The Optimality of Multi-stage Venture Capital Financing: An Option-Theoretic Approach

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For venture capital firms, facing undiversifiable risks, multi-staged financing is an optimal contract which offers significant risk reduction at a cost of only slightly lower potential return. The optimality does not depend on the presence of moral hazard and agency problems. Our theoretical model of multi-stage financing, largely based on Asian option pricing theory, allows us to compute the risk reduction ratio due to multi-staging. The return on a staged financing plan is equivalent to an exchange of a straight equity stake for that acquired through stochastic averaging over time. We compare standard deviation ratios for staged vs. up-front financings as well as across asset classes. We find that risk mitigation due to multi-staging is significant in and of itself and enough to markedly improve venture capital's risk-reward ratios relative to alternatives.

I.   Introduction

Personal investment advisers and individual investors are familiar with the notion of dollar cost averaging. By committing to investing a constant dollar amount, as opposed to a constant number of shares, a stock or mutual fund investors buys more shares when the price drops and fewer shares when the price goes up.

What about a venture capitalist who decides to invest in a private business by repeatedly investing a certain amount of capital? Every time he comes back to invest, the business has become more valuable (product is commercially viable, the business acquires market share,

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etc.), and so for the same dollar amount of capital committed, the venture capitalist acquires fewer shares. He buys more shares when they are cheap, and fewer later when they are expensive. The cost economics of multi-stage venture financing are similar to automatic stock purchase plans of personal investors even though the motivation is quite different. The venture capitalist faces the agency-related moral hazard of private information possessed by the entrepreneur. Wang and Zhou (2004), Jones and Rhodes-Kropf (2003), and Chemmanur and Chen (2003) show that, in the presence of moral hazard, multi-stage financing is an optimal strategy for venture capital to minimize private information costs and incentives problems associated with original owners. The venture capitalist gradually overcomes the private information asymmetry by becoming an insider and by imposing management changes to protect his investment. In this context, each funding stage can be viewed as a joint acquisition of ownership shares and valuable options to acquire more shares at a later stage. Hsu (2002) uses Geske's (1979) compound option approach to value the options inherent in releasing capital in stages. The capitalist holds calls on stocks and calls on calls, i.e. rights to decide later if to acquire more shares if their value increases.

The personal finance literature focuses almost exclusively on the cost savings of the averaged strategy. The venture capital research studies almost exclusively the information asymmetry-induced sources of risks in private equity investments. In this paper, we offer a simplified view of the venture capitalist's position in order to focus, not on the cost savings of his strategy, and not on the origins of the risks in his strategy, but rather on the quantification of the risk reduction inherent in the multi-staging scheme relative to an outright investment. The multi-staging aspect of the strategy leads to a lower standard deviation of the realized return on the investment when taking into account the entry into and exit from the investment. We develop an option pricing-based model of the magnitude of the risk reduction. We relate it to the expected return in order to explicitly compute the risk-return tradeoff. This affords comparisons to other non-private investments. We show that venture capitalists are not necessarily more risk tolerant than other investors as is often thought. They engage in investments that are by nature riskier (low probability of large success). Through multi-staging, they reduce the risks of their investment returns, so that their risk-reward ratio is not very different from ordinary stock investors. The literature abounds in why venture capitalists engage in multi-stage financing. In this paper, we quantify by how much their risks declines. We compare their risk-return ratios to other investments. Very few have attempted this explicit risk quantification up to now.

Let us return to dollar cost averaging of personal finance and the very notion that the averaging reduces costs of acquiring shares and thus enhances the investor's return. But enhances relative to what?

The return is enhanced relative to a strategy in which the investor spends the amount of money equal to the sum of all of his partial investments to buy the stock at an average price over the investment horizon. This is not a fair comparison in that the strategy to buy at an average price is not executable. The stock never trades at an 'average' price, and even if it did, ex ante we would have no way of knowing when that is going to be.

Let us assume that, at the beginning of each of the next N years, we spend a constant dollar amount PMT to buy shares of a stock (or mutual fund, or index basket). By the end of the buying program, we hold the number of shares equal to:

$$\left(\frac{PMT}{S_0} + \frac{PMT}{\tilde{S}_1} + \ldots + \frac{PMT}{\tilde{S}_{N-1}} \right) \tag{1}$$

where $\tilde{S}_t$ is the price of the stock t years from today. Once we have acquired the shares we hold them till some future time $T \geq N-1$ years from today. The value of our investment at that time is:

$$\tilde{V}_T = \left(\frac{PMT}{S_0} + \frac{PMT}{\tilde{S}_1} + \ldots + \frac{PMT}{\tilde{S}_{N-1}} \right) \tilde{S}_T \tag{2}$$

Ex ante, all future stock prices, $\tilde{S}_1, \ldots, \tilde{S}_{N-1}, \tilde{S}_T$, are unknown. In the above-mentioned comparison, this investment plan's value is compared to one where we spend the dollar amount of $N \times PMT$ once to buy stock at an average price of:

$$\frac{1}{N} \sum_{t=0}^{N-1} E\left[\tilde{S}_t\right] \tag{3}$$

where $E[]$ is the expected value operator. As we stated above, the latter is not an executable strategy. A better comparison might be with a strategy where we spend the amount $N \times PMT$ all at once at the beginning to buy all the shares at today's price S_0, or at any other single day's price. An even better comparison would be to a strategy where we spend the sum of the *present values* of the amounts PMT, where the discount rate reflects the cost of borrowing funds.

The purpose of our paper is not to explore further to what extent the multi-stage financing strategy offers a greater *expected* return over some theoretical value. It is something entirely different. And that is to examine the *risk* of the multi-stage financing in quantitative terms. The underlying investment is a risky venture whose value is subject to random fluctuations. The number of ownership shares acquired by the venture capitalist and the terminal value of his investment upon exit are stochastic variables and functions of the path of the underlying enterprise value.

As is common, we will use the standard deviation of the share's (continuously compounded) returns, denoted by σ, as the risk metric. We will posit the log-normality of the share price merely for convenience and clarity of the examples. We do not need it for the analysis. The main focus of our analysis will be the risk of investment alternatives, which we will define as the standard deviation of the terminal dollar amount, $\sigma_{\tilde{V}_T}$. For the multi-stage strategy, this is the standard deviation of the total investment value as defined in (2), at liquidation. For an up-front strategy, where we invest the total present value of the amounts PMT

$$PV_0 = \sum_{t=0}^{N-1} PV_t(PMT) \tag{4}$$

to buy the shares today at price S_0, the risk of the investment will be defined as the standard deviation of the terminal amount:

$$\tilde{V}_T = \frac{PV_0}{S_0} \times \tilde{S}_T \tag{5}$$

For clarity of the numerical exposition, we choose a common benchmark against which to judge the multi-stage venture financing. The benchmark case is for the investor to borrow an amount PV_0 at an annually compounded interest rate i, and to spend that entire amount up front on buying a portfolio of common stocks available in the open market (i.e. non-private equity). The loan is assumed to be repaid in equal amounts PMT over the next N years starting now and ending $N-1$ years from now, i.e. it is an annuity due. In effect, we are taking out an annually serviced margin loan to buy stocks now. We set the total margin loan amount PV_0 equal to the present value of all the amounts used in the multi-stage private venture strategy, as in (4).

We compare: (a) borrowing the amount of money equal to PV_0 in order to buy a portfolio of common stocks up front, by agreeing to make N loan payments PMT, to (b) providing an N-stage financing of a venture investment, by spending once a year over N years the amount PMT to acquire ownership shares in the venture (equal to what the loan payment would have been). The present values of the amounts spent on either investment are identical

In what follows, we will use continuous compounding notation. That is, we will convert the annually compounded borrowing rate i to the continuously compounded equivalent rate r through the relationship $1+i=e^r$. We will also allow the possibility that the investor can borrow at one rate i_B (or equivalently r_B) but invest to earn another rate i_I (or equivalently r_I). The latter is the expected stock price appreciation.

The flow of argument in the paper is organized as follows. In Section 2, we review the relevant average option pricing models with a focus on their implications for multi-stage investments. In Section 3, we review the related Monte Carlo simulation techniques. We provide a summary simulation cookbook for path-dependent investment schemes. In Section 4, we show simulation results comparing the risk of the multi-stage private equity investment to an up-front purchase on margin of common stocks under different assumptions of borrowing/investment rates and volatility, and for different time horizons. In Section 5, we offer a conclusion.

II. Asian Option Pricing Models – Intuitions For Multi-Stage Venture Capital Investments

The main observation of the standard option pricing theory is that a call or put option always has a positive value reflecting the cost of the payoff-replicating strategy of a delta-hedger. This is true for out-of-the-money options, that is, options with no intrinsic value. There are generally three ways of valuing options in practice: (a) a closed-form solution based on the Black-Scholes hedge argument, limited to European cases, (b) a binomial or trinomial tree which approximates the European closed form through a recursive numerical induction, but can be extended to American cases and barrier conditions, or (c) a Monte Carlo simulation which approximates the risk-neutral expected value of the payoff based on a large number of simulated paths, and can handle path-dependent options, but cannot handle American exercise

cases. For the latter two to be deemed numerically accurate, they must produce the same values as the first one for all options for which closed formulae are derivable. Theoretically all three satisfy the same partial differential equation. Our numerical analysis will use (c) Monte Carlo simulation, as we will be naturally dealing with a path-dependent quantity purchased.

To gain some first intuitions about the effect of multi-staging, let us review a few closed-form approaches to average option valuation. Options whose payoff is a function of an average of the stock price over time are called Asian options. They take on two forms: options whose underlying asset is an average, but the strike is not, i.e. having a payoff of:

$$\max\left[S_{Ave} - K, 0\right] \quad or \quad \max\left[K - S_{Ave}, 0\right] \tag{6}$$

and options whose strike is an average, i.e. having a payoff of:

$$\max\left[S_T - S_{Ave}, 0\right] \quad or \quad \max\left[S_{Ave} - S_T, 0\right] \tag{7}$$

In most cases, see Hull (2000), Asian options are cheaper than standard European stock options, because the volatility of the average is always lower than the volatility of the underlying asset. Kemna and Vorst (1991) derive analytic formulae for geometric averages. Closed-form solutions are not derivable for options on arithmetic averages of log-normal variables. However, as Turnbull and Wakeman (1991) show, they can be approximated using a limiting argument. We rely on a less accurate, but more elegant and intuitive argument of Dubil and Dachille (1989). We make every attempt here to keep the math simple.

Any future outcome of the stock price at time t can be written as the sum of today's price, S_0, and non-overlapping contiguous daily increments $\Delta\tilde{S}_t$, i.e.

$$\tilde{S}_t = S_0 + \Delta\tilde{S}_1 + \ldots + \Delta\tilde{S}_t \tag{8}$$

Let us pretend that these increments are i.i.d normally distributed with a daily mean and standard deviation defined in dollars. An arithmetic average of N stock prices starting $T - N - 1$ days from today and ending T days from today can then be defined as:

$$
\begin{aligned}
S_{Ave} &= \frac{1}{N}\sum_{t=T-N-1}^{T}\tilde{S}_t \\
&= \frac{1}{N}\left\{
\begin{array}{l}
S_0 + \Delta\tilde{S}_1 + \ldots \Delta\tilde{S}_{T-N-1} \\
+ S_0 + \Delta\tilde{S}_1 + \ldots \Delta\tilde{S}_{T-N-1} + \Delta\tilde{S}_{T-N} \\
+ \ldots + \\
+ S_0 + \Delta\tilde{S}_1 + \ldots \Delta\tilde{S}_{T-N-1} + \Delta\tilde{S}_{T-N} + \ldots + \Delta\tilde{S}_T
\end{array}
\right\}
\end{aligned} \tag{9}
$$

For example, if an option has 10 days left to maturity and the strike is equal to the average of the last 5 days then the average is equal to $S_{Ave} = \frac{1}{5}\{S_6 + S_7 + S_8 + S_9 + S_{10}\}$. Each component of the average contains the daily price increments of the previous one plus one. *Ex ante*, the

average is a random variable which is a weighted sum of the daily stock price increments between time 0 and time T. The variance of the average can be derived as the variance of the sum of independent normal variables, each having a variance of 1/365 of the annual variance of the stock price. Dubil and Dachille (1989) show that the (annualized) volatility of the average, υ, is related to the (annualized) volatility of the stock price, σ, through the following formula, which is the result of summing the number of increments squared:

$$\upsilon^2 = \left[\frac{T-N}{T} + \frac{(2N+1)(N+1)}{6NT}\right]\sigma^2 \qquad (10)$$

The amount of money needed today to replicate the payoff of the average at maturity T is equal to:

$$Ave_0 = S_0 \times \frac{1}{N}\sum_{t=0}^{N-1} e^{-\frac{rt}{365}} \qquad (11)$$

This formula reflects the strategy of splitting the money into N parts, depositing each part in an account bearing continuously compounded interest r until the time of each of the investment and then buying $\frac{1}{N}$ th of the stock. Once the two variables in (10) and (11) are derived, valuing average options of type (6) boils down to substituting (11) for the stock price and the square root of (10) for the volatility into the Black-Scholes (1973) formula. Valuing average options of type (7) boils down to substituting (11) and (10)into the modified Black-Scholes formula for an option to exchange one asset (stock) for another (average) derived by Margrabe (1978).

Having established the mathematics, let us turn to intuitions. An option on the average of stock prices is less valuable than an equivalent option on a stock price observed once. This is because any fluctuation in the price will be dampened by the averaging, thus lowering the volatility of the stochastic variable underlying the payoff. A 10-day option of type (6) on the average of the last five days with a fixed strike will be less valuable than the same option on the price on Day 10. The longer the averaging relative to the option period, the greater the reduction in the option value due to the volatility dampening. In the extreme example of an old option which has already entered the averaging period, the volatility will be close to zero as the averaging will include the already revealed (and thus constant) prices of the last few days. On Day 9, our 10-day option on a 5-day average will have four of the prices already known and only one to be revealed. Even if the stock is highly volatile, the final average will be unlikely to deviate much from the average so far. The same principle operates prior to the averaging.

We can use Eq. (10) to construct a table of the ratio of the volatility of the average divided by the volatility of the stock for different averaging periods and different times to expirations, by taking the square root of the variance multiplier. The result is noted in Table I. For example, 360 days of averaging in a 720 day option results in a 28.3% reduction (1-0.817) in the volatility of the underlying, relative to a standard option. It is worth noting that the option value change will depend on the in-the-moneyness of the compared options. In most cases, the reduction in the value of the at-the-money calls will actually be greater than the volatility reduction.

Let us turn to the more instructive case of Asian call options of type (7). Their payoff greatly resembles that of a multi-stage private equity investment. We acquire the asset over time incurring the cost of S_{Ave} and exit it through an IPO or a private sale at some point (usually much) after the averaging has stopped. Our total gain is equal to

$$S_T - S_{Ave} \qquad (12)$$

We can think of the two variables in (12) as two different assets. At exit, we exchange the average asset for the market value of the ownership stake at that time. We can use the same approach as before to come up with a formula for the volatility, υ, of the *difference* of the stock at exit and the average as a function of the volatility, σ, of the ownership share itself. The two are related through the following equation:

$$\upsilon^2 = \left[\frac{(2N-1)(N-1)}{6NT} \right] \sigma^2 \qquad (13)$$

Once again, we can construct a table of the volatility ratio, this time of the difference between the average and the stock to the stock. The result is in Table II. In this case, the longer the averaging period, the higher the ratio, reflecting the fact the average will more likely be different from the stock at expiry. The averaging over one day would make the two variables equal, making the ratio and the value of the option equal to zero. The highest volatility is obtained by averaging over the full period. It is always below 60%. Note that the case of the Asian options is different from the most common case of multi-stage financing of private equity in that the averaging in options occurs at the end of the expiry period for the last N days. In the investment plan, it is more likely that a venture capitalist buys ownership shares first over the first N years, then holds it for some time, and then sells it all at once (or also gradually). This implies the averaging at the beginning rather than at the end of the investment period.

III.   Monte Carlo Simulation of Multi-Stage Equity Investments

Cox and Ross (1976) showed that a call option value is equal to its expected payoff discounted by the interest rate where the expectation is taken with respect to a special probability measure. Given this observation, an Asian call option of type (6) or (7) can be priced by evaluating the following expressions:

$$e^{-rT} E\left[\max\left(S_{Ave} - K, 0 \right) \right] \quad or \quad e^{-rT} E\left[\max\left(S_T - S_{Ave}, 0 \right) \right] \qquad (14)$$

The implication is that if we have a numerical method of evaluating the expected values, i.e. probability-weighted averages of the payoff outcomes, then all we need to do is to discount them to today. In practice, the expectation evaluation is most easily performed with the use of a Monte Carlo simulation. The special risk-neutral probability measure requires that for a log-normally distributed traded asset, with the volatility σ, subsequent asset values are generated through the following recursive formula:

$$S_n = S_{n-1} e^{(r-\frac{1}{2}\sigma^2)\Delta t_n + \sigma\sqrt{\Delta t_n}\,\tilde{z}_n} \tag{15}$$

where Δt_n is the time (in years) between the consecutive stock price observations and $\tilde{z}_n$ is a standard normal deviate generated using a random number generator. To generate the first unknown stock price tomorrow, $\tilde{S}_1$, we take today's known price S_0 and one standard normal random number $\tilde{z}_1$, and plug them into (15) to get $\tilde{S}_1 = S_0 e^{(r-\frac{1}{2}\sigma^2)\frac{1}{365}+\sigma\sqrt{\frac{1}{365}}\,\tilde{z}_1}$. Day 2's price is obtained using Day 1's price and another generated standard normal random number, $\tilde{z}_2$, plugged into (15) to get $\tilde{S}_2 = \tilde{S}_1 e^{(r-\frac{1}{2}\sigma^2)\frac{1}{365}+\sigma\sqrt{\frac{1}{365}}\,\tilde{z}_2}$. We generate prices for all days between now and the maturity of the option. We then evaluate the payoff of the option. If the option is a standard European call or put, then all we need is the last price $\tilde{S}_T$ in order to compare it to the strike. In that case, we dispense with all the intermediate prices and use only one step to generate the final price $\tilde{S}_T = S_0 e^{(r-\frac{1}{2}\sigma^2)T+\sigma\sqrt{T}\,\tilde{z}}$, with T defined as a fraction of a year. If the option is Asian, then we do need the stock prices for all the days which are to be included in the average in order to determine the realized average $\tilde{Ave}_T = \frac{1}{N}\sum \tilde{S}_t$ and the final payoff. In both cases, we repeat the path and payoff generation many times and compute the expected payoff value as a simple arithmetic average of the payoff outcomes. The number of paths generated depends on the speed of the convergence of the average to a stable value. There are many techniques to ensure fast convergence. The two most commonly used ones are antithetical variables (one generates a set of random numbers and then simply reverses their signs to generate another path – the mean is then assured to be zero) and conditioning (instead of computing payoffs directly, one computes their deviations from a numerical outcome of a known closed-form value).

Let us turn to the evaluation of a simple venture finance plan with a constant-dollar amount invested over N stages. What we will try to determine is the risk of the plan relative to a one-time purchase. That is, what we want to know is the standard deviation of the terminal value of our investment, assuming that we acquire shares periodically over N purchases and then hold the shares till the final time T. We cannot derive a closed-form solution for the variance of the strategy, but we can compute it using the same Monte Carlo technique we described for average option valuation[†].

We assume that we embark on an N-year financing plan whereby we purchase ownership shares once a year at the beginning of the year by spending a constant dollar amount PMT. At the end of the purchase program, $N-1$ years from now, we hold the number of shares given by Eq. (1). We use Eq. (15) to generate the realized stock prices based on today's starting value. For instance, the price in Year 5 is generated from the price in Year 4 through

$$\tilde{S}_5 = \tilde{S}_4 e^{(r-\frac{1}{2}\sigma^2)\cdot 1 + \sigma\sqrt{1}\,\tilde{z}_5}.$$

[†] Technically, the variance will be computed for the risk-neutral case whose mean may be different from the true mean.

For example, in a 3-stage plan with $PMT = \$10,000,000$ we buy $10,000,000 worth of shares up-front in Year 0, $10,000,000 worth of shares in Year 1, and $10,000,000 worth of shares in Year 2. The last purchase is exactly two years hence. We incur the randomly generated share prices $S_0, \tilde{S}_1, \tilde{S}_2$. We reap the benefits of averaging over 3 purchases, but, in advance, we do not know the average price of acquiring a share or the total number of shares we hold right after the last purchase in Year 2.

We hold the acquired shares from the time of the last purchase in $N-1$ years for $T-N+1$ years until we liquidate the shares in Year T for the price $\tilde{S}_T$ generated again using the correct time interval in Eq. (15). For example, if we plan to liquidate the shares in Year 5, then that means that we hold the acquired shares for additional 3 years after the last purchase in Year 2. We compute the terminal value of the investment plan in Year T using Eq. (2). We repeat the path generation thousands of times until convergence.

IV.   The Reduced Risk of Multi-Stage Venture Financing

We start by describing the investment choices and base case risk and return numbers. Let us consider an equity investor who is able to obtain $PV_0 = \$100,000,000$ in funding. If he is to invest in publicly traded common stocks, his funding comes from an amortizing margin loan with $N = 3$ annual payments starting immediately (annuity due). The borrowing rate is $i_B = 6\%$. The annual payment is thus $PMT = \$35,293,379$. He buys stocks at time 0 with an annual expected return of $i_I = 6\%$, 9% or 12%, and the annualized volatility of $\sigma = 0\%$, 10%, 30%, 50% or 80%. For comparison, the volatility of large cap stocks ranges between 10% and 30% and the volatility of most small caps and technology stocks ranges between 30% and 100%. (We show standard deviations for S&P500 and NASDAQ indices, not individual stocks, in Table III). If the equity investor is a venture capitalist, he does not invest all up-front. Instead every year for $N = 3$ years, starting now, he spends the amount $PMT = \$35,293,379$ to acquire equity ownership shares in a venture. He buys shares in Years 0, 1, 2. In either case, the investor holds the stocks to the withdrawal/exit time $T = 3$ or 5 years.

Tables IV-VI show the expected value and the standard deviation of the investment value $\tilde{V}_T$ at the time of exit for both the up-front investment and the three-stage financing plan assuming that the amount invested at each stage is constant. The tables also show the ratio of the volatilities for the two alternatives. In Table IV, the cost of raising capital and the expected return on investment are the same and equal to 6%. In Table V, the cost of raising capital is 6%, but the expected return on investment is 9%. In Table VI, the return on investment is 12%. The tables contain some striking results.

First, let us consider the issue of one-time financing vs. multi-staging. In all three tables, the standard deviation ratio of the multi-stage scheme to an up-front strategy declines as the volatility of investment increases. For example, in Table VI, it declines from 1.000 to 0.662 as the volatility increases from 0% to 80%. The risk reduction obtained through multi-staging is greatest for the riskiest investments. Any agent with positive risk aversion is more likely to engage in staged investments when investing in new and risky ventures rather than sure bets. This result is irrespective of the expected rate of return on investment (all three tables show the same trend). The risk reduction is slightly greater for ventures with higher expected returns, and we expect private equity to have a higher proportion of growth opportunities in the venture's capital budget portfolio. The volatility decline result is also independent of the exit time, but

the risk reduction is more pronounced for quicker exit horizons (left panels show greater relative reductions than right panels). This is intuitive. A three-stage financier has much lower risk if the exit is scheduled soon after the last financing round rather than in a more distant future.

This finding can also be used to explain the rush of venture capitalists to exits. While the primary motivation for that may be the desire to monetize their accumulated capital gains, the secondary motive may be simply to follow a rational risk reduction strategy in which the exit is planned to follow closely the last financing stage.

Next, let us turn to the comparison of venture capital investment to a listed common stock investment. The first is characterized by a higher standard deviation of the terminal investment value, and presumably a higher expected value (return). The latter is less risky and has a lower expected return. Suppose we look at a five-year investment horizon and we want to attain roughly the same Sharpe ratio from our investments. The common stock investor can be exemplified by the upper right-hand portion of Table IV (or V), say with volatility of 10-30% (compare to Table III). His ratio of return to risk is fairly high, e.g. with $\sigma = 30\%$ and $i_t = 9\%$, it is equal to 153,618,491/ 116,806,861 =1.32. The only way the venture capitalist facing $\sigma = 80\%$ and $i_t = 12\%$ can try to achieve that ratio is by investing in stages. If he invests all up-front, his Sharpe ratio will be 179,159,221 /792,293,731 =0.27, but if he engages is three-staging, he will improve it to 170,155,316 /514,617,120 =0.33. If we believe our assumed expected return and volatility numbers, we can clearly see the extremely risky nature of venture capital investment, and the need for risk reduction methods, like multi-staging or uncorrelated diversification. If we performed the same calculations for the shorter three-year horizon, we would see that while the listed stock investor enjoys the Sharpe ratio of 1.80, the venture capitalist can attempt to increase his from 0.38 to 0.66 by choosing multi-staging. In essence, we might argue that he has no choice but to do it if his reward-to-risk ratio for an individual investment is to look even somewhat attractive relative to listed equity choices.

What emerges from our results as a side outcome is that uncorrelated diversification must be one of the main reasons for investors to consider private equity. The unattractive nature of Sharpe ratios, even if improved through multi-staging, cannot be the first and foremost draw of private equity, unless expected returns can be claimed to be about six times those for listed stocks.

V.   Conclusion

We have developed a simulation methodology for evaluating the risk of a multi-stage venture financing strategy based largely on the Asian option pricing theory. We find that multi-staging may be an optimal risk reducing policy independent of the moral hazard issues discussed in the literature. We show that, irrespective of any agency issues, multi-staging reduces the volatility of the terminal value of the investment. This is particularly so for high risk-high return investments. Multi-staging offers volatility reduction relative to an all-or-nothing plan as well as relative to alternative lower risk-lower return asset classes (listed stocks). We argue that multi-staging may be the only way the Sharpe ratios of venture capital can be made to look attractive relative to other investments.

REFERENCES

Black, F. and M. Scholes, 1973, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81, 637-659.

Chemmanur, Thomas J. and Zhaohui Chen, 2003, Angels, Venture Capitalists and Entrepreneurs: A Dynamic Model of Private equity Investing, Working Paper, AFA Washington, DC, meetings.

Dubil, Robert and Douglas A. Dachille, 1989, The Valuation of Options on Averages, J.P. Morgan Securities, Inc. Working Paper.

Geske, R., 1979, The Valuation of Compound Options, *Journal of Financial Economics*, 7, 63-81.

Hull, John C., 2000, Options, Futures, and Other Derivatives, Fourth Edition, Prentice Hall.

Hsu, Yaowen, 2002, Staging of Venture Capital Investment: A Real Options Analysis, Working Paper, EFMA London meetings.

Jones, Charles and Matthew Rhodes-Kropf, 2003, The Price of Diversifiable Risk in Venture Capital and Private Equity, Working Paper, AFA Washington, DC, meetings.

Kemna, A. and A. Vorst, 1990, A Pricing Method for Options Based on Average Asset Values, *Journal of Banking and Finance*, 14, 113-29.

Margrabe, W., 1978, The Value of Option to Exchange One Asset for Another, *Journal of Finance*, 33, 177-86.

Rubinstein, M., 1991, One for Another, RISK magazine (July-August), 30-32.

Turnbull, S. M. and L. M. Wakeman, 1991, A Quick Algorithm for Pricing European Average Options. *Journal of Financial and Quantitative Analysis*, 26, 377-89.

Wang, Susheng and Hailan F. Zhou, 2004, Staged Financing in Venture Capital: Moral Hazard and Risks. *Journal of Corporate Finance,* 10 (1), 131-155.

Table I

The ratio of the volatility of the average to the volatility of the underlying stock for different averaging periods and times to maturity.

Averaging Days	Time to maturity						
	10	30	90	180	360	720	1800
5	0.849	0.952	0.984	0.992	0.996	0.998	0.999
10	0.620	0.892	0.965	0.983	0.991	0.996	0.998
30		0.592	0.885	0.944	0.973	0.986	0.995
90			0.582	0.818	0.914	0.958	0.983
180				0.580	0.817	0.913	0.966
360					0.579	0.817	0.931
720						0.578	0.857
1800							0.578

Table II

The ratio of the volatility of the difference between the average and the stock to the volatility of the underlying stock for different averaging periods and times to maturity

Averaging days	Time to maturity						
	10	30	90	180	360	720	1800
5	0.346	0.200	0.115	0.082	0.058	0.041	0.026
10	0.534	0.308	0.178	0.126	0.089	0.063	0.040
30		0.563	0.325	0.230	0.162	0.115	0.073
90			0.573	0.405	0.286	0.202	0.128
180				0.575	0.407	0.287	0.182
360					0.576	0.407	0.258
720						0.577	0.365
1800							0.577

Table III

**Average annual returns and annualized
monthly standard deviations of returns.**

	S&P 500 Index		NASDAQ	
	Return	St. Dev.	Return	St. Dev.
1970-1974	-4.11	15.69		
1975-1979	10.51	14.24		
1980-1984	9.90	14.79		
1985-1989	16.53	16.61	13.65	18.12
1990-1994	5.95	11.58	13.21	16.30
1995-1999	26.32	13.30	41.90	21.71
2000-2001	-11.59	18.48	-30.17	45.17

Table IV

**Expected Investment Value, St. Deviation and St. Deviation
Ratio for Up-front and 3-Stage Venture Financing**

Cost of capital=6%, Expected return=6%.   PV up-front=$100,000,000. Periodic investment of $35,293,379.

FV(amt)	Periodic 119,101,600	Up-front 119,101,600	St. Dev Ratio	FV(amt)	Periodic 133,822,558	Up-front 133,822,558	St. Dev Ratio
Exit Year 3				Exit Year 5			
Vol=0 *Exp*	119,101,600	119,101,600		Vol=0 *Exp*	133,822,558	133,822,558	
St. Dev	56	55		*St. Dev*	-	-	
Vol=10 *Exp*	119,090,670	119,071,162		Vol=10 *Exp*	133,810,299	133,781,979	
St. Dev	15,057,832	20,627,339	0.730	*St. Dev*	25,618,918	30,177,240	0.849
Vol=30 *Exp*	119,025,939	118,875,160		Vol=30 *Exp*	133,827,339	133,610,421	
St. Dev	47,164,226	66,221,956	0.712	*St. Dev*	83,881,852	101,593,329	0.826
Vol=50 *Exp*	118,979,666	118,675,952		Vol=50 *Exp*	134,209,111	133,794,704	
St. Dev	86,117,000	128,109,305	0.672	*St. Dev*	167,296,470	216,300,061	0.773
Vol=80 *Exp*	119,255,363	119,203,098		Vol=80 *Exp*	136,103,364	136,043,682	
St. Dev	177,930,304	310,599,000	0.573	*St. Dev*	406,286,996	601,624,388	0.675

Table V

**Expected Investment Value, St. Deviation and St. Deviation
Ratio for Up-front and 3-Stage Venture Financing**

Cost of capital=6%, Expected return=9%.   PV up-front=$100,000,000. Periodic investment of $35,293,379.

FV(amt)	Periodic 126,107,794	Up-front 129,502,900	St. Dev Ratio	FV(amt)	Periodic 149,828,671	Up-front 153,862,395	St. Dev Ratio
Exit Year 3				Exit Year 5			
Vol=0 *Exp*	126,107,794	129,502,900		Vol=0 *Exp*	149,828,671	153,862,395	
St. Dev	58	-		*St. Dev*	58	91	
Vol=10 *Exp*	126,095,908	129,469,803		Vol=10 *Exp*	149,814,462	153,815,740	
St. Dev	16,038,893	22,428,752	0.715	*St. Dev*	28,758,902	34,696,261	0.829
Vol=30 *Exp*	126,025,175	129,256,684		Vol=30 *Exp*	149,830,193	153,618,491	
St. Dev	50,260,105	72,005,207	0.698	*St. Dev*	94,204,018	116,806,861	0.806
Vol=50 *Exp*	125,973,278	129,040,080		Vol=50 *Exp*	150,253,500	153,830,371	
St. Dev	91,871,959	139,297,260	0.660	*St. Dev*	188,084,560	248,690,849	0.756
Vol=80 *Exp*	126,266,795	129,613,262		Vol=80 *Exp*	152,375,456	156,416,132	
St. Dev	190,475,464	337,724,020	0.564	*St. Dev*	457,875,688	691,717,234	0.662

Table VI

**Expected Investment Value, St. Deviation and
St. Deviation Ratio for Up-front and 3-Stage Venture Financing**

Cost of capital=6%, Expected return=12%.   PV up-front=$100,000,000. Periodic investment of $35,293,379.

FV(amt)	Periodic 133,385,254	Up-front 140,492,800	St. Dev Ratio	FV(amt)	Periodic 167,318,462	Up-front 176,234,168	St. Dev Ratio
Exit Year 3				Exit Year 5			
Vol=0 *Exp*	133,385,254	140,492,800		Vol=0 *Exp*	167,318,462	176,234,168	
St. Dev	25	-		*St. Dev*	-	-	
Vol=10 *Exp*	133,372,358	140,456,895		Vol=10 *Exp*	167,302,068	176,180,729	
St. Dev	17,063,143	24,332,105	0.701	*St. Dev*	32,199,126	39,741,138	0.810
Vol=30 *Exp*	133,295,275	140,225,690		Vol=30 *Exp*	167,315,983	175,954,800	
St. Dev	53,493,528	78,115,727	0.685	*St. Dev*	105,518,207	133,790,716	0.789
Vol=50 *Exp*	133,237,399	139,990,704		Vol=50 *Exp*	167,784,227	176,197,488	
St. Dev	97,888,154	151,118,331	0.648	*St. Dev*	210,895,005	284,850,790	0.740
Vol=80 *Exp*	133,549,601	140,612,528		Vol=80 *Exp*	170,155,316	179,159,221	
St. Dev	203,625,011	366,384,021	0.556	*St. Dev*	514,617,120	792,293,731	0.650