtaining financing in order to meet short-term liquidity shortfalls. It is well recognized that small firms experience disadvantages in their relationship with the capital market, i.e., the existence of credit rationing (Stiglitz and Weiss, 1981) and finance gaps (Storey 1994; Mason and Harrison 1996; and Binks and Ennew, 1994, 1996, 1998, among others).

The joint workings of high uncertainty about future liquidity and financing constraints force small firms to be particularly careful in their choice of financing instruments. A highly flexible financing arrangement capable of absorbing large variations in capital requirements is of great value to a small firm. Such flexibility, however, sharply raises the overall financing costs and may appear wasteful.¹

¹ If small firms pledge account receivables, inventory, or other highly liquid assets as a collateral for the line of credit, they may be able to obtain flexible financing that is nearly as cheap as a long-term loan. In such cases, however, the flexible line of credit is still considerably more expensive on a risk-adjusted basis.
The goal of this paper is to examine the value of flexibility in financing arrangements and provide empirical estimates of this value based on the experience of a large number of small firms.

The methodology we employ is analogous to the cash management model of Miller and Orr (1966, 1968) and its numerous applications reviewed by Mullins and Homonoff (1976). Similar to Miller and Orr, we assume cash demands of the firm are stochastic and managers need to make decisions under uncertainty. The difference is in the control variable. While Miller and Orr focus on the liquid cash balance of a firm, we investigate the choice between flexible (lines of credit) and rigid (long-term loans) financing instruments. While these two control variables are very similar conceptually, investigating the financing decision does require some unique methodological choices. Furthermore, as discussed below, the literature on the choice between different financing instruments is surprisingly scarce. In fact, to the best of our knowledge, there is no theory of the demand for flexible versus rigid financing instruments.

The uncertainty about future cash requirements, and therefore our approach to modeling financing choices, is also applicable to large firms. Large firms would also like to have sufficient flexibility to meet their fluctuating cash requirements. The need for flexible financing, however, is not as strong for large firms because of two main reasons. Large firms have more diversified operations, and the uncertainty about cash flows is not as substantial. Furthermore, large firms are more likely to be able to obtain additional financing to meet unexpectedly high cash needs as long as their value remains positive. Thus, we focus our attention on small firms, although many of the conceptual conclusions are applicable to large firms as well.

Small firms are not at all a “small” portion of the economy. The UK Department of Trade and Industry (2003), estimates that there are about 3.8 million businesses operating in the UK economy at the start of 2002 and that close to 99% of these businesses have fewer than 50 employees. Together they account for more than 44% of business employment and 38% of business turnover. Total lending to this sector amounted to about £38 billion and deposits by small businesses with banks totaled £34 billion (British Bankers Association 2000). It was further reported by the BBA that lending on overdraft increased by £300 million to £10.7 billion and that term lending rose by £700 million to a record £27.1 billion in the year 2000. It is worth mentioning that the British Bankers Association further reported (based on a survey of the main British banks) that term loans have replaced overdrafts (lines of credit) as the main source of bank funding for small firms. Term loans now account for 72% of total lending, compared with 51% in 1992, and that three quarters of term lending has a residual maturity of over three years. That amounts to just over half of all lending.

Following a brief review of previous related work, we present the model and examine the optimality characteristics. We then describe the data and present our empirical findings. We conclude with a brief summary and suggestions for future research.

**Previous Work on Financing Flexibility**

The notion of financing flexibility is not new to the small firm literature. Given the existence of liquidity constraint and credit rationing, it appears that firms, especially smaller sized ones, are constantly seeking finance sources that are more flexible. Cosh and Hughes (1994) conclude that small firms for their lack of formal restrictions favor short-term sources of finance such as overdrafts (lines of credit), leasing and hire purchase. It was also observed by

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2 As described below, our data is for UK companies. Small companies play a similar role in most other developed countries.
Soufani (2002a, b) that Factoring and Invoice Financing are gaining increasing recognition as a flexible and a viable alternative for small firm financing.

In explaining small firms’ demand for flexible finance it would be appropriate to examine the position in the pecking order of financial choices. The phenomenon has been recognized for some time but formalization of the concept is attributed to Myers (1984). This hypothesis was originally developed to show the use of capital structure as a signaling device in a market characterized by asymmetric information.

Shyam-Sunder et al. (1999) conclude that the pecking order theory is a better explanatory model than a static trade-off model. Other studies such as Watson and Wilson (2002), Howorth (2001), Helwege and Liang (1996), Jordan et al. (1998), Cosh and Hughes (1994), point that firms demand for finance will be consistent with the pecking order of internal resources, short-term debt, long-term debt, and new equity.

When internally generated cash flow is less than capital expenditures, firms will utilize their cash reserves before seeking new debt and short-term debt will be preferred to long-term debt or new equity; this is because short-term debt is more flexible. It is important to characterize flexible versus rigid credit in terms of the request of financial institutions for less collateral, the ease of getting finance and the cost of finance. It is worth examining how the optimal financing policy of the firm, which encompasses flexible but costly finance, fits the pecking order theory.

Over the years a number of theories (for example, Jensen and Meckling (1976), DeAngelo and Masulis (1980)) have also been proposed to explain the variation in debt ratios across firms. However, the empirical implications of finance theory in the small business sector are seldom discussed (McConnel and Pettit 1984; Pettit and Singer 1985; Ang 1991; Michaelas et al. 1999 are exceptions). In fact, Ang (1991, 1992) points out that modern finance theory has not been developed with the small business sector in mind, and as a result there is no existing theory of finance which can adequately explain small business behavior.

Chittenden and Bragg (1997) conducted a study where they looked at cash requirements and short-term debt of small businesses in the UK, Germany, and France. They argue that because shareholders’ interests and long-term loans are a smaller percentage of a small firm’s equities and liabilities, there appears to be less scope to accommodate cash requirements by increasing equity or long-term debt. Consequently, the two main options available to small firms suffering from cash crunches are to increase short-term bank borrowing, or delay payments to trade creditors. The latter cannot be taken beyond a certain point, therefore, the expectation is for small firms to increase short-term bank borrowing.

However, there is certainly a substantial shortage of academic literature with regard to the flexibility in small firm financing and the optimal choice between short-term and long-term finance. This shortage constitutes an element in the justification for our study.

Optimal Financing Policy

We examine a small firm that can choose between a flexible and a rigid form of financing. The most common flexible instrument is a line of credit, although it can take many forms, such as invoice financing, factoring account receivables or bank overdraft. The advantage of the flexible financing is that the firm pays interest only on the funds actually used each period.

The most typical rigid form of financing is a long-term non-callable bank loan. As discussed in the introduction, rigid financing is substantially cheaper (at least on a risk-adjusted
basis), but the firm has to pay interest on the entire face value, regardless of the funds actually used.

Our goal is to estimate the value of flexibility to the small firm. This also provides an optimal financing policy for the firm, given the costs of the alternative financing forms.

Let \( C \) denote the total financing (cash) requirement of the firm. Similar to Miller and Orr (1966, 1968), this requirement is determined exogenously and is uncertain. We assume it follows a continuous stochastic process:

\[
\frac{dC}{C} = \mu dt + \sigma dz
\]

(1)

where \( \mu \) is the expected drift of the cash requirement, and \( \sigma \) is the standard deviation of the change in the cash requirement. Although this model is probably too simple to fully describe the complex dynamics of cash requirements, data considerations that will become apparent below force us to use this form as a first approximation.\(^3\)

We assume there are two types of financing instruments available to the firm: flexible line of credit, \( R \), and non-callable long-term loan, \( L \). The interest rates on those two instruments are \( r \) and \( l \) respectively. The interest rate on the line of credit is assumed to exceed the interest on the long-term loan (\( r > l \)). Even under absent interest rate risk, investors would still prefer to provide a long-term non-prepayable loan. At the very least this saves on the transaction costs of reinvesting. More importantly, a firm may need funds in difficult times. Thus, a line of credit does not receive a lot of income during good times, but is exposed to lending (and, therefore, credit risk) in difficult times. It is, therefore, not surprising that the interest rate on flexible instruments substantially exceeds that on long-term loans.

Even though we refer to the rigid loan as a long-term security, we will model both instruments as if they have long maturities. In other words, even though the line of credit is available for a long time, it is still intuitively related to short-term borrowing, not because of maturity, but rather because of its flexibility.

Some of the long-term financing is used for purchase of long-term assets, such as capital expenditure programs. For the purposes of this analysis we can exclude such investments from the total financing and focus on the financing of only day-to-day cash requirements through one of the two sources of capital.

\(^3\) A more appropriate model should include mean reversion and a time-varying expectation. Such a model might take the following form:

\[
\frac{dC}{C} = k(M(t) - C)dt + \sigma dz
\]

where

\( k \) = the speed of adjustment coefficient;

\( M(t) \) = expected financing requirement at time \( t \);

\( \sigma \) = instantaneous standard deviation of the change in the cash requirements;
If the firm obtains a loan, it pays interest on the entire face value, regardless of how much of the money is actually being used. The firm can, however, invest the unused portion of the long-term debt at the risk-free rate, \( d \), which is lower than the borrowing rate. If the firm borrows from the line of credit, the firm pays interest only on the actual cash used. Thus, the instantaneous total financing cost of the firm is:

\[
ffd t = |L|L + r \max(C - L, 0) - d \max(L - C, 0)\] dt
\]

The first term in Equation (2) captures the interest paid on the long-term loan, which is a fixed amount regardless of the portion of funds used. If the financing need exceeds the loan amount, \( L \), the firm starts to use the line of credit, and pays interest, \( r \), on any funds used in excess of the loan amount, \( L \). This additional cost of borrowing is denoted by the second term of Equation (2), which is positive if the cash requirements, \( C \), exceed the loan amount, \( L \), and zero otherwise. Finally, if a portion of the long-term loan is not used, the firm can invest the surplus at the risk-free rate, \( d \). This investment reduces the overall borrowing cost through the third term on the right-hand-side of Equation (2).

The optimal financing policy of the firm involves choosing the level of the long-term loan \( L_0(C_0, r, l) \) at time zero to minimize the expected total financing cost, \( F \):

\[
F(C_0, r, l) = \min E \left\{ fe^{-\delta t} \right\} = \min \left\{ \frac{1}{\delta} \int_0^T e^{-\delta t} dt \right\}
\]

where \( \delta \) is the discount factor used by the firm.

The assumption of the firm not being able to adjust the level of the long-term loan beyond the initial period is perhaps too strong. Most of the long-term loans have very steep prepayment penalties, so a firm is unlikely to be able to reduce the rigid financing. Firms can, however, obtain new loans at a certain origination cost. Thus, a more realistic model would allow the firm to adjust the level of rigid financing upward at a certain cost. Since our goal is to develop a theory of flexible versus rigid financing, we take the extreme view that the costs of adjusting the rigid financing are prohibitively high. Clearly, if those costs are low, rigid financing will not be very different from the flexible instruments.

As will be seen in the section describing the empirical results, the assumption that adjusting the rigid financing is prohibitively expensive will favor flexible instruments. Even with this assumption, however, the observed level of flexible financing still exceeds the model prediction for two-thirds of the firms in our sample.

We apply the Black-Scholes formula for a European call and put prices (Hull, 2000, p. 268) to derive the following expression for the total financing cost:

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4 Since we focus on small private firms, only a very small portion of the total capital requirement is financed through equity. Furthermore, unless the company goes public, the potential to raise additional equity is very limited.

5 The firm is assumed to always be able to borrow from the line of credit. Thus, we do not model bankruptcy.
\[ F(1, r, I) = \]
\[
\min_l \left\{ l \left( 1 - \frac{e^{-\delta r}}{\delta} \right) + r \int_0^T e^{-\delta t} \left[ e^{\mu t} N(d_1) - LN(d_2) \right] dt - d \int_0^T e^{-\delta t} \left[ -e^{\mu t} N(-d_1) + LN(-d_2) \right] dt \right\}, \tag{4}
\]

where

\[
d_1 = \frac{\mu t - \ln(L) + \sigma^2 i / 2}{\sigma \sqrt{t}}\]

\[
d_2 = \frac{\mu t - \ln(L) - \sigma^2 i / 2}{\sigma \sqrt{t}}\tag{5}
\]

and \( N(x) \) represents the cumulative probability density function for a normally distributed variable with mean zero and variance one.

Since the integral in expression (4) does not have a closed form solution, we use a Monte-Carlo simulation described below to obtain the optimal financing mix for each firm.

We further define “the value of flexibility” as the marginal increase in total financing cost, \( F \), per small increase in the interest rate on the flexible instrument: \( \frac{\partial F}{\partial r} \). This captures the increase in total expected financing cost due to the increased cost of the flexible financing.

Data

The main data set used in this study was gathered from the FAME Database of UK public and private companies. It is based on the audited accounts submitted to the Companies House.\(^6\)

There are over 1500 firms that satisfy the definitional and data requirements for the research. We selected small, unlisted, independent private companies with less than 200 employees (public companies were excluded as they have access to alternative forms of financing). Summary of the companies characteristics for 1998 is provided in Table 1.

The data was filtered for outliers and mistakes. In an attempt to make the database as representative of the UK small-sized company sector as possible, we selected firms from all the different sectors of the economy. No pretence is made here that the sample is an accurate representation in any ultimate sense. Nevertheless, small firms comprise a material component of the economy and their behavior has inherent importance.

The different sectors and industries in the economy are represented in the database as shown in Table 2. From Table 2 it is evident that Manufacturing and Wholesale account for almost 60% of the database selected, and that the lowest representation is in Services, Transport and Communication.

The original data set comprises the Profit and Loss accounts (income statement) and Balance Sheets for all the firms in the sample for the period covering 1989 to 1998. This gives

\(^6\) In the UK companies are required by law to submit audited accounts to the Companies House.
10 years of times series observation. Since some firms did not exist over the entire period, the data is not considered a complete panel. As will become evident below, our analysis of the optimal choice of flexible financing is performed on a firm-by-firm basis.

All the variables used in the study are based on book values as they are defined below:

- Cash Requirement = Total Liability – Cash and Cash equivalent
- Total Liability = Taken as is from the database
- Cash and cash equivalent = Investments + Trade Debtors + Bank deposits + Other Current Assets
- Flexible Finance = Short-term Loans and Overdrafts + Trade Credits
- Rigid Finance = All Other Long-Term loans + Long-Term Hire Purchase and Leasing

Our measure of cash requirements is the total liability minus the cash and cash equivalent. In a case of perfect flexibility, a firm would never hold cash and cash equivalents and pay interest on loans at the same time. Therefore, in order to make an assessment of the money required by firms from external sources such as banks and other financial institutions, the cash available is deducted from the total liability; where the latter is a figure taken as is from the data base.

We measure flexible financing as short-term loans and overdrafts and trade credits. While these categories may include some rigid forms of financing, they include all lines of credit, bank overdrafts, and trade credits, which all fit our definition of flexible financing. While more detailed breakdown of the liabilities is available cross-sectionally, the database does not contain time series of those finer categories. A descriptive analysis of the data is offered in Table 3.

To put Table 3’s figures into perspective, we report some lending information provided by the British Bankers Association (BBA). Table 4 summarizes the bank lending and borrowing activities of small firms.

Interestingly, while the total lending changes relatively little, term-loans are clearly replacing overdrafts and lines of credit. As will become evident from the empirical estimation below, we find that most firms use more than the optimal level of flexible financing. Perhaps, the trend of replacing flexible with rigid financing in the recent years suggests that firms are improving their decision-making regarding the financing choices available to them.

Table 5 provides the breakdown of loan terms for all fixed term business loans. Let us note again that these figures are representative of the UK economy, not of the sample of small firms we examine. It is interesting to note that most of the term loans have long maturities, which justifies to some extent the assumption we made earlier that the costs of adjusting the rigid component of the firm’s financing are prohibitively high.

**Empirical Findings**

We empirically estimate the stochastic process (1) driving financing requirements for each firm in our sample using data on the total cash requirement, $C$, described above. Since the process is estimated for each firm, we have as many stochastic processes as the number of firms.

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7 Trade credit can be an inexpensive source of financing if all payments are made within the discount period. However, as Peterson and Rajan (1997) point out, many companies do not pay by the discount period, in which case trade credit is a very expensive form of financing.
in the sample. Figure 1 depicts the evolution of financing requirement, $C$, for three firms in our sample. The initial cash requirement is normalized to 1.

Since the financing requirements for small firms are correlated, especially within a single industry, the conceptually correct way of estimating the stochastic processes for $C$ for each firm is to use a disturbance related set of equation technique, such as seemingly unrelated regressions, or Stein-like estimators. Due to the limited time-series observations and the large number of firms in the sample, it is difficult to estimate the residual covariance matrix. Furthermore, considering the size of the firms, disturbances of the cash requirements are largely idiosyncratic and explicit modeling of the correlation between the disturbances is unlikely to improve the estimation.

Given the estimated stochastic process for $C$, we determine the optimal initial level of rigid financing, $L_0$, for each firm as a function of the three interest rates, $r$, $l$, and $d$. The initial date is 1989. The interest rates we use throughout the following examples are 10% for the long-term loan and 15% for the flexible instruments and 2% for the risk-free rate (deposit rate). While the absolute levels of those interest rates have no impact on the conclusions, their relative magnitude matters. Interviews with loan officers and entrepreneurs suggested to us that the interest rates on flexible instruments such as lines of credits and overdrafts are typically twice the interest rate on a standard long-term loan. We note the implication of this assumption when we discuss our empirical findings below.

Given the estimated process of the cash requirement, $C$, for each firm, we use Monte-Carlo simulation to compute the total financing cost subject to some initial level of flexible financing as a percent of total liabilities. More precisely, we generate 10,000 simulation paths over 10 years given the parameters of the process for $C$. We then compute the total financing cost for each path of the simulation using the initial level of flexible financing. The average cost across all paths is an estimate of the expected total financing cost for the particular initial level of flexible financing.

Given the above procedure for estimating the total expected financing cost, we vary the financing mix to minimize this cost. The problem is well behaved and only one-dimensional, so any minimization algorithm, including a direct grid search, quickly finds the level of flexible financing that minimizes the total expected financing costs.

Model Characteristics

Before presenting the main empirical findings, let us illustrate some of the important features of the model. Figure 2 depicts the optimal rigid and flexible financing as functions of the initial capital requirements. The long-term requirement in this example is assumed constant (i.e., $\mu = 0$), and equal to one, as depicted by the flat line on the graph. The 45-degree line on the graph depicts the initial capital requirement, which is also the horizontal axis of the figure. The remaining two curves depict the optimal levels of flexible and rigid financing.

If the initial capital requirement equals the long-term capital requirement (i.e., the intersection point of the two straight lines), then it is optimal to finance 92% of the capital requirement using rigid financing. This number would go down if the difference between the cost of the two financing instruments is smaller. If the initial requirement is below the long-term

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8 The choice of 1989 is arbitrary and driven by data considerations. The underlying assumption is that at the initial date a firm has a choice between flexible and rigid financing. Once locked in, the firm cannot change the level of rigid financing. Since existing firms may have inherited inefficient levels of rigid financing from previous years, this assumption introduces additional noise in the estimation and reduces the explanatory power of the model.
equilibrium level, then an increasingly larger portion of this requirement is financed through rigid debt. If the initial capital requirement is below 90% of the long-term equilibrium, then no flexible financing is used. In fact, managers optimally borrow more through rigid financing then is currently required in anticipation of future needs.

On the other hand, if the initial capital requirement is above the long-term equilibrium, then relatively more flexible financing is used. It takes an extreme initial capital requirement of 150% of the long-term equilibrium to induce flexible financing of 40%.

**Optimal Level of Flexible Financing**

Using the above data we estimate the process driving the cash requirements of each firm given by Equation (1). Thus, we have a separate set of parameters $\mu$ and $\sigma$ for each firm. Given these parameters and our assumptions about the costs of rigid and flexible financing, we estimate the optimal level of flexible financing a firm would hold.

Figure 3 depicts the optimal and the observed proportion of flexible financing for all firms in 1989 as functions of the total cash requirement volatility over the next 10 years. The underlying assumption is that the managers know the parameters of the stochastic process that will drive the cash requirements over the following ten years. We estimate those parameters from the realizations.

The overall shape and level of the theoretically predicted level of flexible financing depicted in Figure 3 are entirely intuitive. If a manager anticipates a low volatility of the future cash flow requirements, he or she would strongly prefer the rigid financing because it is substantially less costly. On the other hand, higher expected volatility of the future cash flows induces higher use of flexible instruments, although it takes substantial volatility of 50% or higher to generate meaningful levels of flexible financing.

The observed use of flexible financing depicted by individual points on the figure, is typically far above the prediction of the model. If the capital needs volatility is below 100%, the vast majority of firms use more flexible financing than predicted by the model.

The correlation between the optimal and actual proportions of flexible financing is 25%. The coefficient estimate of optimal financing in a regression of actual on optimal is highly significant. Considering the limited data available to us to estimate the parameters of the stochastic process driving cash requirements, as well as our inability to observe any inherited financing inefficiency from operations prior to 1989, we find these results very encouraging.

**Actual versus Optimal Levels of Flexible Financing**

While very suggestive, Figure 3 and the related correlation between optimal and actual proportions of flexible financing are only a first step in investigating the financing choices of firms. Table 6 presents the results from linear regression of actual on optimal proportions of flexible financing along with all available control variables: number of employees, total sales, age of the firm, total assets, and industry. While we do not expect all of these variables to have an effect on the choice of financing instruments, we include them in the regression estimation in order to preclude as many alternative explanations as possible.

The regression results reported in Table 6 provide compelling support for the theoretical model of flexible financing developed above. The optimal level of financing has by far the largest coefficient and is highly significant.

Of the remaining control variables included in Table 6, the irrelevancy of the age of the firm presents the largest surprise. As discussed above, one alternative to our explanation for the high use of flexible instruments is that many firms do not have access to long-term loans. We
already discussed the theoretical concerns regarding this proposition. The low significance level of the age of the firm further undermines this alternative explanation. If restricted access to long-term loans had an impact on the choice between flexible and rigid financing, age of the firms would have been an important predictor of the level of flexible instruments. Older, more established firms are more likely to have access to both flexible and rigid financing. If rigid financing is preferable, but difficult to obtain, older firms would tend to use less flexible financing. Since this is not the case, it is unlikely that the observed high use of flexible instruments is driven by the supply of credit.

It is further worth noting that two measures of size of the firm – number of employees and total assets have a negative and significant impact on the use of flexible financing. At the same time, total sales have a positive and also significant impact. These findings are consistent with the possibility that some of the long-term financing is tied to long-term assets. Notice, however, that the presence of these variables does not diminish the significance level of the predicted level of flexible financing. In other words, while a number of other factors contribute to the choice of financing mix, the optimal level of flexible financing derived from our model is a strong predictor of the actual level.

It is interesting that 67% of the firms in our sample have higher than optimal level of flexible financing. Nearly all of those firms have relatively low volatility of cash requirements. There are three likely explanations for the surprisingly high flexible financing for those firms. First, entrepreneurs may overestimate the future volatility of liquidity requirements. This will lead to higher reliance on the flexible financing instruments.

Second, it is easier for firms to obtain flexible forms of financing. Invoice factoring, for instance, may be more available than long-term loans. Thus, firms resort to flexible financing not because they value flexibility, but because of credit rationing in the long-term loan market. In other words, banks ration long-term loans but more readily provide lines of credit. This would be surprising because all the credit-rationing arguments discussed above fully apply to the lines of credit as well. Furthermore, as discussed above, the empirical evidence presented in Table 6 undermines this potential explanation.

Third, our definition of flexible financing is clearly not perfect. It may potentially include short-term financing instruments that are not more expensive than long-term loans. Thus, our measure of flexible financing may be overstating its actual use. However, as discussed above, our measure of long-term financing includes leases and other loans collateralized by specific assets. For the purposes of comparing flexible to rigid financing, these loans should also be excluded. Thus, we likely overstate both the flexible and the rigid financing components.

For firms with high cash requirement volatility, our model predicts higher than observed reliance on flexible instruments. This is not surprising, since various institutional considerations force many firms to have a mix of financing instruments.

Perhaps the most striking result of the empirical estimation is that the average optimal level of flexible financing for the firms in our sample is nearly half of the total liabilities. Therefore, what appears to be irrational and wasteful reliance on flexible and expensive financing instruments turns out to be optimal for many firms.

Value of Flexibility

Our model naturally allows for analyses of the value of flexibility to small firms. We define this value as the increase of total financing costs due to 1% increase of the interest rate on
the flexible instruments. For a firm that does not use any flexible instruments, the increase of total costs will be zero, and therefore flexibility would have no value. This is not surprising considering the firm does not use any flexible financing to start with.

If a firm uses 50% flexible financing, for instance, the total expected financing cost due to a 1% point increase in $r$ would go up by close to .5% point. The increase is slightly less than .5% point because the firm will use more rigid financing and thus soften the impact of higher $r$. However, we find that this substitution effect is minimal, and the increase of financing costs due to higher $r$ closely follows the optimal level of flexible financing depicted in Figure 3. Not surprisingly, firms with higher volatility of cash requirements value flexibility more, choose the financing mix appropriately, and are more sensitive to increases in interest rates on flexible financing.

**Industry Analysis**

As Table 2 indicates, the sample firms selected from the database reasonably represent the main sectors or industries in the UK economy. The manufacturing and wholesale industries constitute the majority of firms in the sample, accounting for 26% and 32% respectively.

Evaluating whether the optimal level of financing between flexible and rigid in the context of our model is industry specific would certainly be an interesting exercise. If the examination of the industries reveal any differences, then this would be useful to suggest some business and policy implications. However, when we computed the correlation between the optimal and actual levels of financing for each industry, the results of the correlation revealed positive values, but there were not any substantial variations to report. Figure 5 provides the optimal and actual levels of flexible financing for all industries.

It is evident from Figure 4 that small firms whose cash requirements are relatively stable use above optimal levels of flexible financing, while the reverse is true for firms with volatile cash needs. Therefore, given the present evidence we cannot infer any conclusive results with regard to whether the choice of flexible financing is industry specific. However, we can certainly infer that the results pertaining to the different industries confirm and are consistent with our overall assessment of the level of optimal financing as depicted in Figure 3. Therefore, the application of the model is evident throughout the whole economy.

**Conclusion**

In this paper we examined the choice between flexible and rigid financing that small firms face. We empirically estimated the stochastic processes driving the cash requirements and provided an optimal financing policy for each firm in our sample. We showed that a high level of flexible (and costly) financing is optimal for most small firms and is dictated by the high uncertainty about future cash requirements. The average optimal level of flexible financing for our sample is nearly 50% of total liabilities. As expected, this level is higher for firms facing high volatility of cash requirements. Even with very limited data and a simple stochastic model of cash requirements, we achieved a correlation of 25% between the optimal and the actual level of flexible financing. Furthermore, the optimal level of flexible financing is an important and significant predictor of the actual level used by firms.

The empirical analysis also rejects several alternative hypotheses for the high reliance on flexible instruments. In particular, our empirical findings are inconsistent with the possibility that limited access to long-term loans dictates the high reliance on flexible financing.
We further estimated the value entrepreneurs place on financing flexibility. Not surprisingly, the owners of small firms with high volatility of cash requirements place a high value on flexibility and choose the financing mix appropriately.

Our findings have important implications for financial institutions and regulators. For instance, the government programs designed to stimulate small businesses tend to be quite rigid and static. Subsidized loans and/or loan guarantees tend to be project-specific and very difficult to adjust once undertaken. Our analysis indicates that these programs may be of substantially greater value to the small business community if they allow continuous adjustment and refinement as economic conditions and business needs change.

More importantly, both governments and private financing institutions may work on developing new products designed specifically to address the need for flexibility by small firms. Lines of credit are expensive because of the potential adverse selection problem. Therefore any institutional or market innovation that can reduce this problem will ultimately help small firms by providing them with enhanced flexibility at a lower cost.

As described above, one possible explanation for the extensive use of flexible instruments is that longer-term loans are harder to obtain. This may be dictated by lender’s incentives to avoid long-term exposure to risky investments. This effect can be greatly reduced through securitization of long-term loans to small businesses.

Finally, our work has implications for the small firms themselves. For instance, we find that many firms with very low volatility of cash requirements use a lot of flexible financing. While not necessary irrational, this behavior might indicate that small firm managers tend to be overly conservative and extremely risk-averse. Therefore, market and institutional innovations designed to alleviate the impact of this high risk-aversion through risk-sharing, for instance, can allow firms to take more aggressively the cheaper long-term fixed loans and ultimately reduce their overall financing costs.
REFERENCES


### Table 1: Major Characteristics of Companies in the Sample

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean and Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of employees</td>
<td>91.9 (60.3)</td>
</tr>
<tr>
<td>Total sales</td>
<td>2895838.9 (3104606.8)</td>
</tr>
<tr>
<td>Age of the firm</td>
<td>34.3 (19.3)</td>
</tr>
<tr>
<td>Total assets</td>
<td>1918502.1 (2302541.3)</td>
</tr>
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</table>

### Table 2: Sector and Industry Distribution

<table>
<thead>
<tr>
<th>Sector</th>
<th>Percentage</th>
</tr>
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<tbody>
<tr>
<td>Agriculture</td>
<td>7.8%</td>
</tr>
<tr>
<td>Mining</td>
<td>4.4%</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>26.2%</td>
</tr>
<tr>
<td>Construction</td>
<td>10.8%</td>
</tr>
<tr>
<td>Wholesale</td>
<td>32.2%</td>
</tr>
<tr>
<td>Retail</td>
<td>6.1%</td>
</tr>
<tr>
<td>Hotels and Restaurants</td>
<td>9.3%</td>
</tr>
<tr>
<td>Services, Transport and Communication</td>
<td>3.2%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
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</table>
Table 3: Means and Standard Deviations of Variables (£000)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Total Liabilities</td>
<td>1,253</td>
<td>1,281</td>
<td>1,265</td>
<td>1,292</td>
<td>1,379</td>
<td>1,526</td>
<td>1,647</td>
<td>1,772</td>
<td>1,923</td>
<td>1,387</td>
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<tr>
<td></td>
<td>(1,526)</td>
<td>(1,547)</td>
<td>(1,407)</td>
<td>(1,401)</td>
<td>(1,412)</td>
<td>(1,583)</td>
<td>(1,750)</td>
<td>(1,899)</td>
<td>(2,114)</td>
<td>(1,690)</td>
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<tr>
<td>Cash &amp; Equivalent</td>
<td>666</td>
<td>667</td>
<td>668</td>
<td>714</td>
<td>782</td>
<td>848</td>
<td>913</td>
<td>1,013</td>
<td>1,013</td>
<td>747</td>
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<tr>
<td></td>
<td>(866)</td>
<td>(853)</td>
<td>(795)</td>
<td>(775)</td>
<td>(803)</td>
<td>(879)</td>
<td>(945)</td>
<td>(1,001)</td>
<td>(1,201)</td>
<td>(973)</td>
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<tr>
<td>Cash Requirement</td>
<td>586</td>
<td>613</td>
<td>598</td>
<td>624</td>
<td>664</td>
<td>744</td>
<td>798</td>
<td>859</td>
<td>909</td>
<td>639</td>
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<tr>
<td></td>
<td>(1,043)</td>
<td>(1,108)</td>
<td>(1,002)</td>
<td>(1,022)</td>
<td>(992)</td>
<td>(1,079)</td>
<td>(1,205)</td>
<td>(1,289)</td>
<td>(1,335)</td>
<td>(1,092)</td>
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<tr>
<td>Flexible Finance</td>
<td>726</td>
<td>700</td>
<td>657</td>
<td>579</td>
<td>575</td>
<td>614</td>
<td>705</td>
<td>764</td>
<td>824</td>
<td>593</td>
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<tr>
<td></td>
<td>(898)</td>
<td>(840)</td>
<td>(844)</td>
<td>(694)</td>
<td>(698)</td>
<td>(759)</td>
<td>(853)</td>
<td>(944)</td>
<td>(1,025)</td>
<td>(768)</td>
</tr>
<tr>
<td>Rigid Finance</td>
<td>526</td>
<td>580</td>
<td>607</td>
<td>712</td>
<td>803</td>
<td>912</td>
<td>942</td>
<td>1,008</td>
<td>1,098</td>
<td>793</td>
</tr>
<tr>
<td></td>
<td>(880)</td>
<td>(1,026)</td>
<td>(876)</td>
<td>(986)</td>
<td>(982)</td>
<td>(1,117)</td>
<td>(1,193)</td>
<td>(1,262)</td>
<td>(1,392)</td>
<td>(1,131)</td>
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</table>

Standard deviations in parentheses.

Table 4: Bank Lending to Small Firms (amount outstanding, £bn)

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<tr>
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</thead>
<tbody>
<tr>
<td>Overdrafts</td>
<td>19.4</td>
<td>16.6</td>
<td>13.9</td>
<td>12.3</td>
<td>11.7</td>
<td>11.4</td>
<td>10.8</td>
<td>10.9</td>
<td>10.7</td>
</tr>
<tr>
<td>Term Loans</td>
<td>20.1</td>
<td>21.6</td>
<td>22.4</td>
<td>22.9</td>
<td>22.3</td>
<td>22.7</td>
<td>25.2</td>
<td>26.4</td>
<td>27.1</td>
</tr>
<tr>
<td>Total Lending</td>
<td>39.5</td>
<td>38.3</td>
<td>36.2</td>
<td>35.3</td>
<td>34.0</td>
<td>34.1</td>
<td>36.0</td>
<td>36.8</td>
<td>37.8</td>
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<tr>
<td>Total Deposits</td>
<td>22.3</td>
<td>23.5</td>
<td>24.8</td>
<td>26.1</td>
<td>27.8</td>
<td>31.6</td>
<td>31.7</td>
<td>35.2</td>
<td>34.4</td>
</tr>
</tbody>
</table>

Source: BBA 2000
Table 5. Breakdown of Term Loans (amounts outstanding, £bn)

<table>
<thead>
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<tbody>
<tr>
<td>&lt;3 years</td>
<td>5.4</td>
<td>5.6</td>
<td>6.1</td>
<td>6.2</td>
<td>6.7</td>
</tr>
<tr>
<td>3-5 years</td>
<td>2.4</td>
<td>2.6</td>
<td>3.1</td>
<td>3.2</td>
<td>3.5</td>
</tr>
<tr>
<td>5-10 years</td>
<td>6.3</td>
<td>6.1</td>
<td>7.1</td>
<td>7.3</td>
<td>7.6</td>
</tr>
<tr>
<td>10+ years</td>
<td>8.2</td>
<td>8.3</td>
<td>8.9</td>
<td>9.4</td>
<td>9.3</td>
</tr>
<tr>
<td>Total</td>
<td>22.4</td>
<td>23.5</td>
<td>24.8</td>
<td>26.4</td>
<td>27.1</td>
</tr>
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Source: BBA 2000
Table 6: Actual and Optimal Proportions of Financing

<table>
<thead>
<tr>
<th>Dependent Variable = Actual Proportion of Flexible Financing</th>
<th>Parameter Estimate</th>
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</thead>
<tbody>
<tr>
<td>Optimal Proportion of Flexible Financing</td>
<td>.14 (9.83)</td>
</tr>
<tr>
<td>Number of Employees (hundreds)</td>
<td>-.03 (-3.87)</td>
</tr>
<tr>
<td>Total Sales (Millions British Pounds)</td>
<td>.007 (2.92)</td>
</tr>
<tr>
<td>Age of the firm (years in operation)</td>
<td>.0001 (.49)</td>
</tr>
<tr>
<td>Total Assets (Millions British Pounds)</td>
<td>-.012 (-3.74)</td>
</tr>
<tr>
<td>Agriculture</td>
<td>.09 (2.02)</td>
</tr>
<tr>
<td>Mining</td>
<td>.06 (2.06)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>.05 (1.8)</td>
</tr>
<tr>
<td>Construction</td>
<td>.1 (3.55)</td>
</tr>
<tr>
<td>Wholesale</td>
<td>.017 (.49)</td>
</tr>
<tr>
<td>Retail</td>
<td>-.006 (-.1)</td>
</tr>
<tr>
<td>Hotels and Restaurants</td>
<td>-.07 (.11)</td>
</tr>
<tr>
<td>Transport and Communications</td>
<td>-.02 (-.73)</td>
</tr>
<tr>
<td>Services</td>
<td>.08 (1.5)</td>
</tr>
</tbody>
</table>

* t-statistic in parentheses, bold if significant at 1% level.
Figure 1: Examples of Evolution of Financing Requirement, $C$

The figure provides three sample evolutions of total capital requirements.
Figure 2: Optimal Flexible and Rigid Financing

The figure depicts the optimal levels of flexible and rigid financing for various levels of initial capital requirement. The long-term capital requirement is normalized to 1 and is depicted by the horizontal line in the figure. The initial capital requirement is depicted by the 45 degree line. The remaining two curves depict the optimal rigid and flexible financing levels. Other parameters are $\mu = 0$, $\sigma = .5$, $r = 15\%$, $l = 10\%$, $d = 2\%$. 

![Figure 2: Optimal Flexible vs. Rigid Financing](image)
Figure 3: Optimal vs. Actual Flexible Financing

The figure depicts the optimal proportion of flexible financing as a function of the standard deviation. It also depicts the proportion of observed flexible financing for each firm in our sample and the average proportion of observed flexible financing. The horizontal axis is the capital requirement annual volatility. All other parameters are as estimated from our data.
Figure 4: Industry Analysis