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Cash Reserve and Venture Business Survival Probability

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Both solid business ventures and those not on as firm a footing can fail because they do not manage risk properly. This study shows that start-ups with a positive NPV project can fail because of inadequate cash reserves. We apply the first-hitting time model to analyze the effect of a cash reserve on the business failure density function and the cumulative failure probability for a specific business venture. The analysis of this model shows that business ventures have a much higher

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survival probability when they reduce their future cash-flow volatility. It is also shown that when risks cannot be controlled or are too expensive to be controlled, then business ventures need to have adequate cash reserves if they are to reduce failure density and cumulative failure probability.

I. Introduction

According to a Dun & Bradstreet report, "Businesses with fewer than 20 employees have only a 37% chance of surviving four years and only a 9% chance of surviving 10 years." It is particularly striking that the failure rate for new businesses seems to be around 70% to 80% in the first year and that only half of those who survive the first year remain in business the next five years.[a] Of the failed businesses, some fail because they do not have a viable business model while others have solid business plans but do not do a good job of managing their business resources and business risks.

This is reinforced by the findings that nine out of ten business failures in the United States are caused by a lack of general business management skills and planning.[b] Given the odds of a start-up surviving to its tenth year in business, risk management skills are especially important for an entrepreneur.

New business ventures are vulnerable to uncontrollable and unfavorable business environments (Gertler and Gilchrist, 1994). Entrepreneurs starting these companies need to be aware of these risks and plan accordingly (Kihlstrom and Laffont. 1979, Cressy , 2000; Hopenhayn, 2002). Any failures to do so will only contribute to the high failure rate of venture businesses. Just as Governor Bernacke of the Federal Reserve Bank said, being struck by a lightning is a rare event. However, if you are in a field that is struck by lightning often, then you should have a robust plan to mitigate the harm done by lightning. For the entrepreneur, risk management is not luxury but a must.

Yet this commonsense notion is lost in entrepreneurs who often decide to take more risk than needed. At the same time, due to self-financing, their financial resources are less for their projects than more established entrepreneurs (Evan, 1987, Evans and Jovanovic. 1989, Dunne, Roberts, and Samuelson, 1989, Davis and Haltiwanger, 1992). Add on top of this fact, start-up ventures face borrowing constraints especially in a prolonged unfavorable business environment (Gomes, 2001, Albuquerque and Hopenhayn, 2002, Clementi and Hopenhayn, 2002) and entrepreneurs have circumstances that limit the ability of their firms to survive to their tenth year. When the business environment is unfavorable and the entrepreneurs have difficulty borrowing money, the prudent thing to do is to set-aside funds to cushion against business uncertainties. This raises the question of how much entrepreneurs should set-aside as a reserve for an uncontrollable bad business environment. Based our knowledge, this study is the first to examine this problem as a risk management issue for venture businesses.

In this paper, we show that, in the traditional capital budgeting approach, the risk of failure is underestimated for venture businesses. Entrepreneurs need to do separate risk analysis for their venture businesses. We apply the first-hitting time model based on Brownian motion to estimate the venture failure density function and the cumulative failure probability for a venture business. We show that there exists a peak value for the venture failure density function that entrepreneurs should pay attention to when deciding how to allocate their resources. We also estimate the

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a The Business Plan, Small Business Development Center, Bradley University, Peoria, IL.
b Small Business Development Center, Troy State University, Troy, Alabama.

cumulative failure probability for a venture business. We show that the cumulative failure probability, like the venture failure density, is sensitive to the entrepreneurs' cash reserves. Increasing cash reserve can significantly reduce both the maximum failure density and the cumulative failure probability for a given business venture.

The study is organized as follows. Section II shows using a traditional capital budgeting approach that entrepreneurs underestimate failure risk. Section III applies the first-hitting time approach of Brownian motion to analyze the venture failure density and the cumulative failure probability. This analysis forms the basis for Section IV where we discuss risk management issues faced by the entrepreneur. Finally, Section V provides a conclusion to our study.

II. Capital budget

Before we apply the continuous time approach or Brownian motion model to our setting, we adopt a discrete model to describe the circumstances faced by an entrepreneur. The discrete model is intuitive and can be understood with limited mathematical tools, however, it does not offer simple closed-form solutions that enable easy identification of the venture failure probability. While this is clearly a disadvantage, the use of a discrete model allows us to make the results associated with the continuous time model more intuitive. Once this intuition is gained, we can then take advantage of the closed-form solutions allowed by continuous time models to describe a venture's failure probability.

For both the discrete and continuous setting, we assume that a startup firm has a good business model and an adequate business plan. In the following, we analyze why, in the traditional capital budgeting approach, entrepreneurs will often underestimate failure risk. To simplify our presentation, we make some assumptions in the model. Nevertheless, the model can be expanded to cover more sophisticated business environments after some adjustments. The extended model in Section IV serves as an example. Next, we list the assumptions used in this section:

1. The future annual cash flow for year i, which is denoted as C_i, has two possible values, either high (H) or low (L). L can be negative, which means that the entrepreneur will lose money in that year. The probability of a high cash flow is denoted p, and the probability of a low cash low is denoted *(1-p).*
2. The venture business risk is characterized by a beta coefficient □, which is the customary risk measure embedded within the Capital Asset Pricing Model.
3. The entrepreneur's investment is denoted by I and represents the funds the entrepreneur raises to start her new venture. It includes the initial investment and the present value of any future investment.
4. Besides the investment I, the entrepreneur has a limited amount of cash reserve or some maximum amount of funds available to him/her in the case of an emergency. We denote this amount by R.
5. The firm's investment horizon is infinite, that is, the entrepreneur plans to pass her ownership to heirs.
6. The venture is assumed to have zero debt.[c]

c. Again, we make this assumption just for simplicity. In this case, the weighted average cost of capital (WACC) is the cost of equity, which can be calculated from CAPM.

Given assumptions 1 and 6, we follow the traditional capital budgeting approach to making the investment decision. Under traditional capital budgeting, the risk of the business is captured by the beta coefficient. Thus, the net present value (NPV) is determined by discounting the future expected cash flows by the risk adjusted weighted average cost of capital (WACC).

Under assumption (1), the expected cash flow per year can be computed as:

$$E[C(t)] = pH + (1-p)L \tag{1}$$

Since the venture capital is assumed to have no debt component, the WACC is the cost of equity, whose value is computed using CAPM.

$$WACC = r_f + \beta(r_M - r_f) \tag{2}$$

where r_f is the risk-free interest rate and r_M is the return to the market portfolio. Combining (1) and (2) and factoring in the entrepreneur's investment, the NPV of the venture can be computed as in (3).

$$NPV = \frac{E(C_i)}{r_f + \beta(r_M - r_f)} - I \tag{3}$$

where r_f represents the risk-free rate of interest and I the present value of the entrepreneur's investment.

In conventional capital budgeting approaches, we accept the project if *NPV*>0. For a venture business with limited resources, however, we need to reconsider this conventional approach. The reason is that the risk modeled in (3) does not adequately describe the risk faced by the business venture. In (3), the beta coefficient captures the systematic risk associated with the project and ignores the nonsystematic risk associated with most start-ups (Heaton and Lucas. 2000). With CAPM, nonsystematic risks are assumed to be diversified away such that the risks unique to a particular firm are cancelled out by the negatively correlated returns of other investments. However, this diversification is generally not available to entrepreneurs since they typically tie up all of their wealth in their business ventures. For entrepreneurs, the risk represented by the β □coefficient represents only a portion of the stand-alone business risk, which we denote as σ□ or as the variance of the future cash flows:

$$\sigma^2 = p[H - E(C_i)]^2 + (1-p)[L - E(C_i)]^2$$

If an entrepreneur does not manage the stand-alone risk properly, the result will be detrimental even if the business plan is sound and the NPV in (3) is positive. We discuss this issue further in the next section.

III. Failure probability for new ventures

Unlike established companies, new ventures quite often do not have ample cash reserves to shield them against bad business decisions. Also, in contrast to the large companies that have dedicated risk management teams, new ventures generally have no expertise in risk management. Yet, risk management is especially crucial for new ventures and for those starting these companies. To illustrate this, we modify the value at risk (VaR) approach to new venture business and apply it to the risk management of a business venture.

Under this approach, the entrepreneur is assumed to have a given risk tolerance, characterized by her cash reserve *R*. Under the VaR framework, the entrepreneur needs to estimate the probability of losing more than R dollars within a given time period. This probability represents the probability of business failure. This definition allows for business failure to happen to a well planned venture just as a not well-thought one. As such, we allow good start-ups to fail when unlucky circumstances cause the cash constraint to strangle an otherwise successful business venture.

If there are *n* good years and *m* bad years for the entrepreneur within *N* years, then the venture would fail if:

$$nH + R < -mL \tag{4}$$

Given that the venture survives up to year *N*-1, the condition for business failure in year *N* is

$$\sum_{i=1}^{N} C(i) + R < 0, N \in Q^{+} \tag{5}$$

where Q^{+} represents the set of nonnegative integers. The interpretation is as follows. Given that the venture survives up to year *N*-1, if year N is a good year, then the entrepreneur survives. On the other hand, if the entrepreneur's reserve minus the cash burned before year *N* is not enough to cover the losses in a bad year, then the venture cannot survive. In other words,

$$\sum_{i=1}^{N} C(i) + R = 0$$

is the absorbing boundary, on which the venture is dead and there is no chance for it to bounce back.

The computations for the discrete case are very intensive. Because of this, we need to chop the time periods into small intervals and let the number of periods becomes very large. By doing this, the above discrete random walk model becomes the continuous-time model, or Brownian motion model. As we approach a continuous model, we are able to apply the first-hitting time model to analyze the probability that the new venture will fail.

To apply the first-hitting time model with Brownian motion to estimate the probability of venture failure, we calculate the probability that the Brownian motion reaches the zero boundary. To do this, the cumulative cash flows are viewed as a stochastic process *C*(*t*), whose arithmetic Brownian motion is described as in (6).

$$dC(t) = \mu dt + \sigma dW \tag{6}$$

where μ is the drift and σ is the standard deviation of the cash flows. The movements of the cumulative cash flows in the continuous time setting are related to those in the discrete model by:

$$\mu = E(C_i) \tag{7}$$

$$\sigma = p(H - E(C_i))^2 + (1-p)(L - E(C_i))^2 \tag{8}$$

It is assumed that the cash flow process has an initial level $C(0) = R > 0$, which is the entrepreneur's cash reserves. Business failure will occur in either a discrete or continuous setting whenever the path of cumulative cash flows hits zero, which we refer to as the absorbing threshold. We denote this first passage time by T. Thus, $T = \min\{t : C(t) \leq 0\}$. Such an occurrence is illustrated by the path in Figure 1. It should be noted that the path is one of numerous possibility and it does not mean that the sample path will reach the absorbing threshold by a given time period.

The cumulative cash flow process and the possibilities of hitting the absorbing threshold are determined by three parameters, namely the initial cash level R, the mean rate of cash flow per year, μ and the standard deviation σ of the Brownian motion. The value R sets the starting point for the trajectory. In contrast, σ represents the inherent variability per unit time of the sample path and gives the model its random or stochastic behavior. The smaller σ is the more predictable the cash flow outcome.

Jointly the three parameters, R, μ, and σ determine the business venture's survival probability. Applying the first-hitting time model, we have the following result.

Proposition 1 Let $f(s|R.\mu,\sigma)$ denote the probability of the first time failure for a business venture within an infinitesimal time interval *ds*, $P(T \in ds)$, given parameters R, μ and σ, the probability density function of business failure is given by

$$f(s|R.\mu,\sigma) = \frac{R}{\sqrt{2\pi\sigma^2 s^3}} \exp\left[-\frac{(R+\mu s)^2}{2\sigma^2 s}\right] \tag{9}$$

The failure function of $F(s|R.\mu,\sigma)$ represents the cumulative failure probability for the business venture within time s, $P(T< s)$ and will have the form

$$F(s|R,\mu,\sigma) = \Phi\left(\frac{\mu s + R}{\sigma\sqrt{s}}\right) - \exp\left[-\frac{2R\mu}{\sigma^2}\right]\Phi\left(\frac{\mu s - R}{\sigma\sqrt{s}}\right) \tag{10}$$

Proof: See Harrison (1985).

In Figure 2 we plot the failure density (9) for *s<5*, assuming $\mu = 0.25, \sigma = 0.5625$ (corresponding to the discrete case with $H = 1, L = -0.5, p = 0.5$) and for *R*=0.5, 1, and 1.5 respectively. The failure density gives the probability of business failure in an infinitesimal time interval *dT*. The most instructive feature of the figure is that there is a maximum failure density for each different cash reserve. For very short time periods measured from the start-up of the business, the failure density is small. It increases and reaches a peak value, then declines. The reason for the existence of peak failure density is as follows. The entrepreneur has a certain amount of cash reserve. At start-up, even if the uncontrollable business condition is bad, the cash reserve is able to cover the business losses. Thus the probability of failure is small. However, when bad conditions extend to a longer period, the cash reserve runs out. As these cash reserves dwindle, the probability of business failure in a given time interval increases.

While initial cash reserves provide a shield against business failure, the positive NPV feature of the chosen project will also strengthen the entrepreneur's cash positions over the long-term. As these returns are realized, the probability of business failure in a given time interval will decline after reaching its peak in earlier years.

If we are interested in the cumulative failure probability within a certain time horizon, we can integrate (9) to get (10). Figure 3 plots the cumulative failure probability for different cash reserves. It shows that the cash reserve has a significant impact on the cumulative failure probability for a venture business. For R=0.5, the cumulative failure probability approaches more than 40%, while for R=1.0, it is cut in half and approaches 20%. For R=1.5, it is cut in half again from that found when R=1.

We can also compute the mean failure time under various scenarios. The expected time to fail for failed business ventures is the mean failure time, which is the expected failure time *T* conditional on the venture fails $\{T < \infty\}$. This can be expressed as:

$$E\left(T \middle| \{T < \infty\}\right) = \frac{R}{\mu} \tag{11}$$

The failure density of venture businesses is nonlinear and there is a maximum failure density, f^*, for each different cash reserve. The first-hitting-time model also allows us to compute the time, t^*, at which the failure probability to reach a maximum. The following proposition provides the explicit formulas for f^* and t^*.

Proposition 3 Given the probability density function of business failure

$$f\left(s \middle| R.\mu, \sigma\right) = \frac{R}{\sqrt{2\pi\sigma^2 s^3}} \exp\left[-\frac{\left(R + \mu s\right)^2}{2\sigma^2 s}\right]$$

The time at which the failure probability to reach a maximum, t^*, is given by

$$t^* = \frac{B}{2\mu^2} \tag{12}$$

where

$$B = \sqrt{4\mu^2 R^2 + 9\sigma^4} - 3\sigma^2$$

and the maximum failure density f^* (at t^*) is given by

$$f^* = \frac{2\,\mathrm{Re}\,xp\left[\dfrac{(2\mu R + B)^2}{4\sigma^2 B}\right]}{\sqrt{\pi}\,\sigma\left(\dfrac{B}{\mu}\right)^{3/2}} \qquad (13)$$

Proof: Take the derivative of (9) with respect to time *t*, then set the derivative to zero to solve for *t*. It yields (12). Substituting (12) back into (9) yields (13).

Figure 4 plots the maximum business failure density at different cash reserve levels. It shows that the impact of cash reserves on the maximum business failure density function is nonlinear. A small amount of reserve significantly reduces the maximum failure density. Entrepreneurs should consider this when deciding how to allocate funds between investment and cash reserves.

It should be noted that, in this example, we assume that the business venture will generate positive cash flows immediately after the business starts. In many cases, businesses may experience many years in the red and, without proper risk management, the probability of failure will be much higher. Second, in real cases, the cash flows are estimated from detailed information and the future cash flows will vary from those in the above example. To adapt our results to these circumstances, the failure probability would need to be computed based on the future cash flows for a particular venture business.

IV. Cash reserve and risk management

We have shown that, to increase the chances of success, an entrepreneur needs to have not only a solid business model, but also good risk management skills. An entrepreneur should apply risk management approaches to minimize her failure probabilities. An immediate result from the above analysis is that an entrepreneur should hold cash reserves that are consistent with the risks associated with the start-up. Doing this will have a significant dampening effect on the failure probability.

Entrepreneurs should identify the major risks their businesses face and use the appropriate risk management techniques to reduce, transfer, share, or eliminate those risks. By managing risks properly, the business venture can greatly improve their chances to survive both short-term and long-term into the future. For those risks that cannot be dealt with by other means, keeping a reasonable amount of cash reserve on hand can be an effective way to reduce the business failure probability. The problem with relying on this approach is that it limits the amount of cash available for reinvestment in the business. To avoid having to starve operations to maintain cash balances, companies are using risk management techniques to reduce business failure probability. By doing this, these companies are not put into the position of having to choose between hoarding or operating with insufficient cash reserves. Instead they proactively manage risks so that the cash reserves can be used to grow the business.

In the above analysis, we assume that the growth rate and the volatility of future cash flows, μ and σ are constant. The size of investment in a business venture often affects the growth rate and the volatility of the business. For example, a larger investment can make the business more competitive. For a simple linear case, we can model these economies by letting the growth rate be a multiple of the investment.

$$\mu = aI = a(D - R) \tag{14}$$

The volatility of future cash flows often increases with the size of investment. A simple effect of investment on volatility of future cash flows can be captured by:

$$\sigma^2 = bI = b(D - R) \tag{15}$$

Substituting (14) and (15) into (13), we can estimate the maximum failure density under different future cash-flow volatilities. Figure 5 plots some numerical results for *b*=0.1, 0.2, and 0.3 respectively, while keeping *a*=0.5. As we expect, a decrease in *b* reduces the business failure density. What this says is that entrepreneurs who want to limit the maximum business failure density should take steps to transfer, share, reduce or eliminate some or all of the future cash flow risks using risk management techniques, when those techniques are not expensive to implement. Barring this, the entrepreneur needs to maintain the cash reserve level needed to achieve the goal of controlling the failure density. For the business venture in the example, if the entrepreneur can reduce her future cash flow volatility from 0.3 to 0.1, she can reduce the required cash reserve from about 55% down to 25% while maintaining the maximum failure density at 0.1.

While our focus has been on the failure density, an entrepreneur may also want to make decisions based on the cumulative business failure probability. Substituting (14) and (15) into (10), we can estimate the cumulative failure probability under different volatilities. Figure 6 plots some numerical results for *b*=0.1, 0.2, and 0.3 respectively, while keeping *a*=0.5 and *t*=5 years. Similar to the results in Figure 5, a decrease in *b* reduces the cumulative business failure probability. For the business venture used in the example, if the entrepreneur can reduce her future cash flow volatility from 0.3 to 0.1, she can reduce the required cash reserves from about 50% down to 20% while maintaining a cumulative failure probability over the five years of 0.2.

V. Conclusions

Both solid business ventures and those not on as firm a footing can fail because they do not manage risk properly. As a consequence, risk management is not a plus but a must for both new and existing business ventures. Entrepreneurs need to realistically estimate their business risk, especially, the business failure density function and the cumulative failure probability for their specific venture businesses. Entrepreneurs need to control the maximum failure density or the cumulative failure probability to levels consistent with their "risk appetites".

In the model presented in this paper, the venture failure density is not a constant. It goes up and reaches a maximum value and then goes down. The maximum value indicates the maximum failure probability in a small time interval. Entrepreneurs should carefully estimate this failure probability for their businesses and consider ways to reduce this probability. Both failure density and cumulative failure probability are sensitive to the cash reserves. Increasing cash reserves can significantly reduce both failure density and cumulative failure probability. Or in other words, increasing reserves significantly increases a business venture's chances of survival.

In addition to considering the need for a cash reserve, entrepreneurs should also identify the major risks of their business and how to manage these risks using proper risk management approaches, especially financial approaches like insurance, hedging, etc. This topic has been discussed in many papers and books and we do not repeat it here. By doing this, the company can free up cash reserves while maintaining risk levels at desired levels. These freed up funds can in turn be reinvested in the business, allowing for enhanced expected returns and greater chances for survival.

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Figure 1
A sample path of the cash flow process, *C(t)*

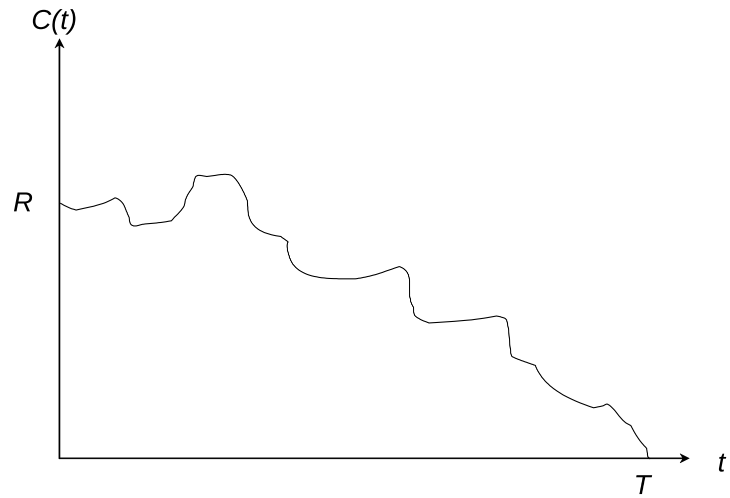

Figure 2
Business failure density for different cash reserves

Thick line: R=0.5; dashline: R=1.0; thin line: R=1.5

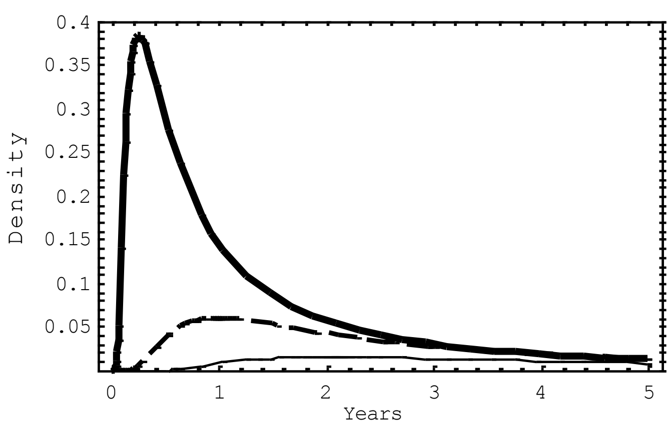

Figure 3
Cumulative business failure probability for different cash reserves

Thick line: R=0.5; dashline: R=1.0; thin line: R=1.5

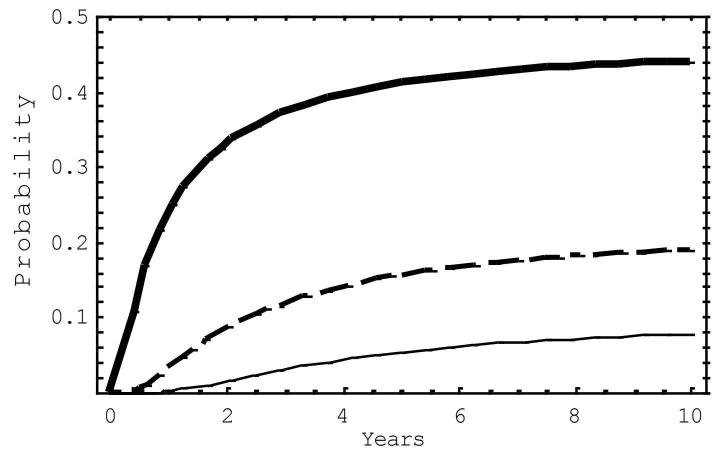

Figure 4
Maximum business failure density for different cash reserves

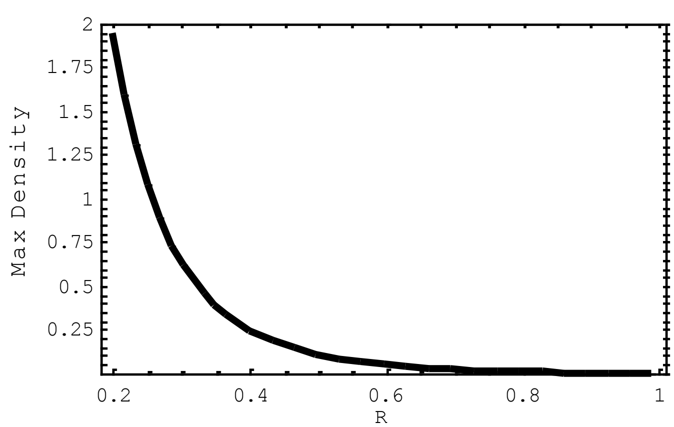

Figure 5
Maximum failure density as a function of cash reserve for different *b*

Thick solid line: *b* =0.1, dashed line: *b* =0.2; thin solid line: *b* =0.3

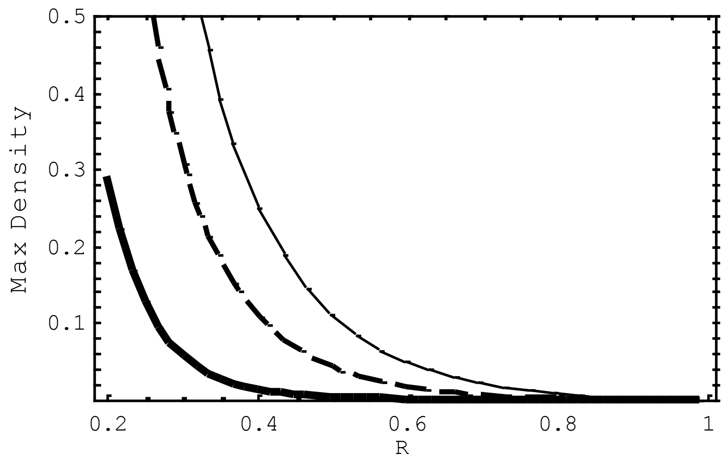

Figure 6
Cumulative failure probability as a function of cash reserve for different *b*

Thick solid line: *b* =0.1, dashed line: *b* =0.2; thin solid line: *b* =0.3