The Journal of Entrepreneurial Finance

Volume 12
Issue 1 *Spring 2007*

Article 6

December 2007

Why Would Financial Bubbles Evolve After New Technologies?

Haim Kedar-Levy
Ben-Gurion University of the Negev

Why Would Financial Bubbles Evolve After New Technologies?*

Haim Kedar-Levy**
Ben-Gurion University of the Negev

This paper presents an equity market where the value of a new technology is infrequently observable while the equity claim of the asset is continuously traded. We clear the stock market between two optimal asset allocation strategies, speculative vs. fundamental, adopted by risk-averse investors who differ in their risk-aversion. The stock price path maintains a potential for endogenous bubbles or under-pricing vs. the asset as a function of total funds invested in the stock by each investor type. Bubbles grow exponentially if speculation dominates but if the fundamental strategy dominates, the stock's growth rate and its volatility will decline.

Introduction

Observed market prices of real and financial securities alike are probably not stationary and, according to many researchers diverge from fundamental valuation for periods that are too long to be explained by response to external shocks (Shiller, 1981, Summers, 1986). Such endogenous deviations, termed "bubbles," "booms and crashes" or "cycles" were attributed by the first researchers of the issue to speculation, which has been considered rational. Keynes, 1930 and Hicks, 1946 addressed speculation as a byproduct of differences in risk aversion, where agents who are more risk-averse wish to "sell" some of the risk to less risk-averse ones. The more risk-averse agents will trade based on fundamental values of the risky asset while the less risk-averse agents will adopt speculative strategies. It turns out then that speculation is a tool to reallocate risk among trading agents, as our model captures.

---

* I am indebted to Profs. Dan Galai, Arie Gavius, Ben Z. Schreiber, Itzik Venezia and Zvi Weiner for helpful discussions. All remaining errors are mine.

** Dr. Kedar-Levy received his Ph.D. in Finance from the Hebrew University, Jerusalem Israel in 1999. Since 1998 he has been affiliated with Ben-Gurion University in Beer Sheva, Israel. He spent the period of September 2002 to August 2004 visiting Temple University, Philadelphia. He is currently involved with the research of financial market anomalies, both theoretically and empirically. Since June of 2006 he has served as the Academic Director of the Honor MBA program at Ben-Gurion University.

We present a model in which the real profitability rate of a new technology is unknown to investors, yet they need to assess the asset's value in the stock market. The real asset reveals observations of its profitability infrequently, while the stock is traded more frequently. We find that there is a singular composition of weighted average risk aversions between speculation and fundamental strategies, for which the variability and return of the share price will be equal to those of the underlying asset. The solution complies with well-known dynamic models such as Merton, 1971, where price-taking agents rebalance their portfolio when the price of the risky asset is stochastic. We solve the stock price periodically through trade between speculators and fundamental traders. The sequence of equilibrium prices generates a price process that evolves into bubbles if there is excess speculation or to under-pricing if fundamental strategies are dominant.

A rigorous analysis of economies with unobservable fundamentals has been presented by Kurz, 1996. According to Kurz, agents do not possess complete structural knowledge of the environment, and structural changes are not observable (p. 352). Agents can hardly predict changes in technology, tastes, economic institutions, or international balance of power. Thus, there is a question whether such changes are random deviations around a fixed mean value function of a stationary process, or whether they reflect changes in the mean value function itself.

Rational expectations models of bubbles are attractive as they adhere to core economic theory. This is probably the reason why these models were the first to appear in the late '70s - early '80s (e.g., Blanchard, 1979, Flood and Garber 1980, Blanchard and Watson 1982, Tirole, 1982). Blanchard comes to two clear-cut conclusions about rational bubbles, which are best summarized in his words: *"...speculative bubbles followed by market crashes, are consistent with the assumption of rational expectations. ...speculative bubbles may take all kinds of shapes. Detecting their presence or rejecting their existence is likely to prove very hard"* (P. 387.) Bubbles in these models evolve based on a self-fulfilling rational expectations mechanism, which supports the bubble as long as it exists, and the price crashes when the mechanism disappears. The models do not specify what endogenous conditions must be satisfied for a bubble to evolve and under what inherent terms will the bubble blast.

Starting from the mid '80s, efforts to map real life observations into equilibrium models of bubbles typically involve explicit or implicit assumptions of irrational behavior by a certain group of agents. Some models assume noise trading (e.g., Kyle, 1985, Black, 1986, De-long, Shleifer, Summers and Waldmann, 1990, and Binswanger, 1999), others assume overreaction to news (Jegadeesh and Titman, 1995, De-Bondt and Thaler, 1987, Kent, Hirshleifer and Subrahmanyam, 1998) or inside information signaling between investors who have asymmetric information (Allen and Gale, 1992, Allen, Morris and Postlewaite, 1993). The economic meaning of such assumptions is that either fundamental information is available to some of the agents but not to others (asymmetric information), and/or information is processed differently by different agents (the noise-trader approach). Binswanger, 1999 extends De-Long et al.'s model by allowing dynamic changes of the fundamentals due to technological changes. Zeira, 1999, asserts that fundamental information about the real economy is not acknowledged continuously by agents, thus when the true size of the economy is revealed share prices either jump or crash to reflect it. Such informational mismatch is assumed in this paper.

The economic setting is described in Section I, where optimal investment rules are defined for individual agents and institutions. In Section II marginal supply and demand functions for shares are derived, and in Section III equilibrium price for the stock is calculated and bubbles and under-pricing are reasoned. Section IV concludes the paper.

I. The Economic Setting

Consider an economy in which atomistic, expected utility maximizing individual agents deposit their wealth with institutional agents for the purpose of portfolio management. The individual agents can be grouped based on similarity of tastes vs. risk as if each group is represented by its own "representative agent." Average tastes differ between groups in a way formalized below. The investment opportunity set is comprised of two assets, a riskless bond yielding exogenous, fixed rate of return r and a single stock for the single real capital asset in the economy. The total return generating process (i.e., including dividends) of the real capital asset is assumed by all agents to follow a Geometric Brownian Motion with drift μ and standard deviation σ.

When fundamental information becomes available, the stock and the real capital asset values must be equal. If the shares are priced higher than the real asset, they should decline toward it, and if it was priced lower, it will jump once investors acknowledge the mis-pricing. These adjustments are assumed external to the model presented below; our focus is on the pricing processes that results above, at or below the value of the real asset through the trades of speculators and fundamentals. Finally, assume that there are no transactions costs, including payments for portfolio management, taxes, or restrictions on short sales, and total stock and bond holdings are, in aggregate, positive.

A. Individual investors

All agents in the economy make their investment decisions based on the expected μ and σ as the only relevant parameters of the new technology. Information (innovation) dz_t is observed when agents value the traded security. If the Geometric Brownian Motion of the traded security is

$$dP_t = P_{t-dt}(\mu dt + \sigma dz_t)\,, \tag{1}$$

then the following is the wealth-accumulation process for each investor group *K*, (*K=F,S*, as will be elaborated below),

$$W_{K,t+dt} = W_{K,t}\big(1 + (r + \alpha_{K,t}(\mu - r))\big)dt + \alpha_{K,t}W_{K,t}\sigma dz_t\,, \tag{2}$$

where, *dt* is a short time interval, *W* is the market value of both securities, $\alpha_{K,t}$ represents the proportion of wealth group *K* holds in the risky asset at time *t*. Using a Taylor series expansion to estimate $U(W_{K,t+dt})$, where *U* is a von-Neumann-Morgenstern utility function defined over total wealth, and applying the expectation operator, we get

$$E\big(U(W_{K,t+dt})\big) = U(W_{K,t}) + U'(W_{K,t})W_{K,t}\big(r + \alpha_{K,t}(\mu - r)\big)dt + \frac{1}{2}U''(W_{K,t})W_{K,t}^2\alpha_{K,t}^2\sigma^2 dt\,. \tag{3}$$

In order to find the optimal proportion each group should invest in the risky asset, the first order condition of (3) with respect to $\alpha_{K,t}$ is,

$$\frac{dE(U)}{d\alpha_{K,t}} = U'(W_{K,t})W_{K,t}(\mu - r) + U''(W_{K,t})W_{K,t}^2\alpha_{K,t}\sigma^2 = 0\,. \tag{4}$$

Solving for $\alpha_{K,t}$ from (4) we get

$$\alpha_{K,t} = -\frac{U'(W_{K,t})}{U''(W_{K,t})W_{K,t}}\frac{\mu - r}{\sigma^2}, \tag{5}$$

and using the Arrow-Pratt measure of relative risk aversion, $R_{W,K,t} \equiv -\frac{U''(W_{K,t})W_{K,t}}{U'(W_{K,t})}$, we obtain the well known relationship (e.g., Merton, 1971)

$$\alpha_{K,t} = \frac{1}{R_{W,K,t}}\frac{\mu - r}{\sigma^2}. \tag{6}$$

Now assume all individuals' utility function is hyperbolic in wealth, taking the form of the following HARA type

$$U(W,t) = e^{-\rho t}\frac{1-\gamma}{\gamma}\left(\frac{W}{1-\gamma} + \eta\right)^{\gamma}. \tag{7}$$

It is advantageous to use this type of utility functions since we compare agents with different attitudes toward risk, which are readily available here. This function can produce relative or absolute measures of risk aversion, that both can be constant, decreasing or increasing in wealth. To obtain the explicit measures of risk aversion we find the first two derivatives

$U' = e^{-\rho t}\left(\frac{W}{1-\gamma} + \eta\right)^{\gamma-1}$, $U'' = -e^{-\rho t}\left(\frac{W}{1-\gamma} + \eta\right)^{\gamma-2}$, thus the measure of absolute risk aversion is $A_W = \frac{1}{\frac{W}{1-\gamma} + \eta}$, and the measure of relative risk aversion is $R_W = \frac{W}{\frac{W}{1-\gamma} + \eta}$. Fixing R_W in (6) we obtain the optimal investment rule for agents in group *K*, as a function of taste parameters γ_K, η_K,

$$\alpha^*_{K,t}W_{K,t} = \frac{\lambda}{\delta_K}\left(W_{K,t} + \eta_K\delta_K\right), \tag{8}$$

where $\delta_K = 1 - \gamma_K$ and $\lambda = \frac{\mu - r}{\sigma^2}$.

This type of utility functions embodies a displacement factor, $\eta_K\delta_K$ that, if negative, represents a demand for a minimum portfolio value required by the investor. Models based on a single representative agent conclude that (8) is the solution for the asset allocation problem, which may be referred to as an "investment rule" or "strategy." In models with atomistic investors, the stochastic price path is exogenous, observable and unaffected by the agent's trade, thus the equilibrium path will be the stochastic process itself. In this model however, as long as expected utility maximizing investors determine the equilibrium price, it will not be stochastic. For a two agents case, termed *S* and *F* (Speculative and Fundamental investors as elaborated below), (8) will be $E(P_t^*) = \frac{\lambda}{N}\left(\frac{E(W_{S,t})}{\delta_S} + \frac{E(W_{F,t})}{\delta_F} + \eta_S + \eta_F\right)$, when $E(P_t^*)N = \sum_K \alpha_{K,t}^* W_{K,t}$, N is the total number of shares in the market, thus the expected share price will be a function of the absolute differences of both δ_K from λ, together with the aggregate effect of $\eta_S + \eta_F$. If the weighted average of risk tolerance (the reciprocal of risk aversion) is less than λ (weighted by wealth) then the price process $E(P_t^*)$ will grow at an exponential rate greater than μ, and vice versa. Only when the weighted average of risk tolerance equals λ, will the value of the stock market grow at rate μ. We will show below that $\delta_S < \lambda$, $\delta_F > \lambda$, and for the simple case when displacement factors are zero we obtain an amplifying effect of stock prices when *S* are dominant and a dimming effect when *F* are dominant. Figure 1 illustrates these three generic possibilities.

B. Institutional Investors

We classify institutional agents by two rebalancing strategies. The first institutional agent will increases holdings of the risky asset with an increase in the value of its portfolio denoted the "Speculative" (*S*), strategy (Badrinath and Wahal, 1999, Goetzmann and Massa, 2000). The general form of this strategy in equilibrium is

$$S_t = m(W_t - F), \tag{9}$$

where *S* is the amount held in the risky asset, *m* is a multiplier that must be strictly greater than unity, *W* is total assets value and *F* is the required protection level for total assets (Floor), which may be a function of time. By presenting *W* as the sum of stock (*S*) and riskless debt (*D*) as a multiplication of the number of securities held at *t-dt* ($N_{S,t-dt}$ and $Q_{S,t-dt}$ respectively) by their current prices (P_t and B_t respectively), we obtain the rebalancing rule of the Speculative strategy,

$$N_{S,t}P_t = m_S(N_{S,t-dt}P_t + Q_{S,t-dt}B_t - F_S). \tag{10}$$

The second institutional agent decreases exposure to the risky asset with an increase in the value of its managed assets, being the "Fundamental" (*F*) strategy. Its dynamic rebalancing rule is

$$N_{F,t}P_t = m_F(N_{F,t-dt}P_t + Q_{F,t-dt}B_t + F_F). \tag{11}$$

Thus, at each period, the institutional agents *S* and *F* need to rebalance the portfolios they manage based on their strategy. However, since the real capital assets' price is not observable and because the institutions are not price-takers, they price the stock by matching supply and demand functions for units of shares due to their rebalancing requirements. They do so by submitting a vector of quantity of shares matched by a set of prices to a clearing agency, assumed to operate in a standard tâtonnement procedure. The tâtonnement will clear excess demand and set an equilibrium price for which only *marginal* rebalancing requirements will be traded.

II. Marginal Supply and Demand Functions for Shares

Comparing (10) and (11) with (8) we may conclude by structural resemblance that the coefficients for the Speculative and Fundamental strategies matches the optimal investment strategy of individual investor K if $m_K = \dfrac{\lambda}{\delta_K}$, $F_K = \eta_K \delta_K$. In order to establish clientele, the institutional investors must adjust their strategy parameters to match the tastes of the specific group of individual investors. For each strategy, we now define specifically those taste parameters that satisfy a concave payoff schedule for F and a convex one for S. Incrementing (2) backwards in time and replacing it with $W_{K,t}$ in (8) we obtain the wealth accumulation process of the institutional investors

$$N_{K,t}P_t^f = \frac{\lambda}{\delta_K}\left\{W_{K,t-dt}\left[1+\left(r+\alpha_{K,t-dt}(\mu - r)\right)dt + \alpha_{K,t-dt}\sigma dz_t\right] + \eta_K \delta_K\right\}, \tag{12}$$

for K=S,F and f=s,d for the marginal supply and demand functions that will be constructed for both agents. Let $S_{K,t} = N_{k,t}P_t, \forall t$, $D_{K,t} = Q_{K,t}B_t$, $\forall t$, $\alpha_{K,t-dt} = S_{K,t-dt}/W_{K,t-dt}$, $1\text{-}\alpha_{K,t-dt} = D_{K,t-dt}/W_{K,t-dt}$, then (12) becomes

$$N_{K,t}P_t^f = \frac{\lambda}{\delta_K}\left(N_{K,t-dt}P_t + Q_{K,t-dt}B_t + \eta_K\delta_K\right). \tag{13}$$

(13) is equivalent to equation [10] in Merton's 1971 discrete time prolog to the continuous time model. In the next section, we solve for P_t using the marginal supply and demand functions for shares that we develop below and match them with individual tastes.

Denote the total relevant number of shares in the market $N = \sum_K N_{K,t}$ $\forall t, K = S, F$ (new issues and buy-back of shares are ignored). By definition, the number of shares held by K at t equal the number of shares they held at t-dt plus an optimal, unknown at that stage, marginal trade over dt, $N_{K,t} - N_{K,t-dt} = dN_{K,t}$. Marginal supply and demand functions for shares can now be formed; a marginal supply function by agents S and a marginal demand function by agents F, in the plane $P_t^f, dN_{K,t}$.

Type F Agents:

By replacing $N_{F,t} = N_{F,t-dt} + dN_{F,t}$ in (13) and solving for $dN_{F,t}$ we obtain

$$dN_{F,t} = \frac{\lambda}{P_t^d \delta_F}\left(\tilde{D}_{F,t} + \eta_F\delta_F\right) + N_{F,t-dt}\left(\frac{\lambda}{\delta_F} - 1\right). \tag{14}$$

where $\tilde{D}_{K,t} = Q_{K,t-dt}B_t$ and P_t^d represents the marginal demand function for shares. The marginal demand function can be represented as a function of price in the following manner,

$$P_t^d = \frac{\frac{\lambda}{\delta_F}\left(\tilde{D}_{F,t} + \eta_F \delta_F\right)}{N_{F,t-dt}\left(1 - \frac{\lambda}{\delta_F}\right) + dN_{F,t}}. \tag{15}$$

Define type *F* individual agents as agents whose taste parameters δ_F, η_F result is a concave payoff function, i.e., as implementing an investment rule whereby an increase in the stock price results in *selling* some shares and vice versa. The marginal demand function for shares must therefore maintain a negative slope, i.e., we require

$$\frac{\partial P_t^d}{\partial(dN_{F,t})} = -\frac{\frac{\lambda}{\delta_F}\left(\tilde{D}_{F,t} + \eta_F \delta_F\right)}{\left(N_{F,t-dt}\left(1 - \frac{\lambda}{\delta_F}\right) + dN_{F,t}\right)^2} < 0. \tag{16}$$

Since the market price for risk is ex-ante positive and δ_F must be strictly positive to assure risk aversion, than $\tilde{D}_{F,t} + \eta_F \delta_F > 0$ (or $\eta_F > -\tilde{D}_{F,t} / \delta_F$) must hold. By restricting η_F, we implicitly limit δ_F as well. Solving (8), the continuous time version of the asset allocation problem, for $\frac{\lambda}{\delta_F}$ and replacing $W_{F,t} = S_{F,t} + D_{F,t}$ and $\alpha_{F,t} W_{F,t} = S_{F,t}$, we find that

$$\frac{\lambda}{\delta_F} = \frac{S_{F,t}}{S_{F,t} + D_{F,t} + \eta_F \delta_F} < 1. \tag{17}$$

That is, $D_{F,t} + \eta_F \delta_F > 0$ imply $\delta_F > \lambda$. Convexity of the marginal demand function for shares will be satisfied if the second derivative is positive,

$$\frac{\partial P_t^d}{\partial(dN_{F,t}^2)} = \frac{2\frac{\lambda}{\delta_F}\left(\tilde{D}_{F,t} + \eta_F \delta_F\right)}{\left(N_{F,t-dt}\left(1 - \frac{\lambda}{\delta_F}\right) + dN_{F,t}\right)^3} > 0. \tag{18}$$

This condition will be satisfied iff $1 + \frac{dN_{F,t}}{N_{F,t-dt}} > \frac{\lambda}{\delta_F}$, which holds for all positive marginal changes, but will be altered when the marginal trade is negative and greater in absolute terms than $\frac{\lambda}{\delta_F} - 1$. We can now propose the following relationship between strategy and tastes for agent *F*:

Proposition 1 (Marginal demand function for stock)

a) Assume the economy is structured as illustrated above whereby some of the individual agents, termed Type-F, posses a HARA type utility function as in (7), and their tastes parameters satisfy $\delta_F > \lambda$, and $\eta_F > -\frac{\tilde{D}_{F,t}}{\delta_F}$. Then, their strategy implies a preference for the monotone marginal demand function for shares (15) offered by institutional agent F with a negative slope as in (16) that is strictly convex iff $1 + \frac{dN_{F,t}}{N_{F,t-dt}} > \frac{\lambda}{\delta_F}$. Type-F individual agents may therefore posses DRRA and DARA attitude toward risk if $\left\{\begin{matrix} \eta_F < 0 \\ \delta_F > \lambda \end{matrix}\right\}$, DARA alone if $\left\{\begin{matrix} \eta_F > 0 \\ \delta_F > \lambda \end{matrix}\right\}$, and if $\left\{\begin{matrix} \eta_F = 0 \\ \delta_F > \lambda \end{matrix}\right\}$, their risk aversion will satisfy CRRA.

b) By clientele preferences, all individual investors of Type-F will deposit their wealth for management with institutional investor F, if its strategy's parameters equal the individual's tastes.

Type S Agents:

Let type *S* individual agents have a convex payoff schedule i.e., they desire an investment rule whereby an increasing share price result in an increasing exposure to shares in their portfolio, and vice versa. Following the same procedure as above, their marginal trade preference is

$$dN_{S,t} = \frac{\lambda}{P_t^s \delta_S}\left(\tilde{D}_{S,t} + \eta_S \delta_S\right) + N_{S,t-dt}\left(\frac{\lambda}{\delta_S} - 1\right), \tag{19}$$

where P_t^s is the marginal supply function for shares that may be phrased as a function of price,

$$P_t^s = \frac{\frac{\lambda}{\delta_S}\left(\tilde{D}_{S,t} + \eta_S \delta_S\right)}{N_{S,t-dt}\left(1 - \frac{\lambda}{\delta_S}\right) + dN_{S,t}}. \tag{20}$$

Here we wish to define those taste parameters that satisfy a Speculative investment strategy. Upward slope of the payoff schedule requires $\frac{\partial P_t^s}{\partial dN_{S,t}} > 0$, i.e.,

$$\frac{\partial P_t^s}{\partial (dN_{S,t})} = -\frac{\frac{\lambda}{\delta_S}\left(\tilde{D}_{S,t} + \eta_S \delta_S\right)}{\left(N_{S,t-dt}\left(1 - \frac{\lambda}{\delta_S}\right) + dN_{S,t}\right)^2} > 0, \tag{21}$$

which implies $\tilde{D}_{S,t} + \eta_S \delta_S < 0$ for a positive measure of risk aversion δ_S. If agents S are net borrowers in aggregate terms, than η_S must be strictly negative, and if they are net lenders it may be either negative or positive, as long as $\eta_S < -\tilde{D}_{S,t} / \delta_S$. For simplicity and without loss of generality, these investors will be referred to herein as net borrowers, unless otherwise indicated. In order to define the conditions on δ_S, solve (8) for $\frac{\lambda}{\delta_S}$

$$\frac{\lambda}{\delta_S} = \frac{S_{S,t}}{S_{S,t} + D_{S,t} + \eta_S \delta_S} > 1. \tag{22}$$

Based on $D_{S,t} + \eta_S \delta_S < 0$, the fraction $\frac{\lambda}{\delta_S}$ must be greater than unity, which implies $\delta_S < \lambda$, and by the assumption of risk aversion, it must be positive. In order to assess convexity, the derivative of (21) must be positive,

$$\frac{\partial P_t^s}{\partial (dN_{S,t}^2)} = \frac{2\frac{\lambda}{\delta_S}\left(\tilde{D}_{S,t} + \eta_S \delta_S\right)}{\left(N_{S,t-dt}\left(1 - \frac{\lambda}{\delta_S}\right) + dN_{S,t}\right)^3} > 0. \tag{23}$$

This condition on the second derivative of the supply function for shares is satisfied when $1 + \frac{dN_{S,t}}{N_{S,t-dt}} < \frac{\lambda}{\delta_S}$, a condition that will hold for all negative trades but might be violated if the proportional marginal trade exceeds $\frac{\lambda}{\delta_S} - 1$. The following proposition can therefore be phrased.

Proposition 2 (Marginal supply function for shares)

a) Assume the economy is structured as illustrated above whereby some of the individual agents, termed Type-S, posses a HARA type utility function as in (7), and their tastes parameters satisfy $\delta_S < \lambda$ and $\eta_S < -\frac{\tilde{D}_{S,t}}{\delta_S}$. With such taste parameters the speculative strategy offered by institutional agent S yield a monotone marginal supply function for shares with a positive slope as in (21) that is strictly convex iff $1+\frac{dN_{S,t}}{N_{S,t-dt}}<\frac{\lambda}{\delta_S}$. Type-S individual agents' measure of risk aversion complies with DRRA and DARA attitude toward risk since $\begin{Bmatrix}\eta_S<0\\ \delta_S>\lambda\end{Bmatrix}$ but it will not be CRRA or CARA.

b) By clientele preferences, all Type-S individual investors will deposit their wealth for management with institutional investor S, iff the institutional investors' strategy parameters comply with their taste parameters.

The marginal supply and demand functions for shares, being the tool for conveying information regarding asset re-allocation preferences by the institutional investors, provide the basis for trade. Trade will be executed between the institutional agents, though they have identical expectations about ex-ante return μ and variability σ. We now turn to establish Walrasian equilibrium and demonstrate the conditions for bubbles and under-pricing.

III. Equilibrium and Mis-Pricing

A. Equilibrium

At each t both institutional agents must trade in order to optimally rebalance their portfolio according to their clients' tastes. Walrasian equilibrium conditions are

$$\begin{Bmatrix}P_t^d=P_t^s\\ dN_{F,t}=-dN_{S,t}\end{Bmatrix}. \tag{24}$$

By using $N_{F,t}+N_{S,t}=N,\forall t$ in (14) and (19) we can solve for the equilibrium price of shares P_t^*,

$$P_t^*=\frac{\lambda}{N}\left(\left(\frac{S_{S,t-dt}}{\delta_S}+\frac{S_{F,t-dt}}{\delta_F}\right)(1+\mu dt+\sigma dz_t)+\left(\frac{D_{S,t-dt}}{\delta_S}+\frac{D_{F,t-dt}}{\delta_F}\right)(1+rdt)+\eta_S+\eta_F\right). \tag{25}$$

Note that the equilibrium price depends on the amount of shares held by each agent, weighted by their measure of risk aversion. The variance of the stock price process, (25) is

$$Var(P_t^*)=\left(\frac{\lambda}{N}\left(\frac{S_{S,t-dt}}{\delta_S}+\frac{S_{F,t-dt}}{\delta_F}\right)\right)^2\sigma^2,$$

or, by replacing $S_{F,t-dt}=S_{t-dt}-S_{S,t-dt}$, and $\alpha_{K,t-dt}=\frac{N_{K,t-dt}}{N}$, it may be rewritten as

$$Var(P_t^*)=\left(\lambda P_{t-dt}\left(\frac{1}{\delta_F}+\alpha_{S,t-dt}\left(\frac{1}{\delta_S}-\frac{1}{\delta_F}\right)\right)\right)^2\sigma^2. \tag{26}$$

The positive relationship between the proportion of shares held by agent *S* to the stock price variance (since $\left(\frac{1}{\delta_S}-\frac{1}{\delta_F}\right)$>0) implies that the larger the proportion of speculative agents operating in the market, the higher will be the variance of the share price. If we replaced the terms above to show how the variance corresponds to the presence of agents *F*, we will find that the variance declines with their presence in the economy. The effect on equilibrium price will be an amplification of the stochastic information process in the first case and it will be dimmed in the second case. Since *S* buy shares from *F* as price increases, *S* will crowd-out *F* and both share price and its variance will grow in time. The base stochastic process variability will not be altered when $\delta_S=\delta_F=\lambda$.

Taking (25) to the limit ($dt\to 0$), the market value of the stock in continuous time becomes

$$P_t^* N=\lambda\left(\frac{W_{S,t}}{\delta_S}+\frac{W_{F,t}}{\delta_F}+\eta_S+\eta_F\right). \tag{27}$$

If $\eta_S=\eta_F=0$ then, by Merton (1971), (27) reduces to the CRRA case. Thus, this solution for the HARA-type utility function is a generalization of Merton (1971) for multiple agents. Further, if we assume that all assets are held by a single agent, ($\delta\equiv\delta_S=\delta_F$; $\eta\equiv\eta_S=\eta_F$), (27) reduces to Merton's result of optimal investment rules (ibid. equation 49). Though (27) is similar in form to the ones derived under the assumptions of exogenous price process with a single, price-taking investor, it differs in its meaning. It shows that the equilibrium price of the traded share satisfies the individual investors' optimum asset-allocation rules but this price might differ from the real capital asset value under incomplete information. As we shall see, the traded asset price may be lower, equal to, or higher than the real capital asset value under different attitudes toward risk, mainly as measured by their absolute dispersion from λ.

B. Long Term Paths

In order to evaluate the effect of dissimilar tastes on the expected equilibrium price, take the expected value of (25) and find the change in the expected equilibrium price with respect to time,

$$\frac{E(dP_t^*)}{dt}=\frac{\lambda}{N}\left(\left(\frac{S_{S,t-dt}}{\delta_S}+\frac{S_{F,t-dt}}{\delta_F}\right)\mu+\left(\frac{D_{S,t-dt}}{\delta_S}+\frac{D_{F,t-dt}}{\delta_F}\right)r\right). \tag{28}$$

Assume for simplicity that the information process remains with stable parameters, i.e., there are no fundamental changes in the real economy. Than, (28) set the basis to compare expected growth rate of the traded stock with that of the real asset. Since $D_{K,t-dt}$ changes with the amount held in shares, it must be replaced with $D_{K,t-dt}=S_{K,t-dt}\left(\frac{\delta_K}{\lambda}-1\right)-\eta_K\delta_K$, which is a reorganization of (8), being the investment strategy denoted in terms of the amount held in bonds. Replacing this strategy into (28) after multiplying by *dt* and dividing through by P_{t-dt}, we obtain[1]

[1] Kedar-Levy (2002) analyzed the dynamic stability properties of a similar structure in a different model.

$$\frac{E(dP_t^*)}{P_{t-dt}} = \frac{\lambda}{S_{t-dt}}\left(\left(\frac{S_{S,t-dt}}{\delta_S} + \frac{S_{F,t-dt}}{\delta_F}\right)\mu dt + \left(\frac{S_t}{\lambda} - \frac{S_{S,t-dt}}{\delta_S} - \frac{S_{F,t-dt}}{\delta_F} - \eta_S - \eta_F\right)rdt\right). \tag{29}$$

If the traded share is priced equal to the real capital asset, (29) must be equal to μdt. There are three specific cases of interest with that regard:

a) Assume both measures of risk aversion are equal to the market price of risk, $\delta_S = \delta_F = \lambda$ and displacement parameters unspecified. Thus, (29) become

$$\frac{E(dP_t)}{P_{t-dt}} = \mu dt - \frac{(\eta_S - \eta_F)}{S_{t-dt}} rdt \tag{30}$$

which implies the following Proposition:

Proposition 3:

3.a) If $\delta_S = \delta_F = \lambda$ and there is no excess displacement in both utility functions, ($\eta_S + \eta_F = 0$), than the growth path of the traded stock will be equal to that of the real capital asset, μdt.

3.b) In this particular case only, the proportional share holdings of both agents will be fixed in time, equal to those at t=0 and there will be no trade since these conditions reduce the economy to a single representative agent.

3.c) If there are non-null displacements to the utility functions as required by both optimal strategies ($\eta_S < 0, \eta_F > 0$) than the expected growth rate of the stock will be higher than that of the real capital asset if $|\eta_S| > \eta_F$ and lower than μdt if $|\eta_S| < \eta_F$.

Conditions $\delta_S = \delta_F = \lambda$ and $\eta_S = \eta_F = 0$ are simulated in Figure 2a, conditions $\delta_S = \delta_F = \lambda$ and $|\eta_S| > \eta_F$ in Figure 2b and conditions $\delta_S = \delta_F = \lambda$ and $|\eta_S| < \eta_F$ in Figure 2c.

b) Assume $\delta_F = \lambda,\ \delta_S < \lambda$, that is, the speculative strategy is the dominant trading rule in the market. After replacing $S_{F,t-dt} = S_{t-dt} - S_{S,t-dt}$, (29) become

$$\frac{E(dP_t^*)}{P_{t-dt}} = \mu dt + \alpha_{S,t-dt}\left(\frac{\lambda}{\delta_S} - 1\right)(\mu - r)dt - \left(\frac{\lambda(\eta_S + \eta_F)}{S_{t-dt}}\right)rdt \tag{31}$$

where $\alpha_{S,t-dt} = \frac{N_{S,t-dt}}{N} = \frac{S_{S,t-dt}}{S_{t-dt}}$. (31) implies:

b.1) The coefficient of *(μ–r)dt* is strictly greater than unity when *S* hold the stock long since $\lambda/\delta_S > 1,\ 0 < \alpha_{S,t-dt} < 1$.

b.2) The coefficient of *rdt* may be positive if $|\eta_S| < \eta_F$ or negative if $|\eta_S| > \eta_F$ (disregarding the preceding "-" sign).

From b.1 and b.2 we get Proposition 4:

Proposition 4:

When the individual agents' risk aversions are such that speculation dominates the market, $\delta_F = \lambda, \delta_S < \lambda$ and $|\eta_S| > \eta_F$, than expected return on the traded share will be higher than that of the real capital asset. If $\eta_S + \eta_F = 0$, the fundamental strategy is more dominant, thus the gap between the return on the traded stock and μdt will decline, possibly to levels lower than μdt. The traded stock variability will be higher than that of the real capital asset regardless of displacement factors.

The case where $\delta_F = \lambda,\ \delta_S < \lambda$ and $\eta_S + \eta_F = 0$, illustrating over-pricing of the stock vs. the real asset, is presented in Figure 3a and in Figure 3b we show that if the displacements satisfy $\eta_S + \eta_F > 0$, this difference offsets the bubble presented in Figure 3a.

c) Assume $\delta_F > \lambda,\ \delta_S = \lambda$, i.e., the fundamental strategy is the dominant one among traders in the market. Replace $S_{S,t-dt} = S_{t-dt} - S_{F,t-dt}$ in (29) and find that

$$\frac{E(dP_t^*)}{P_{t-dt}} = \mu dt - \alpha_{F,t-dt}\left(1 - \frac{\lambda}{\delta_F}\right)(\mu - r)dt - \left(\frac{\lambda(\eta_S + \eta_F)}{S_{t-dt}}\right)rdt \qquad (32)$$

From (32) we can draw the following conclusions:

c.1) The coefficient of *(μ–r)dt* is strictly positive (excluding the preceding "-" sign) for long position of shares by agents *F* since $\lambda/\delta_F < 1, 0 < \alpha_{F,t-dt} < 1$.

c.2) The coefficient of *rdt* (excluding the preceding "-" sign) may be either positive or negative. If $|\eta_S| > \eta_F$ the coefficient will be negative and the stock's expected return will increase, and if $|\eta_S| < \eta_F$ it will be positive, thus the expected stock return will decline.

Based on c.1 and c.2 we draw Proposition 5:

Proposition 5:

For $\delta_F > \lambda,\ \delta_S = \lambda$, let the condition $|\eta_S| < \eta_F$ be a benchmark. Then, dominance of the fundamental strategy results in a lower expected growth rate of the traded asset vs. the real asset. From this benchmark, increase $|\eta_S|$ and see that it increases the expected growth rate of the stock and may exceed that of the real capital asset.

Both cases are presented in Figures 4a and 4b.

IV. Concluding Remarks

The model suggested in this paper combines individual and institutional investors in a dynamic asset market where a new technology real asset should be priced in a continuously operating stock exchange. Since the value of the asset is not observable between revelations of fundamental information, the value of the stock price might diverge from the value of the real asset. This diversion will be upward, a positive bubble, if low risk-averse investors apply an optimal speculative dynamic asset allocation strategy, in which case return volatility will increase as well.

However, if fundamental strategists dominate the market, their optimal asset allocation strategies will act to reduce the rate of return volatility and its average growth rate. We conclude that frequent revelations of the fundamental value of new technology assets will serve best the goal of fair valuations in the stock market.

REFERENCES

Allen F., and D. Gale, 1992. Stock Price Manipulation. *Review of Financial Studies,* 5: 503-29.

Allen F., S. Morris and A Postlewaite, 1993. Finite Bubbles with Short Sale Constraints and Asymmetric Information. *Journal of Economic Theory*, 61, 206-29.

Badrinath, S. G., and S. Wahal, 1999. Momentum Trading by Institutions, *Working Paper*.

Binswanger M. 1999. *Stock Markets, Speculative Bubbles and Economic Growth*. Edward Elgar Publishing Ltd., MA, USA.

Blanchard O.J., 1979. Speculative Bubbles, Crashes and Rational Expectations. *Economic Letters*, 3: 387-89.

Blanchard, O.J. and Watson, M. 1982. Bubbles, Rational Expectations and Financial Markets. In P. Wachtel (Ed.), Crises in the Economic and Financial Structure, 295-315. Lexington, MA: Lexington Books.

De-Bondt W. and R. Thaler, (1987). Further Evidence on Investor Overreaction and Stock Market Seasonality. *Journal of Finance,* 42: 557-81.

De-Long, B., A. Shleifer, L. Summers and R. Waldmann (1990). "Noise Trader Risk in Financial Markets." *Journal of Political Economy,* 98: 703-38.

Flood, R. and Garber, P. (1980.) Market Fundamentals Versus Price-Level Bubbles: The first Tests, *Journal of Political Economy*.

Hicks, J. 1946. *Value and Capital.* 2nd Edn., London: Oxford University Press.

Goetzmann, W. N., and M. Massa, 2000. Daily Momentum and Contrarian Behavior of Index Fund Investors. *Yale, International Center for Finance, Working Paper No. 99-13.*

Jegadeesh N., and S. Titman, (1995): Overreaction, Delayed Reaction, and Contrarian Profits. *The Review of Financial Studies, Vol. 8, No. 4, pp. 973-993.*

Kedar-Levy, H., Price Bubbles of New Technology IPOs, *The Journal of Entrepreneurial Finance and Business Ventures*, 7, 2, 2002, 11-31.

Kent, D., D. Hirshleifer and A. Subrahmanyam (1998). A Theory of Overconfidence, Self-Attribution, and Securities Markets Under and Over Reactions. *Journal of Finance,*

Keynes, J. M. 1930. *A Treatise on Money.* London: Macmillan.

Kurz, M. 1996. Rational Preferences and Rational Beliefs, 339-361, in Arrow, J. K., E. Colombatto, and M. Perlman (Eds.), *The Rational Foundations of Economic Behaviour*. (1996), Macmillan Press Ltd., London.

Kyle, A., 1985. Continuous Auctions and Insider Trading, *Econometrica*, Vol. 53, pp. 1315-1336.

Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny, 1992. The Impact of Institutional Investors on Stock Prices. *Journal of Financial Economics*, 32, 23-43.

Lakonishok, Josef, Andrei Shleifer, Richard H. Thaler and Robert W. Vishny, 1991: Window dressing by pension fund managers, *American Economic Review* 81, 227-231.

Merton, R. C., 1971. Optimum Consumption and Portfolio Rules in a Continuous-Time Model. *Journal of Economic Theory*, 3, 373-413.

Shiller, R. J. 1981. Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *American Economic Review,* 71 (3), 421-36.

Summers, L. H. 1986. Does the Stock Market Rationally Reflect Fundamental Values? *Journal of Finance*, 41, 591-601.

Tirole, J. 1982. On the Possibility of Speculation Under Rational Expectations. *Econometrica*, 50. 1163-81.

Zeira, J., 1999. Informational overshooting, booms and crashes. *Journal of Monetary Economics* 43. 237-57.

Figure 1
Expected Share Price with Individual Agents Only

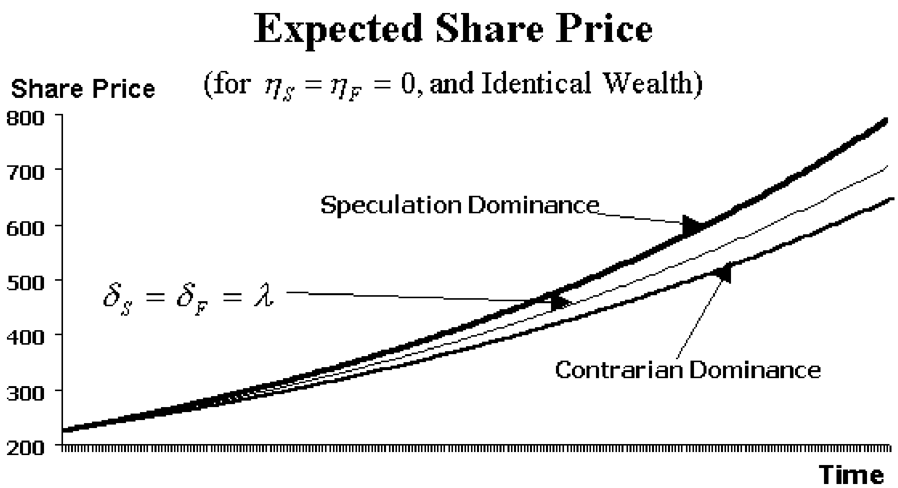
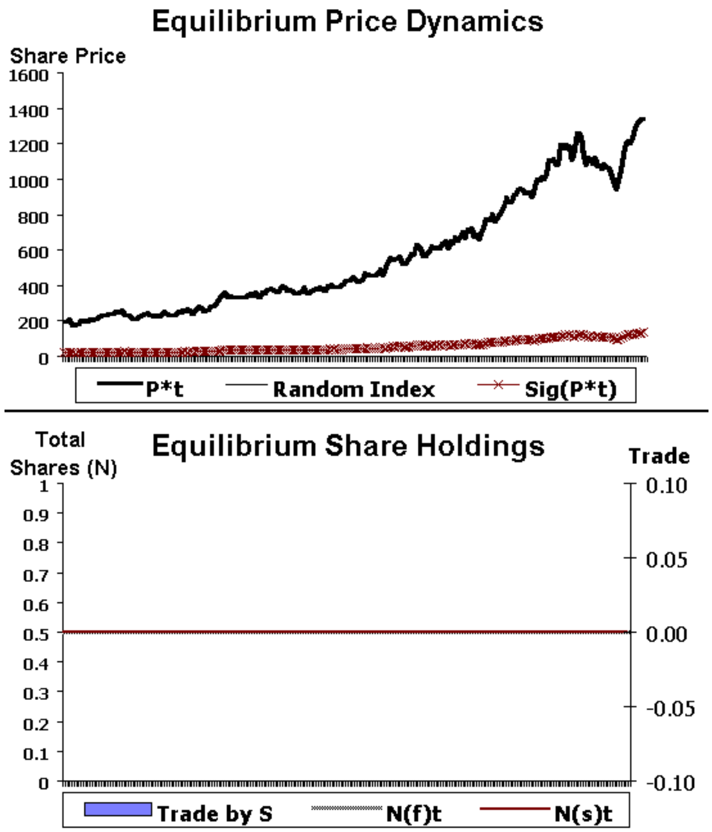


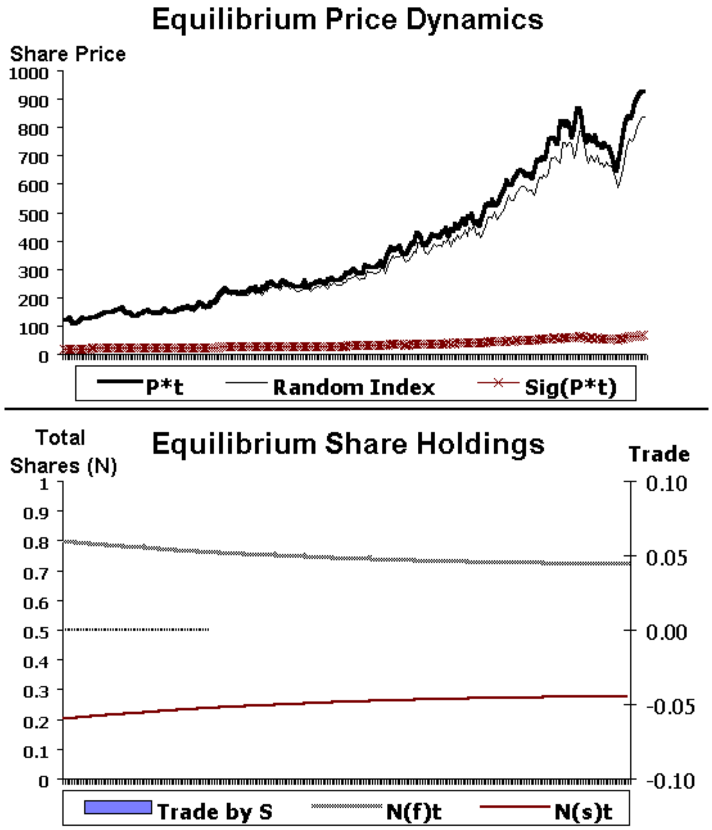
Figure 1 presents the expected share price path that will prevail when either group dominates.

Figure 2a
Equilibrium Price Dynamics (top) and
Share Holdings and Trade (bottom) for
Tastes: $\delta_S = \delta_F = \lambda$ and $\eta_S = \eta_F = 0$



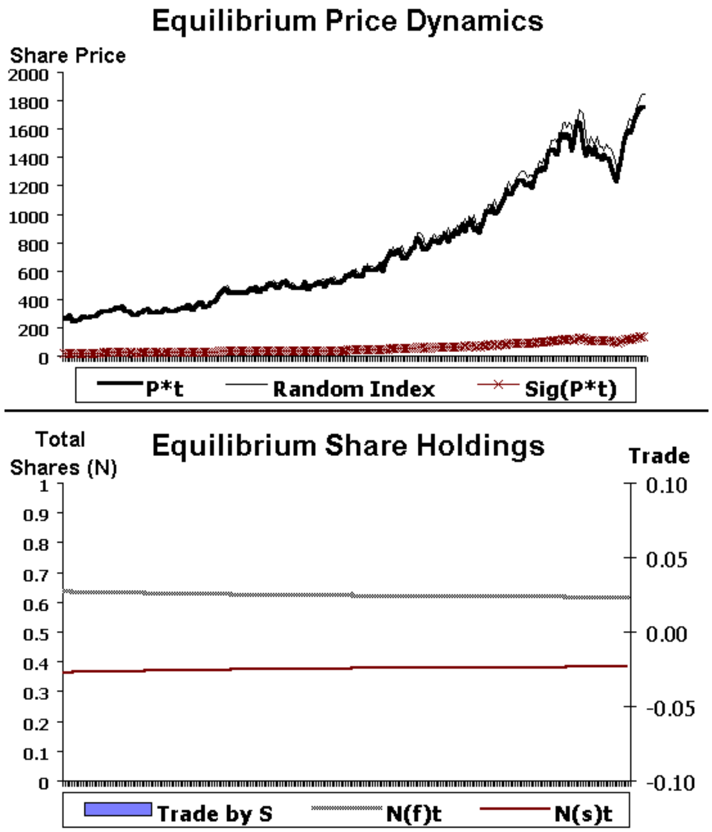
In this simulation, a vector of random numbers drawn from a standard normal distribution has been used to generate a path that represents the real capital asset value (Random Index). The calculated stock price (P*t) coincides with the Random Index thus a single line is visible. In the lower figure, we see that the proportional holdings between both agents are fixed throughout the simulation and there is no trade. Other parameters are: $\lambda = 1.875, N_S = N_F = 0.5, D_S = D_F = 0, S_S = S_F = 100$.

Figure 2b
Equilibrium Price Dynamics (top) and
Share Holdings and Trade (bottom) for
Tastes: $\delta_S = \delta_F = \lambda$ and $|\eta_S| > \eta_F$



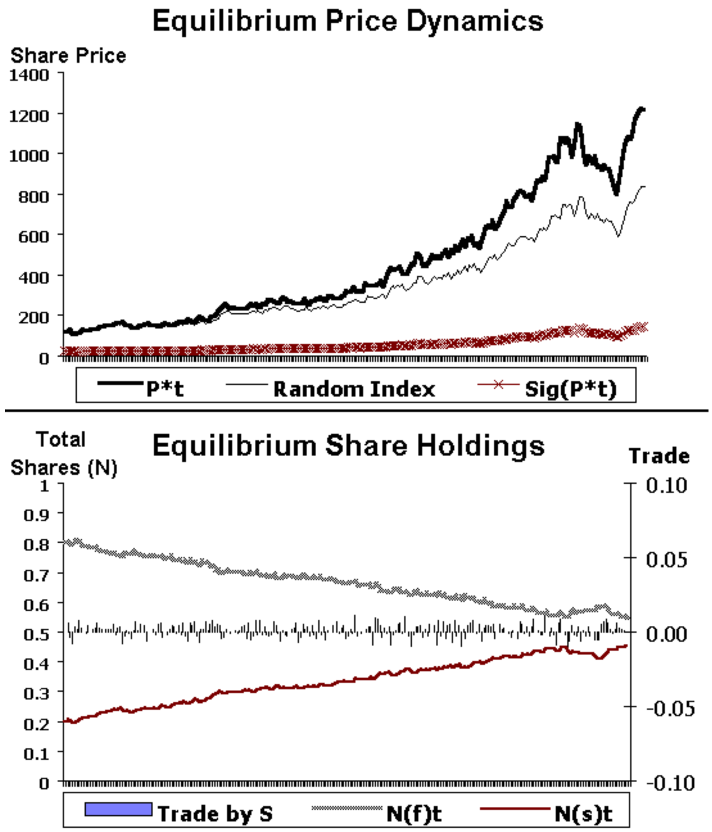
The same vector of random numbers as in Figure 2a that represents the real capital asset value (Random Index) has been used here. The calculated stock price (P*t) is higher than the Random Index, representing a higher average rate of return on the stock, as compared with that of the real capital asset. In the lower figure, we see that the proportional holdings between both agents are not fixed in time, yet asymptotically get stable. There is little trade at the beginning of the simulation, but as holdings become stable, trade vanishes. Other parameters include the following: $\lambda = 1.875, \eta_S = -40, \eta_F = 0.$ Note: Trade is presented in quantity of shares traded by agent *S*.

Figure 2c
Equilibrium Price Dynamics (top) and Share Holdings and Trade (bottom) for Tastes: $\delta_S = \delta_F = \lambda$ and $|\eta_S| < \eta_F$



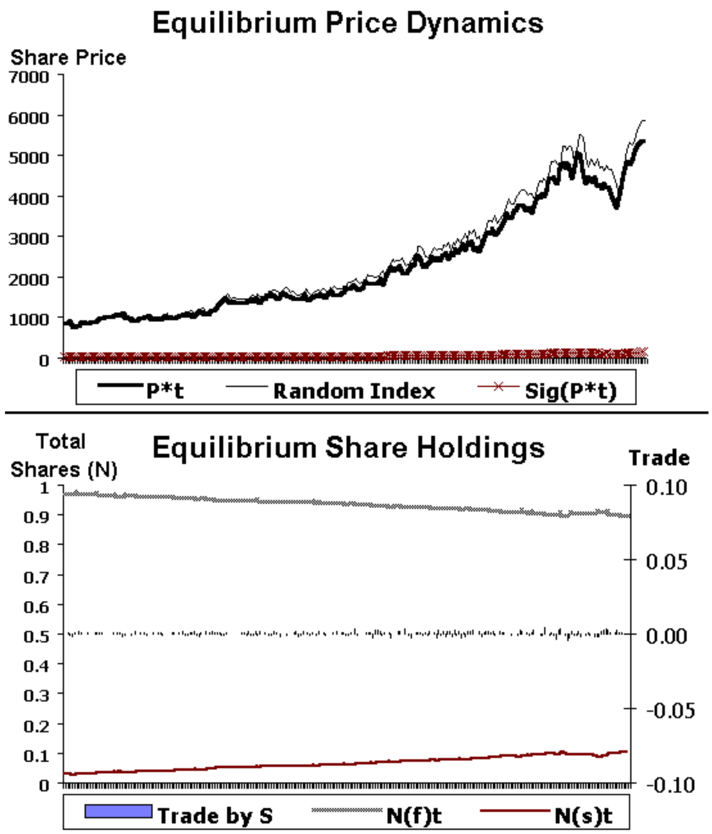
This figure uses the same vector of random numbers as in Figures 2a and 2b to represent the real capital asset value (Random Index). The calculated stock price (P*t) here is lower than the Random Index, representing a lower average rate of return on the stock, as compared with that of the real capital asset since the speculative strategy dominates the stock market. In the lower figure, we see that the proportional holdings between both agents, again, change in time, and asymptotically get stable. There is little trade, which is not visible graphically. Main parameters include: $\lambda = 1.875, \eta_S = 0, \eta_F = 40.$ Trade is presented in quantity of shares traded by agent *S*.

Figure 3a
Equilibrium Price Dynamics (top) and Share Holdings and Trade (bottom) for Tastes: $\delta_F = \lambda, \delta_S < \lambda$ and $\eta_S + \eta_F = 0$



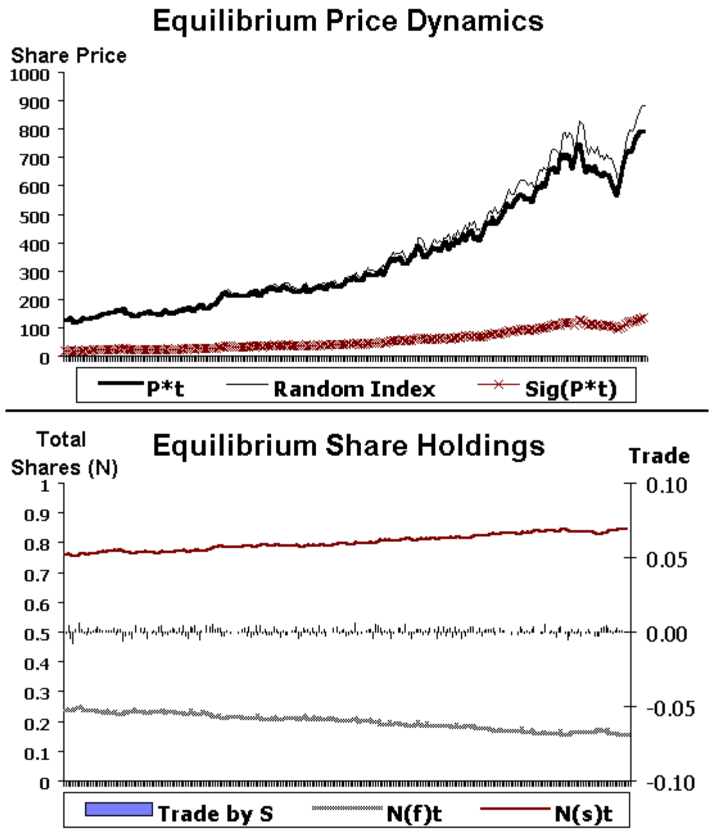
In this figure, we use the same vector of random numbers as in Figures 2a-c, representing the real capital asset value (Random Index). The calculated stock price (P*t) here is significantly higher than the Random Index, representing a speculative bubble. This bubble is more significant than the one presented in Figure 2a since the excess supply of speculation that stems from the diversion $\delta_S < \lambda$ increases the impact on trade. This can be seen in the lower figure, where the proportional holdings between the agents changes faster with much more trade. The main parameters here are: $\lambda = 1.875, \delta_S = 1.25, \delta_F = 1.875, \eta_S = 66.67, \eta_F = 0.$ Trade is presented in quantity of shares traded by agent *S*.

Figure 3b
Equilibrium Price Dynamics (top) and
Share Holdings and Trade (bottom) for
Tastes: $\delta_F = \lambda, \delta_S < \lambda$ **and** $\eta_S + \eta_F > 0$



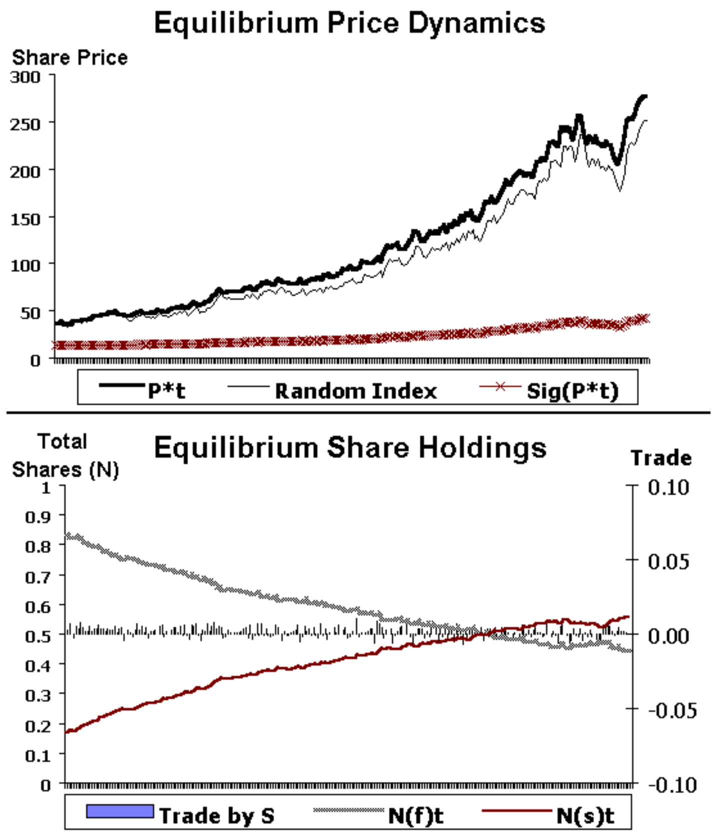
Using all data as in Figure 3a but $\eta_F = 400$, we see that as Proposition 4 states, $\eta_S + \eta_F > 0$ may reduce the bubble below the real capital asset path (Random Index). Trade volume (measured in quantity of shares) reduces significantly.

Figure 4a
Equilibrium Price Dynamics (top) and Share Holdings and Trade (bottom) for Tastes: $\delta_F > \lambda, \delta_S = \lambda$ and $\eta_S + \eta_F = 0$



As in all simulations, we use the same random path for the real capital asset as presented in previous figures. Here we see that dominance of the fundamental strategy reduces the expected growth rate and volatility of the traded stock. The parameters here are: $\lambda = 1.875, \delta_S = 1.875, \delta_F = 3, \eta_S = 0, \eta_F = 0.$

Figure 4b
Equilibrium Price Dynamics (top) and
Share Holdings and Trade (bottom) for
Tastes: $\delta_F > \lambda, \delta_S = \lambda$ **and** $\eta_S + \eta_F < 0$



Using the same random path for the real capital asset as before, we see that although $\delta_F > \lambda, \delta_S = \lambda$, there exist values $\eta_S + \eta_F < 0$ for which the speculative strategy more than offsets the effect of the fundamental strategy through the displacement factor. The parameters here are: $\lambda = 1.875, \delta_S = 1.875, \delta_F = 3, \eta_S = -50, \eta_F = 0$.