WHAT MAKES A 'GOOD' DEFINITION: ANALYZING STUDENTS' CONCEPTIONS OF MATHEMATICAL DEFINITIONS AND THE PEDAGOGY OF DEFINITIONS IN MATH EDUCATION

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Mathematical Definitions

- There is a well-documented difference between the way students view the nature of mathematical definitions and the way their professors and professional mathematicians do.

- Models have been constructed by many Mathematical Education researchers toward modeling this phenomenon.
  - Stipulated vs. Extracted Definitions (Edwards & Ward)
  - DMA Framework (Zandieh & Rasmussen)
  - Onion-Layer model (Parameswaran)

CHAIR

Definition: SIMPLE CHAIR

An object that has four legs and two planar regions (one parallel to these legs and one perpendicular to them) that allows for a person to sit upon it.
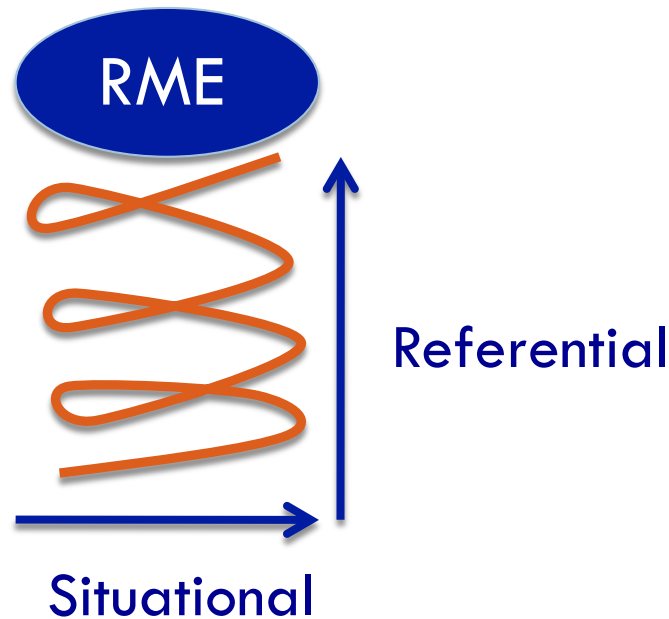
SIMPLE CHAIR

Terminology

- Extracted/lexical definition: "definitions that are based on examples of actual usage, definitions *extracted* from a body of evidence."
  - Descriptive

- Stipulated definition: "the explicit and self-conscious setting up of the meaning-relation between some word and some object, the act of assigning an object to a name."
  - Prescriptive

Barbara S. Edwards and Michael B. Ward. "Surprises from Mathematics Education Research: Student (Mis)use of Mathematical Definitions." *The American Mathematical Monthly*, Vol. 111, No. 5 (May, 2004), pp. 411-424. http://www.jstor.org/stable/4145268

Realistic Mathematics Education (RME)

Zandieh, Michelle, and Chris Rasmussen. "Defining as a Mathematical Activity: A Framework for Characterizing Progress from Informal to More Formal Ways of Reasoning." *Elsevier: The Journal of Mathematical Behavior,* n.d. Web. 11 Aug. 2013.

DMA Framework (modified)

- Defining as a Mathematical Activity

DMA	DMA*

DMA

Situational

Concept-Image → Definition

Referential

Definition → Concept-Image

DMA*

☐ Original: observed throughout multiple contexts.

☐ Modified: observed within a single context.

Zandieh, Michelle, and Chris Rasmussen. "Defining as a Mathematical Activity: A Framework for Characterizing Progress from Informal to More Formal Ways of Reasoning." *Elsevier: The Journal of Mathematical Behavior,* n.d. Web. 11 Aug. 2013.

Guided Reconstruction 2012

The definition for Sequence Convergence

Our Task

- Students are asked in their Transition to Abstract Mathematics course to reconstruct a mathematical definition with limited guidance from their professor.

- The definition in our reconstruction project is that for sequence convergence.

- Students were filmed during their reconstruction and our task is to analyze these videos for meaningful changes in students views of definitions.

"GROUP 1"

All names presented in this project are pseudonyms

Properties of a 'Good' Definition

- Mike speaks for group 1 when he posits the following
  - A "Good" Definition"
    - Includes all aspects of the concept at hand
    - Doesn't leave any parts out
    - Short, simple and sweet (CONCISION)
    - Specific enough so that it's nothing else but it can be applied to multiple things (GENERALIZES)
      - Inherently stipulated

Situational Activity

- The group exhibits *situational activity* for much of the beginning of the activity: the members focus on altering their definition to suite their example space.

  - "Something like… 'consecutively get closer' to 5…"—Laura

Situational Activity

"But *this* one goes away, it does not get closer to 5"—Tony
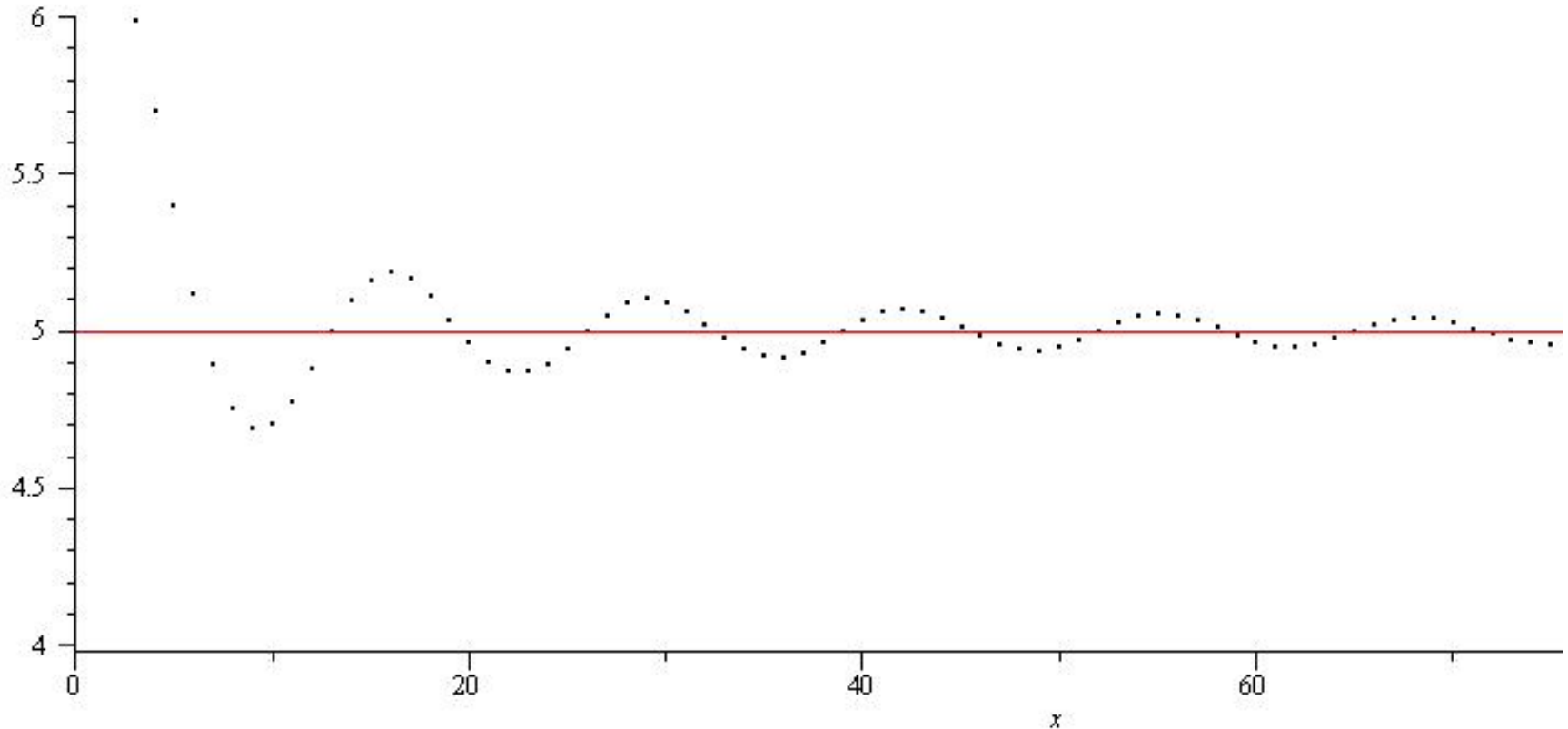
Situational Activity

- "So the sequence converges to 5 if, the last element of a_n equals 5… except cosines and sines…"—Laura

- In this moment the group attempts to alter their definition to exclude the phenomenon they intuitively agree are non-examples.
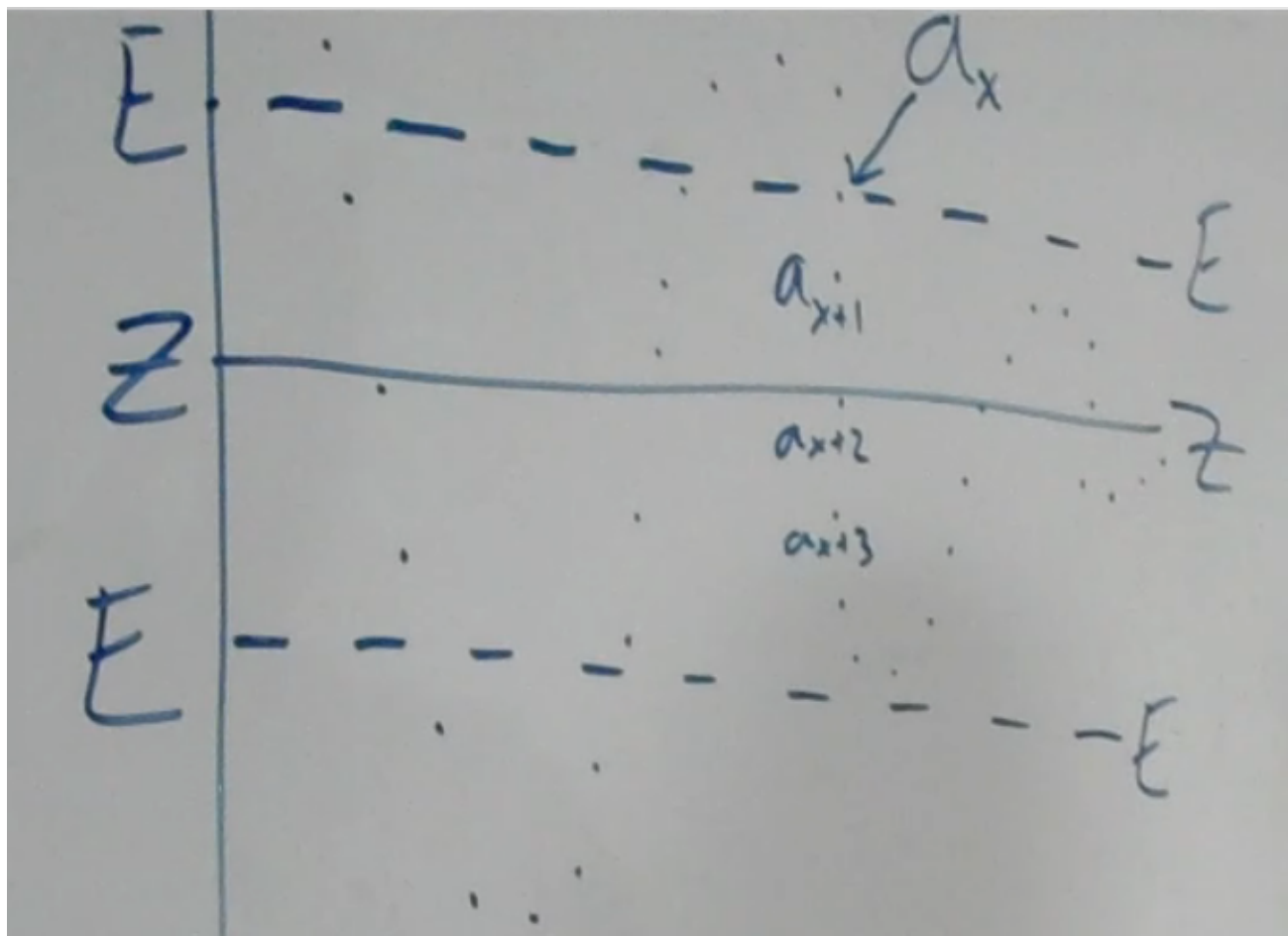
Referential Activity

□ The group demonstrates *referential activity* when they begin to alter their example space to suite their agreed upon definition.

- In particular we note how the group begins to notice structures within their definition and, right or wrong, they superimpose these structures onto their example space
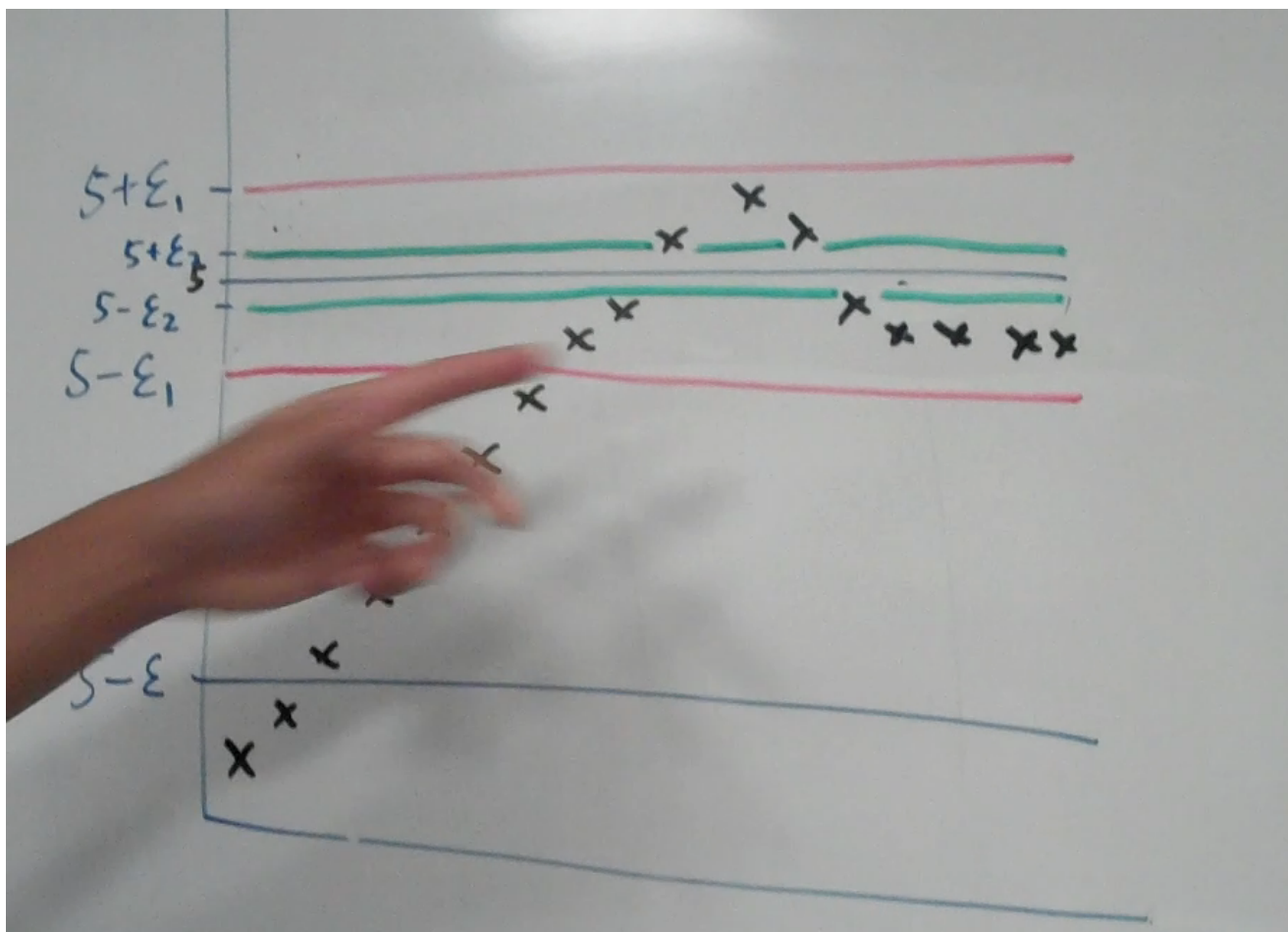
- Notion of Boundary

Referential

Definition ➙ Concept-Image

Referential Activity

Referential Activity

Conclusion

- Applying DMA* to the reconstruction process is a valid method of interpreting student's conceptions of definitions and provides insight into how learning occurs.

Situational

Concept-Image $\longrightarrow$ Definition

Referential

Definition $\longrightarrow$ Concept-Image

THE END

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