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George W. Blazenko
Simon Fraser University

Andrey D. Pavlov
The Wharton School, University of Pennsylvania

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Investment Timing for New Business Ventures

George W. Blazenko[a,]* Andrey D. Pavlov[b]

[a]*Faculty of Business Administration, Simon Fraser University,8888 University Way, Burnaby, BC, Canada, V5A 1S6*

[b]*The Wharton School, University of Pennsylvania*

Abstract

A key requirement for the start of many entrepreneurial businesses is private equity or venture capital financing. In the traditional approach to entrepreneurial investment analysis, an entrepreneur starts a new venture and a venture capitalist finances the new venture when business return exceeds the financial opportunity cost for comparable risk—the cost of capital for the new venture. The real options literature recommends that entrepreneurs delay business start due to investment irreversibility until business return reaches a threshold greater than the cost of capital. In this paper, we show that for new ventures with modest earnings volatility, an entrepreneur starts his/her business before return exceeds the cost of capital. We identify the circumstances in which the cost of capital is an unduly conservative return benchmark for the start of a new business and discuss the empirical implications of our findings.

Keywords: New ventures, business start, corporate investment.

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[b] Contact information: *Tel.: +1 778 782 4959; fax: +1 778 782 4920; *E-mail address:* blazenko@sfu.ca, apavlov@wharton.upenn.edu

Introduction

The discovery of business opportunities is an essential feature of entrepreneurship (Shane, 2003). Audretsch (1995) reports evidence that entrepreneurs often identify these opportunities from knowledge that university researchers or R&D units of large corporations generate but leave uncommercialized due to resource constraints or other impediments. Capital costs often exceed the financial resources of entrepreneurs in manufacturing and in innovative growth industries such as information and biotechnology (Shane, 2008). In these industries, private equity or venture capital financing is a key requirement for starting a new business. In traditional entrepreneurial investment analysis, an entrepreneur starts a new business and a venture capitalist finances the new business when return exceeds the financial opportunity cost for comparable risk—the cost of capital for the new venture (Metrick, 2006). In this approach, the entrepreneur and the venture capitalist have the same outlook on business start which belies the complexity of the relation between them.

Our research identifies a key difference in risk perception between entrepreneurs and venture capitalists that impedes the start of new businesses. Entrepreneurs and venture capitalists see risk in different ways. In fact, their views are so different as to be opposite. For the entrepreneur, risk decreases the appeal of starting a new business and increases the return threshold for doing so. On the other hand, risk increases the value of a new venture's option features which increases the price that a venture capitalist pays for fractional equity ownership and decreases the expected return he/she will accept. The cost of capital embodies the venture capitalist's financial perspective on risk but not the entrepreneur's business investment perspective that includes an assessment of the option to time the start of the new business.

The primary decision maker that we investigate is the entrepreneur for start of a new business. We presume, for the most part, that the venture capitalist is ready to invest in the equity of a new venture at the request of the entrepreneur at a price that just compensates for risk. The supply of entrepreneurial opportunities is limited and venture capital funding is unconstrained. Alternatively, as either entrepreneurial ideas or the supply of venture capital funds waxes and wanes in the economy, relative bargaining power shifts to or from the entrepreneur relative to the venture capitalist. Because of their different views on risk, we identify the types of new ventures most likely to be funded and in what circumstances.

Unlike the traditional approach to entrepreneurial investment analysis that requires the cost of capital as a return benchmark for new business start, the real options[1] literature recommends that entrepreneurs delay business start until return exceeds the cost of capital. McDonald and Siegal (1986) and Dixit and Pindyck (1994) base their investment timing analysis on economic

[1] Because the timing of business investments and disinvestments in a dynamic setting is equivalent to exercising financial options, the study of business investments and disinvestments with modern tools of financial economics is referred to as "real options." See McDonald and Siegal (1986) for early research on investment timing and Dixit and Pindyck (1994) for an introduction to real options methods.

models with an exogenous new venture cost of capital that is invariant to profit volatility. They model the start decision for a new business as the exercise of a perpetual call option. Investment irreversibility forces the entrepreneur to delay business start until return exceeds the cost of capital and net present value (NPV) is positive. However, standard call option pricing (Galai and Masulis, 1976) shows that volatility increases the expected payoff relative to the expected cost of buying an asset through the exercise of a call option. An increase in payoff relative to cost is a leverage (risk) reduction that decreases expected option return. Because a new venture is an option to start an operating business, its expected return—the cost of capital for the new venture—decreases with volatility. This decrease represents the venture capitalist's attraction to risk which contrasts with the entrepreneur's aversion to risk that a new business creates. Our approach differs from McDonald and Siegel (1986) and Dixit and Pindyck (1994) because we determine the cost of capital endogenously rather than impose it exogenously.

Since options features are a primary determinant of a new venture's value, the relation between the cost of capital and earnings volatility is of critical importance in the entrepreneurial decision to start a new business. We find that when earnings volatility is modest, the cost of capital for the new venture is great and exceeds the return threshold for new business start that the entrepreneur sets to maximize value.[2] An entrepreneur should not delay business start until return exceeds the cost of capital. On this point, our results contradict current recommendations in the finance literature and show that both the real options and the traditional approaches to entrepreneurial business start decisions are improperly conservative.

If the cost of capital is an unreliable benchmark for the start of a new business, is there better guidance available for entrepreneurs? One possibility is for an entrepreneur to mimic the expansion decisions of otherwise similar operating businesses. To investigate this benchmark, we compare the value maximizing return threshold for start of a new business with the value maximizing return threshold for expansion of an otherwise similar already operating business.[3]

We find that the motivation to invest is stronger for managers of already operating businesses in their expansion decisions compared to entrepreneurs in their business start decisions. If managers of operating businesses do not expand, they face a reduction in the dollar value of future expansion investments upon improved profitability. Today's expansion investments, even if only modestly profitable, make the business larger which reduces constraints, such as financing, on future expansion investments. This "size imperative" encourages current investment. Since there are no assets, new ventures effectively have no size and no size imperative. Thus, an entrepreneur has stronger aversion to investment irreversibility for the start of an operating business than does the manager of an already operating business for

[2] The entrepreneur delays the start of an operating business until the hypothetical return to that business equals a return threshold. In this paper, we refer to that return threshold as the entrepreneur's value maximizing return threshold for business start.

[3] The manager of an already operating business expands profitability with incremental investment when business return exceeds the "value maximizing return threshold for business expansion." On the other hand, if business return falls below this threshold, the manager suspends growth until profitability improves.

expansion. Under the same conditions, an entrepreneur delays the start of a new business when the manager of an already operating business expands.

Dixit and Pindyck (1994) show that value maximizing profit thresholds for investment are greater in earlier compared to later stages of a sequential investment. While our model of timing the start of a new business and expansion of an already operating business also has this feature, it arises from different economic forces. Dixit and Pindyck investigate a sequentially staged investment, which, when *all* stages are complete, generates operating income. They give the example of oil production where oil is first discovered through exploration and then oil is recovered with wells and pipelines before the investment earns a profit. With only one remaining development stage, the investment cost from the first stage is sunk and does not impact the decision to complete the investment in the second stage. Because the remaining investment cost in the second stage is lower than the sum of investment costs in the two stages, the decision to proceed with the investment in the second stage does not require dollar profit as great. The fact that the dollar profit threshold is not normalized with remaining investment costs limits their findings. Instead, we compare *return* thresholds between the start of a new business and expansion of an already operating business which does not have this limitation.

The literature on new venture start identifies various factors that influence the investment decisions of entrepreneurs: partial reversibility,[4] the price of capital,[5] learning,[6] capital stock adjustment costs,[7] construction lags,[8] resolution of uncertainty,[9] agency costs,[10] growth constraints including financing,[11] and strategic investment induced by competition.[12] All these factors modify the degree to which an entrepreneur defers an irreversible investment. However, apart from Blazenko and Pavlov (2009), none of these factors reverses the deferral option to induce investment earlier than recommended by cost of capital analysis in a dynamic setting.

Blazenko and Pavlov (2009) show that managers of operating businesses accelerate an indefinite sequence of expansion investments that have constant returns to scale and that neither displace nor impair future returns. The value maximizing return threshold for expansion of an already operating business is *always* less than the cost of capital. Managers expand their businesses before return reaches the cost of capital. Our results differ because we study a new venture which, unlike an already operating business, is not subject to the size imperative. Since a new venture does not yet have operations, the start option for the new venture is a larger portion of value than is the expansion option for an already operating business. This greater option value makes the cost of capital for a new venture exceptionally sensitive to earnings volatility. When

[4] Kandel and Pearson (2002).
[5] Abel et al. (1996).
[6] Bergemann and Hege (1998), Grenadier and Weiss (1997), Lévesque et al. (2009).
[7] Abel and Eberly (1994).
[8] Aguerrevere (2003).
[9] Berk et al. (2004), MacMillan et al. (2006), Choi et al. (2008).
[10] Bergemann and Hege (1998), Grenadier and Wang (2005).
[11] Boyle and Guthrie (2003), Mauer and Triantis (2005), Hirth and Uhrig-Homburg (2010), Blazenko and Pavlov (2009), Pavlov et al. (2004).
[12] Cottrell and Sick (2002), Dewit and Leahy (2006).

earnings volatility is great and the entrepreneur's aversion to business start is correspondingly high, the entrepreneur delays business start.

In section I, we model a new venture as a perpetual call option on an operating business. We derive the return threshold for start of a new business and compare it to the expansion threshold of an otherwise similar operating business. In Section II, we derive the new venture cost of capital and compare it to the value maximizing return threshold for start of a new business. Section III presents comparative static results and discusses empirical implications. Section IV concludes and offers directions for future research.

I. Return Thresholds for the Start of a New Business Versus Expansion

Shane (2003) reviews the evidence from the entrepreneurship literature that individual characteristics such as employment, income, and marital status are primary determinants of the opportunity costs that entrepreneurs face in starting a new business. In the context of the knowledge spillover theory of entrepreneurship (Acs et al., 2009), a prospective entrepreneur might be employed in a new knowledge institution, like a research university or the R&D unit of a large firm, and weighs his/her opportunity costs with the possibility of embarking on a business prospect left uncommercialized by his/her employer. Opportunities arise exogenously with little or no cost to the entrepreneur who awaits minimum profitability before starting a new business.

For our purposes, a new venture is a business opportunity not yet economically viable and the "entrepreneur" is the economic agent that directs the new venture. In contrast, the "manager" is the economic agent that has already made the irreversible investment that starts an operating business that converts potential earnings into actual earnings. Thereafter, the manager has an indefinite option to expand his/her business.

A. The Operating Business

Because a new venture is a call option on an operating business, before we determine the value of the new venture, we first find the value of the operating business that would exist if the entrepreneur hypothetically started the business immediately. We use Blazenko and Pavlov's (2009) model of an operating business where the manager has an indefinite sequence of expansion options. Profit growth requires these expansion investments.[13] The manager can suspend business expansion at any time upon inadequate profitability and then restart upon improved profitability. This model has the attraction that it has a closed-form solution for the value maximizing return threshold for expansion of the operating business.

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[13] Our use of Blazenko and Pavlov's (2009) model for an operating business does not restrict the generality of our results. Our conclusions arise from the primary result that the cost of capital for a new venture increases without bound as volatility of potential earnings approaches zero (see, section II B). Because this result has an analogy in standard option pricing for call options, our results are robust to changing the underlying operating business that one might use. We uncover novel results not because of the nature of the operating business that we model, but because we endogenously determine the cost of capital rather than adopt it as an exogenous constant.

A manager controls the level of a firm's capital stock,[14] $B_t > 0$, by undertaking irreversible expansion investments at the maximum instantaneous rate $0 \le dB_t / B_t \le gdt$. The maximum uncompounded per annum rate of capital growth is the parameter *g*. A constant returns to scale technology with stochastic return on capital, Y_t, generates operating cash flow that we denote as X_t. That is, $X_t = Y_t B_t$. When the manager expands the business, the growth rate of cash flow equals that of capital *g*. On the other hand, when the manager suspends growth, then neither capital, *B*, nor cash flow, X_t, grows.

The return on capital follows a non-growing geometric diffusion, $dY_t / Y_t = \sigma dz$, where, σ is volatility of both the return on capital and operating cash flow, X_t, and dz is a Wiener process. There is no growth in capital efficiency for expansion investments or the existing business. That is, the return on capital does not grow,[15] $E\left[\tilde{Y}_t\right] = Y_0$. Irreversible investment is the sole cost of growth without restriction or other costs.

Business return is the return on capital, cash flow divided by capital, $Y \equiv \dfrac{X}{B}$, rather than the return on capital plus a growth factor *Y+g*. Return on capital plus a growth factor is the business return for a hypothetical investment with spontaneous profit growth—like a stand of timber that does not require ongoing capital investment.[16] However, this is not the nature of the investment we study. In our case, profit growth requires capital growth. Either in-place assets or expansion investments generate a non-growing perpetual stream of expected profit, *Y*, per dollar of capital. Regardless of the magnitude of the constraint on capital investment, *g*, business return, the internal rate of return (IRR), satisfies Y/IRR-1=0 which means that IRR=Y.

V(X) denotes the value of the operating business—its sellable value after the start of the new business by the entrepreneur. This amount is equivalent to present value (PV) as opposed to net present value (NPV) in static investment analysis. The value of the operating business, *V(X)*, is capital, *B*, times the market/book ratio which is a function of the return on capital, *Y*. Denote the market/book ratio for the operating business as $m(Y)$. Then, *V(X)=B** $m(Y)$. The value maximizing return threshold for expansion, ξ^*, separates the market/book ratio that we report in Appendix A, into growth and no growth branches. Figure I depicts the market/book ratio, $m(Y)$, for a numerical example. The manager expands profitability, *X*, with incremental capital,

[14] Capital stock includes, for example, working capital and depreciable assets. Corporate process improvements that enhance profitability require human capital investments. While capital stock, *B*, can conceptually include these investments, it then diverges from commonplace accounting for corporate assets.

[15] Abel et. al (1996) show that managers invest early when they expect the price of capital to increase.

[16] The static environment illustrates the point. If permanent profit growth at the rate g requires capital growth at the rate g, then, the IRR satisfies (X-g*B)/(IRR-g)-B=0, and, IRR=Y regardless of the growth factor, g. For comparison purposes, for spontaneous profit growth, the IRR satisfies X/(IRR-g)-B=0, and, IRR = Y+g.

B, at the rate gdt when the return on capital exceeds the expansion boundary, $Y \geq \xi^*$. On the other hand, if the return on capital falls below the expansion boundary, $Y < \xi^*$, then manager suspends growth until profitability improves.

B. The New Venture

In this subsection, we determine the value of a new venture as a perpetual call option on the operating business from the previous subsection. We then determine the value maximizing return threshold that the entrepreneur uses to start the operating business. Last, we compare the return threshold for start of the operating business with the return threshold that the manager of the operating business uses for expansion (that is, ξ^* from the previous section).

Prior to the start of the operating business, earnings, *X*, are potential earnings—earnings that would accrue if the entrepreneur hypothetically started the business immediately. Because earnings growth requires capital growth and because the entrepreneur undertakes no capital investment prior to starting the business, we presume no earnings growth,[17] $\frac{dX}{X} = \sigma dz$.

Let $I > 0$ be the required irreversible investment that the entrepreneur uses to start the operating business. Return on capital upon business start is $Y = \frac{X}{I}$. Once the entrepreneur starts the business, capital of the now operating business is this amount, $B = I$. Thereafter, both earnings and capital of the operating business grow at the discretion of the manager.

Let $P(X,I)$ be the value of the new venture prior to start, which is equivalent to NPV in a static investment analysis. Then, $P(X,I) = I\pi(Y)$, where $\pi(Y)$ is the market/book[18] ratio for the new venture that depends on the return capital, *Y*. Appendix B describes how to find the market/book ratio which is,

$$\pi(Y) = cY^{\alpha} \tag{1}$$

where c is an arbitrary constant and Equation (A2) in Appendix A gives the value of the parameter α.

C. The Value Maximizing Return Threshold for the Start of a New Business

Denote the value maximizing return threshold for start of the new business as ψ^*. In the first instance, the return on capital is below the start boundary, $Y < \psi^*$. When the return on capital increases to the start boundary, $Y = \psi^*$, the entrepreneur irrevocably starts the new business by making the dollar investment *I*. He/she then becomes the manager of the new

[17] Alternatively, we could presume earnings growth without capital growth, like a stand of timber, without changing the substance of our results. However, this spontaneous growth is unrealistic for most industrial and commercial enterprises.

[18] Prior to investment, "book" is the investment cost *I*.

operating business. The manager expands the operating business when the return on capital exceeds the expansion boundary, $Y \geq \xi^*$, and suspends expansion otherwise. When the return on capital increases once more above the expansion boundary, the manager recommences growth. While growth investments are irreversible, the manager's decision to grow or not is provisional and reversible.

Appendix B describes the value matching and smooth pasting conditions that jointly determine c, the arbitrary constant in Equation (1), and ψ^*, the value maximizing return threshold for start of the new business. This return threshold satisfies the equation,[19]

$$\psi^* = \frac{(r^* - g)}{r-g} \frac{\alpha}{\alpha - 1}\left[r - \frac{g}{(1-\lambda)}\left(\frac{\psi^*}{\xi^*}\right)^{\lambda}\right] \tag{2}$$

where Equation (A2) and Equation (A3) in Appendix A give expressions for the parameters α, λ, and the value maximizing expansion boundary for the operating business, ξ^*. The parameter, *r*, is risk free interest rate. The parameter *r** is the risk adjusted rate[20] for an operating business that hypothetically never grows. Equation (2) has no closed-form solution for ψ^*, except in the special case when the operating business never grows, *g*=0. Alternatively, we determine the return threshold for start of a new business, ψ^*, numerically for our analysis that follows.

For a numerical example, Figure I plots one plus[21] the market/book ratio for the new venture prior to business start, $1+\pi(Y)$, and the market/book ratio for the otherwise similar operating business, $m(Y)$. As the return on capital, *Y*, increases from the left, convergence of the first function to the second illustrates that the return threshold for start of the new business, $\psi^* = 0.126$, exceeds the return threshold for expansion of the operating business, $\xi^* = 0.0927$, by over 300 basis points.

D. Comparing the Return Thresholds

In this subsection, we compare the value maximizing return threshold that the entrepreneur uses to start the operating business, ψ^*, to the value maximizing return threshold that the manager of the operating business uses for expansion, ξ^*.

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[19] Substitute Equation (A5) into Equation (A6) to find Equation (2).

[20] The risk adjusted rate for a firm that permanently does not growth is $r^* \equiv r + \theta\sigma_{x,c}$ which is the risk free interest rate *r* plus a risk premium $\theta\sigma_{x,c}$. The parameter θ is constant relative risk aversion for a representative investor. The parameter $\sigma_{x,c}$ measures business risk and equals the covariance of log earnings with the log of aggregate consumption in the economy. For expositional simplicity, we presume a positive risk premium, $\theta\sigma_{x,c} > 0$.

[21] Value matching in Equation (A5) says that the value of the new venture immediately prior to the start of the new business equals the net value of the operating business immediately after the start (per one dollar of capital).

Because the market/book ratio for the operating business, *m(Y)*, has two branches in Equation (A1)—for growth and no-growth—it is not obvious which branch jointly determines c and ψ^* in Equations (A5) and (A6). In Appendix C, we prove that the return threshold for start of the new business, ψ^*, is on the growth branch (presuming the contrary leads to a mathematical contradiction). Because the return threshold for expansion of the operating business, ξ^*, separates the two branches (growth and no growth), this result establishes that the return threshold, ψ^*, for the start of the new business exceeds the return threshold for expansion of the already operating business, $\psi^* \geq \xi^*$.

E. Discussion

Even though expansion is an identical scaled copy of the investment that starts the business, the value maximizing return threshold for new business start exceeds the value maximizing return threshold for expansion of the operating business, $\psi^* \geq \xi^*$. An entrepreneur delays the start of the new business under the same conditions that the manager of the operating business expands. Equivalently, the manager expands a newly started business even when profitability falls immediately after business start, $\psi^* > Y \geq \xi^*$. The manager expands a newly started business even though in an earlier period—as the entrepreneur of the new venture—he/she delayed business start.

The restriction on growth, *g*, encourages expansion of the operating business in order to increase the size and dollar value of future growth investments. Future growth investments are proportional to capital at that time. Future capital is greater and the dollar value of future growth investments is greater, if the manager makes expansion investments now. A low expansion boundary reflects the benefit of today's expansion investment for future expansion investments (that is, $\xi^* \leq \psi^*$).

The entrepreneur of the new venture does not have the same imperative to start the business that the manager of the operating business has to expand. For the new venture, the likelihood of any sequence of earnings increments after business start does not change with the start date of the new business. The reason for this probabilistic feature of earnings increments is the "memoryless" property of all Markov stochastic processes. The memoryless property says that the probability of future earnings increments does not depend on past earnings increments. So, firm size (that is, the capital of the operating business), is probabilistically the same at any fixed time interval after business start. Because in-place assets do not yet exist for a new venture, there is no imperative for the entrepreneur to start the business in the first instance. Rather, the entrepreneur delays the start of the new business in order to avoid downside earnings risk, which the manager of the operating business cannot avoid, having irreversibly started the business that now generates risky earnings.

F. Volatility and Return Thresholds

Figure II plots the entrepreneur's return threshold for start of a business, ψ^*, against earnings volatility, σ. Volatility decreases the appeal of business start to the entrepreneur for the same reason that start boundaries increase with volatility in McDonald and Siegel (1986) and Dixit and Pindyck (1994). The entrepreneur avoids disappointing future earnings outcomes and protects the value of the entrepreneurial opportunity with the delay of business start. At the same time, delay does not prevent the entrepreneur from starting the business in the future when the earnings potential of the new venture improves. Because downside protection is more important than upside earnings potential for the entrepreneur, and because downside risk increases with volatility, the return threshold for start of the business, ψ^*, increases with earnings volatility, σ.

In Appendix D, we prove a number of results related to earnings volatility. First, we prove $\lim_{\sigma \to 0+} \psi^* = r^*$. This result says that for new ventures without earnings volatility, the return start boundary is the same as the cost of capital for a hypothetical business that permanently does not grow, r^*. With no volatility, there is no value protection from the delay of new venture start and the new venture return start threshold falls (see, Figure II). In addition, with no volatility, the value maximizing expansion threshold for the already operating business also approaches the cost of capital for a permanently non-growing business. That is, $\lim_{\sigma \to 0+} \xi^* = r^*$. Without volatility, there is no size benefit to expansion investment for the operating business. Without this attraction, the expansion boundary increases (see, Figure II). Combining these two results, the return threshold for start of the new business converges to the return threshold for expansion of the already operating business when volatility approaches zero. That is, $\lim_{\sigma \to 0+} \psi^* = \lim_{\sigma \to 0+} \xi^* = r^*$.

Second, we prove that the return threshold for start of a new business becomes unboundedly great with earnings volatility. That is, $\lim_{\sigma \to \infty} \psi^* = \infty$. This result says that value protection from delaying business start increases with volatility.

These two results are essential pieces in the argument for why an entrepreneur starts a new business when return is less than the cost of capital for businesses with low earnings volatility. The completion of the argument requires that we derive the cost of capital for the new venture which we do next.

II. New Venture Start Boundary Versus the New Venture Cost of Capital

A. New Venture Cost of Capital

Since Modigliani and Miller's (1958) seminal investigation of the impact of corporate financial structure on hurdle rates for business investment, academics and practitioners have measured the cost of capital as the expected return on the market value of an asset. Modigliani and Miller analyze a static business environment in which a manager permanently commits to business expansion at a fixed per annum rate. In the dynamic business setting that we investigate, the manager makes moment-by-moment decisions to expand the business and the entrepreneur makes moment-by-moment decisions to start a new business. Consequently, an entrepreneur trained in traditional investment analysis (a "cost of capital" entrepreneur), compares business return to the cost of capital every instant as a hurdle rate for the start of a new business.

The cost of capital is the expected return on a venture capitalist's financial investment in a new venture prior to business start and is also the average return that a venture capitalist earns on a portfolio of similar new venture investments. Therefore, the cost of capital can be calculated or estimated from observed financial market returns. In this section, we investigate the investment errors that an entrepreneur makes by using this observable financial market return as a benchmark to start a new business.

For a new venture prior to start, there is no business income, no capital growth, and no expected growth in business income. Thus, the cost of capital is the expected capital gain from the convexity of the new venture value function divided by the value of the new venture in Equation (1). Convexity represents the option features of the new venture. We use the symbol, ω, to represent the cost of capital for the new venture. Then,[22]

$$\omega = \frac{\frac{1}{2}\frac{\partial^2\left[cIY^{\alpha}\right]}{\partial Y^2}Y^2\sigma^2}{cIY^{\alpha}} = \alpha(\alpha-1)\sigma^2/2$$

$$= r + \frac{\theta\sigma_{x,c}}{2} + \left(\frac{\theta\sigma_{x,c}}{\sigma}\right)^2 + \theta\sigma_{x,c}\sqrt{\frac{2r}{\sigma^2} + \left(\frac{1}{2} + \frac{\theta\sigma_{x,c}}{\sigma^2}\right)^2} \quad (3)$$

From Equation (3), we can establish a number of features of the cost of capital for the new venture.

B. New Venture Cost of Capital Features

We identify seven features of the cost of capital for the new venture. First, the cost of capital, ω, decreases with earnings volatility, σ. The expected return on the new venture prior to start is the expected return on a call option: the option to start the business. In standard option pricing (Galai and Masulis, 1976), volatility decreases call option risk which decreases expected return. Thus, volatility dissipates leverage and decreases the new venture's cost of capital, ω.

This inverse relation between volatility and return is critical to the performance evaluation of venture capital portfolios. Skewness and volatility are related because in a lognormal distribution—such as the distribution for the return on capital, $\tilde{Y}$—volatility increases positive skewness. A large literature documents positive skewness for new venture returns and interprets positive skewness as great risk and, therefore, looks for high returns (Grabowski and Vernon 1994, 1999; Grabowski et al., 2002; Kolbe et al., 1991). However, our analysis indicates that because positive skewness arises from volatility, new ventures with great earnings volatility have low leverage and should have low returns. For a venture capital fund with a fixed dollar amount to invest, a large number of low volatility, low priced, new venture investments should have a

[22] Use Ito's lemma, and substitute Equation (1) to find Equation (3).

greater return than a small number of high volatility, high priced, and highly skewed new venture investments.

Second, for unbounded volatility, the cost of capital for the new venture approaches the cost of capital for a hypothetical non-growing business. That is, $\lim_{\sigma\to\infty}\omega = r^*$. Figure II depicts this result for a numerical example. Unbounded volatility effectively eliminates leverage and decreases the risk of a new venture to that of a permanently non-growing business.

Third, as volatility approaches zero, the cost of capital becomes unboundedly large. That is, $\lim_{\sigma\to 0+}\omega = \infty$. Figure II depicts this result for a numerical example. Both the expected capital gain from the curvature of the value function (the numerator of ω) and business value (the denominator of ω) approach zero as volatility approaches zero. Value approaches zero more rapidly than does dollar risk and, therefore, the cost of capital increases without bound as volatility approaches zero. Low volatility decreases business value which makes risk per dollar invested exceptionally great.

Fourth, the return on capital for the new venture, *Y*, does not change the cost of capital. The return on capital changes dollar risk, the numerator of Equation (3), in exact proportion with option value, the denominator of Equation (3), which leaves the cost of capital unchanged. This observation means that latent profitability has no impact on risk prior to the start of a new business. This result is quite distinct from the underlying operating business. Blazenko and Pavlov (2009) show that the cost of capital for the operating business increases as the return on capital *Y* increases from zero but decreases with the return on capital, *Y*, when the return on capital is high.

Fifth, the cost of capital exceeds the riskless rate, $\omega \ge r$, and equals the riskless rate only for zero covariance risk. Of course, this result means that the venture capitalist requires compensation for covariance risk, $\sigma_{x,c} > 0$.

Sixth, capital to start the business, *I*, has no impact on the cost of capital. Equivalently, investment irreversibility is not a determinant of the cost of capital. The reason for this result is that the new venture entrepreneur can time the start of the new business and faces investment irreversibility only at his/her discretion.

Finally,[23] $\dfrac{\partial\omega}{\partial\left[\theta\sigma_{x,c}\right]} > 0$ and $\dfrac{\partial^2\omega}{\partial\left[\theta\sigma_{x,c}\right]^2} > 0$, which says that business risk increases the cost of capital for the new venture more than proportionately. If the operating business is risky, then option leverage makes the new venture dramatically more risky. As an illustration of this leverage, at the far right side of Figure III, when covariance risk is 10%, the cost of capital for a non-growing operating business is $r^* \equiv r + \theta\sigma_{x,c} = 15\%$, whereas, the cost of capital for the new venture is close to 70%. The option features of a new venture increase both risk and required return appreciably over those of an operating business. However, a great financial return does not

[23] Apart from their signs, these derivatives are of little economic interest and, therefore, we do not report them.

mean that the return threshold for start of a new business should be correspondingly great. We compare the value maximizing return threshold for start of a new business with its cost of capital in the next subsection.

C. New Venture Start Boundary Versus the Cost of Capital

In this subsection, we compare the value maximizing return threshold for start of a new business with its cost of capital. First, we make this comparison for new ventures with modest volatility. In Appendix D, we show that $\lim_{\sigma \to 0+} \psi^* = r^*$ and $\lim_{\sigma \to 0+} \omega = \infty$. These two results means that for modest earnings volatility, σ, the cost of capital for the new venture exceeds the return start threshold for start of the new business. That is, $\omega > \psi^*$. The numerical example in Figure II illustrates that the cost of capital for the new venture, ω, exceeds the return threshold for start of the new business, ψ^*, when earnings volatility σ is less than 36.5% per annum.

Second, we compare the return threshold for start of a new business with the cost of capital for the new venture when earnings volatility is great. In Appendix D, we show that $\lim_{\sigma \to \infty} \psi^* = \infty$ and $\lim_{\sigma \to \infty} \omega =$ r*. These results mean that for new ventures with high earnings volatility, σ, the return threshold for start of the new business exceeds the cost of capital for the new venture. That is, $\psi^* > \omega$. The value maximizing entrepreneur delays the start of the new business compared to an entrepreneur who uses the cost of capital as a return benchmark when starting a new business.

The combination of these two sets of results indicates that for modest profit volatility, a new venture entrepreneur starts a new business before return reaches the cost of capital. For great profit volatility, an entrepreneur delays the start of a new business until return exceeds the cost of capital. The first result is opposite to the recommendation from the real options literature to defer the start of a new business due to investment irreversibility (McDonald and Siegal, 1986; and Dixit and Pindyck, 1994). The second result indicates that our previous finding (Blazenko and Pavlov, 2009) that managers expand their businesses before the return on capital reaches the cost of capital does not apply to the start of new businesses. With high earnings volatility, a value maximizing entrepreneur delays the new start of a new business until the return on capital exceeds the cost of capital.

D. Determinants of the Cost of Capital and the Return Threshold for New Venture Start

Different economic forces determine the cost of capital for the new venture, ω, and the return threshold for start of the new business, ψ^*. The primary determinant of the return threshold for start of the new business, ψ^*, is irreversibility of the initial investment, *I*. Earnings volatility, σ, increases both downside earnings risk and upside earnings potential but because the entrepreneur is more concerned with downside risk (which he/she avoids with the delay of business start), the return threshold for start of the new business, ψ^*, increases with earnings volatility, σ. On the other hand, the primary determinant of the cost of capital for the new venture is leverage. Volatility, σ, dissipates leverage which decreases the cost of capital, ω.

Because volatility decreases the cost of capital but increases the return threshold for start of the new business and because these schedules cross, the cost of capital, ω, exceeds the return threshold for start of the new business, ψ^*, for modest volatility, σ. The entrepreneur starts the new business before return exceeds the cost of capital. On the other hand, the return threshold for start of the new business, ψ^*, exceeds the cost of capital, ω, when earnings volatility is great. In this case, the entrepreneur defers the start of the new business until business return exceeds the cost of capital.

E. Discussion

Why does a traditional "cost of capital" entrepreneur erroneously compare business return, Y, to the cost of capital, ω, when making decision to start a new business and then become a value maximizing manager when expanding the business (as implied in the valuation of the already operating business in section I)? The cost of capital for the new venture, ω, is independent of the manager's business expansion strategy during the operating phase of the business. The value of the new venture has the form given by Equation (1) regardless of the manager's investment strategy in the operating phase of the business, which determines only the parameter "c." Smooth pasting and value matching conditions in Equations (A5) and (A6) determine "c" for the value maximizing start of a new business that is subsequently expanded using a value maximizing return threshold for expansion. A non-value-maximizing start with a non-value-maximizing subsequent expansion will have a different set of requirements which together determine the parameter "c" and the return threshold for business start. Nonetheless, the constant "c" cancels in the numerator and the denominator in the determination of the cost of capital for the new venture, ω, in Equation (3). This cancelation means that the cost of capital for the new venture, ω, is invariant to the manager's investment strategy in the operating phase of the business. This discussion highlights the fact that our primary result does not depend on the model we use for the operating business.

F. Hurdles Rates for Business Stages

In this subsection, we discuss the business development stages that we model and the hurdle rates between them. Figure II represents our model of three distinct business stages: venture capital investment, start of the new business, and expansion of the operating business. These business stages require distinct hurdle rates between them. First, a venture capitalist uses the cost of capital as a financial criterion for equity investment in the new venture. Because the venture capitalist makes a financial investment, the hurdle rate for venture capital investment is a financial rate—the cost of capital. Figure II illustrates that for a prospective new business with earnings volatility of 20% per annum, the venture capital rate is 25.5%. Venture capitalists expect this rate of return to purchase shares of the new venture from the entrepreneur. Next, at the point of starting the business, the return threshold for business start is ψ^*=12.6%. Low volatility makes the cost of capital great and the start boundary low, 25.5%>12.6%. Last, the hurdle rate for expansion of the operating business is less than the return threshold for start of the new business, $\psi^*=12.6\%>\xi^*=9.27\%$. The entrepreneur can start the new business only once but can expand it any time thereafter and, thus, the return threshold for start of the new business is more demanding than is the return threshold for expansion of the operating business, $\psi^*\geq\xi^*$.

G. Empirical Estimates of Earnings Volatility

Whether or not the value maximizing return threshold for start of a new business, ψ^*, exceeds the cost of capital, ω, depends on the characteristics of the operating business and the economic environment. Figure II illustrates that the cost of capital for the new venture exceeds the return threshold for start of the new business when volatility, σ, is less than 36.5%. It strikes us that this is a relatively large number and, therefore, we expect that a large fraction of entrepreneurs start their new businesses before return reaches the cost of capital. In order to benchmark this number, we undertake a modest empirical analysis using annual earnings before interest tax depreciation and amortization (EBITDA) for fiscal years 1950 to 2008 from the COMPUSTAT database of NYSE/AMEX/NASDAQ firms. Firms on the COMPUTSTAT database are primarily operating businesses with positive revenues rather than firms in their development phase. Thus, in using COMPUTSTAT companies, we make the presumption that an entrepreneur analyses the start decision anticipating that his/her new venture will become an operating business.

We discard firms that do not have at least five consecutive years of positive EBITDA, leaving a sample of 4492 firms. Firm by firm, we calculate from ordinary least squares (OLS) regression, residual standard deviation from detrended logarithmic EBIDTA as our estimate[24] of earnings volatility, $\hat{\sigma}$. We find that the median value of earnings volatility, $\hat{\sigma}$, across the 4492 firms is 46.0% and decile break points are 15.4, 22.7, 29.9, 37.6, 46.0, 56.6, 67.7, 80.5, and 103.3%. Interpolating, approximately 38.5% of these firms have an estimated earnings volatility, $\hat{\sigma}$, less than 36.5%.

III. Comparative Statics and Empirical Implications

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[24] The regression for firm *i* is, $\log(EBITDA_{i,t})=a+g*t+\varepsilon_{i,t}$ where *t* is an index that represents the fiscal year. The standard deviation of the error term is our estimate of earnings volatility, $\hat{\sigma}_i=\sqrt{\sum_{t=1}^{T}\hat{\varepsilon}_{i,t}^2\Big/(T-2)}$, where *T* is the number of years (from 5 to 59) in the time series for firm *i*. Operating businesses seldom have negative *EBITDA* and, thus, the requirement for positive *EBITDA* is not critical to the conclusions that we make. We find little evidence that $\hat{\sigma}$ varies with *T* (results not reported) which suggests that earnings volatility does not depend on the age of a business and that the earnings volatility of COMPUSTAT firms is not an unreasonable proxy for the earnings volatility that an entrepreneur can expect upon the start of a new business.

In this section, we combine comparative statics with testable implications from our new business start study. Often, empirical tests of real options models of investment timing have been undertaken without due care. For example, because profit thresholds for new venture start increase with profit volatility, Folta et al. (2003) conclude that volatility discourages the start of new businesses. However, this conclusion does not recognize that volatility impacts not only the profit threshold for start of a new business, but also the stochastic process for profitability changes. For realistic levels of profit volatility, Sarkar (2000) shows that volatility increases the probability of reaching a value maximizing profit threshold that itself increases with volatility. This result means that volatility encourages rather than discourages the start of new businesses.

In prior sections, we show that differences exist for investment timing between new businesses and existing businesses. Based on these findings, in the current section, we develop testable implications for the study of new businesses when benchmarked against incumbent businesses in an industry. These implications cannot be derived from the existing real options literature which assumes that new ventures and operating businesses are essentially the same.

A. Industry Entry

A number of testable implications arise from our finding that the return threshold for start of a new business exceeds the return threshold for expansion of an already operating business, $\psi^* \geq \xi^*$. First, because incumbent firms in an industry accept more marginal investments while entrepreneurs of new ventures hold out for greater profitability before business start, when an entrepreneur does start a new business, it is on average more profitable than an otherwise similar incumbent in the industry. Equivalently, because the market/book ratio depends on profitability, the market/book ratio for a newly started business should exceed that of an otherwise similar incumbent in the industry.

Second, as an industry recovers from a recession and corporate earnings improve from depressed levels, incumbent firms expand before industry entry by new businesses becomes commonplace. Empirically, the ratio of operating profit (as well as other measures of business activity) for new entrants compared to incumbents should fall in the early stages of the business cycle, as incumbent businesses expand and entrepreneurs of new ventures defer industry entry. This ratio increases only as the industry approaches the peak of the business cycle and new business entry becomes more frequent.

B. Volatility and the Difference in Return Thresholds

In Figure II, the increasing difference between the return threshold for the start of a business, ψ^*, and the return threshold for expansion of an already operating business, ξ^*, as earnings volatility, σ, increases has a number of testable implications. First, the profitability difference between new businesses and incumbent businesses in an industry should be increasing in the earnings volatility of the industry.

Second, the increasing difference between these return thresholds indicates that volatility encourages the expansion of operating businesses relative to the start of new businesses. Empirically, the ratio of industry growth from new businesses relative to incumbent businesses should be decreasing in the earnings volatility of that industry. Similarly, if earnings volatility is not constant (as we assume in our model) but stochastic, then in periods of high earnings volatility, industry growth should arise to a relatively greater extent from incumbent businesses compared to new businesses.

C. The New Venture Cost of Capital Versus Volatility

We hypothesize an inverse relation between the new venture cost of capital and volatility. This hypothesis arises from the option pricing result of Galai and Masulis (1976) that says that call option returns relate inversely to volatility. Since this result is not widely recognized in the finance literature, it is not surprising that it has not been tested for venture capital investments. However, the venture capital market represents a natural experiment for this hypothesis since a larger fraction of new venture value is option dependent than is the case for already operating businesses. According to Kolbe et al. (1991), venture capitalists prefer positively skewed returns with the potential for high payoff. Because skewness arises from volatility, option pricing theory suggests that venture capitalists will pay more for the common equity of new ventures that have high volatility. A high price implies a low rate of return which is the inverse relation between volatility and return that Galai and Masulis propose. This discussion suggests the empirical proposition that returns to venture capital funds should be high for low priced investments with low volatility and low positive skewness. A difficulty in testing this proposition in the venture capital market is the measurement of profit volatility. Since a new venture prior to commercialization may have expenses but no revenues, earnings volatility cannot be calculated or estimated. Thus, testing requires a proxy for profit volatility. One possibility is the ex-post earnings volatility of otherwise similar operating businesses as, for example, we estimate in section II.

In periods of constrained venture capital financing, the views of venture capitalists on risk will tend to dominate those of entrepreneurs in the venture capital market to determine the type of entrepreneurial businesses that are financed and started. The decrease in the cost of capital, ω, with respect to profit volatility, σ, in Figure II indicates that venture capitalists have a preference for high volatility new ventures that have returns with high positive skewness. Consequently, in periods of constrained venture capital financing, we expect that venture capitalists will focus their financing on industries with high profit volatility. On the other hand, in periods when venture capital financing is more available, the views of entrepreneurs on risk will have relatively greater influence on the types of new ventures that are financed. The increase in the return threshold for new venture start, ψ^*, with respect to profit volatility, σ, indicates that entrepreneurs have a preference for low volatility new ventures. Thus, in periods of less constrained access to venture capital financing, we expect that venture capitalists will finance relatively more new ventures from industries with low profit volatility.

D. Business Risk and Credit Constraints

The difference in return thresholds for start of a new business versus expansion of an already operating business in Figure III as business risk, $\theta\sigma_{x,c}$, increases suggests testable hypotheses.

Recall that θ measures risk aversion for the financial economy and $\sigma_{x,c}$ is a covariance risk specific to a particular business. The product, $\theta\sigma_{x,c}$, is a risk premium that venture capitalists demand for risk. Both the return threshold for start of a new business, ψ^*, and the return threshold for expansion of an already operating business, ξ^*, increase with business risk, $\theta\sigma_{x,c}$. For a greater risk premium, $\theta\sigma_{x,c}$, the underlying operating business is less attractive to venture capitalists and both the return threshold for start of the new business and the return threshold for expansion of the operating business increase. Except when the risk premium, $\theta\sigma_{x,c}$, is close to zero, the curves that depict these return thresholds versus the risk premium, $\theta\sigma_{x,c}$, in Figure III are close to parallel. This observation means that the type of risk that venture capitalists price in financial markets has little impact on relative industry growth between incumbents and new entrants. For example, in a period where investors are more risk averse (that is, θ is great), a period of constrained credit possibly, relative industry growth does not change between incumbents and new entrants in the industry.

E. Growth

In Figure IV, the return threshold for start of a new business, ψ^*, as well as the return threshold for expansion of an operating business, ξ^*, decrease with growth, g. Growth increases the value of expansion options, which increases the attraction of business start to the entrepreneur and expansion to the manager of the already operating business. Other than for exceptionally high growth rates, the curves depicting these return thresholds in Figure IV are close to parallel. Empirically, this observation means that regardless of whether an industry is high or low growth, the relative increase in asset size over a period for incumbent firms versus new entrants in the industry should be the same.

F. The Riskless Interest Rate

In Figure V, the return threshold for start of a new business, ψ^*, and the return threshold for expansion of an already operating business, ξ^*, both increase with the riskless rate, *r*. The riskless rate increases investors' opportunity costs which decreases the attraction of the underlying business and increases both the return threshold for the start of a new business and the return threshold for expansion of an already operating business. However, because the riskless rate in our model, *r*, is in real rather than nominal form, and because real rates historically do not vary as significantly as suggested by the horizontal axis of Figure V, the effect of interest rates on the start of new businesses versus expansion of already operating businesses is unlikely to be empirically observable.

IV. Conclusion and Future Research

Graham and Harvey (2001, 2002) and Farragher et al. (1999) suggest that while corporate managers predominantly use traditional investment decision tools such as NPV and IRR, they also recognize the real options features of their investments. Our research suggests that because new ventures have more extensive option features than operating businesses, entrepreneurs who rely on traditional investment analysis to make a start decision for a new business more likely make investment errors than mangers of already operating businesses. The timing option makes financial returns exceptionally great and, in many cases, unduly conservative as hurdle rates for the start of a new business. For example, for an operating business with an earnings volatility of 20% per annum (see Figure II), the expected financial return for the new venture is 25.5% and the value maximizing return start threshold is 12.6%. The financial return is a poor approximation to the return threshold for the start of a new business and impedes rather than promotes value maximizing entrepreneurial investment.

The results of our study offer insights for practicing entrepreneurs looking for guidance and heuristics on when to start a new business. First, entrepreneurs should not use venture capital returns as a benchmark for the start of a new business. In many instances, financial returns are unduly high due to options leverage. The return threshold for the start of a new business does not require a premium for option leverage. Second, and for the same reason, any reasonable financial market return plus a risk premium (for greater risk of new ventures compared to already operating businesses) is also unduly high as a benchmark since the risk premium for options leverage is unnecessary. Third, unless earnings volatility is extreme so that the force of deferral for the new business is great (Figure II), an entrepreneur can look beyond the new venture stage of his/her business and calculate the cost of capital for the operating business as a return threshold for start of a new business. Because this return does not have a risk premium for the start option, it is close to the value maximizing return threshold for start of the new business. The average financial return for an operating business is closer to the return threshold for new business start than is the rate of return on a portfolio of venture capital investments. Finally, in cases of great profit volatility, the return threshold to start a new business requires a substantial deferral factor to represent the combination of investment irreversibility and downside risk on the entrepreneur's decision to start a new business.

Our study focuses on conventional investments (e.g., plant property and equipment) without development risk in a business environment where opportunities arise exogenously. An entrepreneur passively awaits minimum profitability before starting a new business. In current work (Blazenko et al., 2009), we investigate the conditions under which a new venture entrepreneur undertakes innovative R&D investment prior to commercialization of a new business. If development risk is great, a new venture entrepreneur endogenously invests in R&D to enhance *latent earnings* prior to commercialization that only then generates *actual earnings* for the business. We also determine the conditions under which an entrepreneur invests in R&D

to enhance latent earnings before the manager of an otherwise similar already operating business invests in R&D to increase actual earnings.

APPENDIX

Appendix A

Blazenko and Pavlov (2009) determine the market to book ratio, $m(Y)$, for an operating business that makes growth investments up to a fraction of existing capital. The entrepreneur has an indefinite pair of real options to commence and suspend growth at any future time. The cost of commencing growth is irreversible capital investment. The market to book ratio, $m(Y)$, with both growth and non-growth branches is,

$$m(Y) = \begin{cases} \dfrac{Y}{r^*} - \dfrac{g\lambda}{(r-g)(\lambda-\alpha)(1-\alpha)}\left(\dfrac{Y}{\xi^*}\right)^{\alpha}, & \textit{no growth, } Y<\xi^* \\ \\ \dfrac{Y}{r^*-g} + \dfrac{g\alpha\lambda}{(r-g)(\alpha-\lambda)(1-\lambda)}\left(\dfrac{Y}{\xi^*}\right)^{\lambda} - \dfrac{g}{r-g}\left[1-\dfrac{\alpha}{\alpha-\lambda}\left(\dfrac{Y}{\xi^*}\right)^{\lambda}\right], & \textit{growth, } Y \geq \xi^* \end{cases} \tag{A1}$$

where, ξ^* is the value maximizing return threshold for expansion of the operating business that we describe in Equation (A3) below. The return $r^* \equiv r + \theta\sigma_{x,c}$ is the cost of capital for a hypothetical business that permanently commits to no-grow where *r* is the riskless rate of interest[25] and $\theta\sigma_{x,c}$ is the risk premium for such a business. The risk premium is the product of $\theta \geq 0$ (relative risk aversion for the financial economy) and $\sigma_{x,c}$ (a business specific covariance risk between operating profit, *X*, and aggregate consumption in the economy, *C*). Smith and Smith (2003) discuss the importance of covariance risk for the valuation of new ventures. Other than new venture option effects, because venture capitalists make financial investments in new ventures as part of diversified portfolios, the venture capital market "prices" covariance risk, $\sigma_{x,c}$, rather than volatility, σ. Covariance, $\sigma_{x,c}$, represents the operating risk for the business once it is started and is a constant that does not depend upon earnings volatility, σ. For expositional purposes, we presume positive covariance, $\sigma_{x,c} \geq 0$, which implies a positive risk premium, $\theta\sigma_{x,c} \geq 0$. The parameters α and λ equal,

[25] Real options models commonly presume a constant riskless interest rate. A primary determinant of stochastic nominal interest rates is changes in inflation expectations. The riskless rate, *r*, in our model is real rather than nominal, and therefore, more likely constant. In this case, consistency between cash flows and opportunity cost rates of return requires that corporate earnings, *X*, are also real rather than nominal.

$$\alpha \equiv \frac{1}{2}+\frac{\theta\sigma_{xc}}{\sigma^2}+\sqrt{\frac{2r}{\sigma^2}+\left(\frac{1}{2}+\frac{\theta\sigma_{xc}}{\sigma^2}\right)^2}$$
$$\lambda \equiv \frac{1}{2}+\frac{\theta\sigma_{xc}}{\sigma^2}-\sqrt{\frac{2(r-g)}{\sigma^2}+\left(\frac{1}{2}+\frac{\theta\sigma_{xc}}{\sigma^2}\right)^2}, \qquad \text{(A2)}$$

The first term on the right hand side of the market/book ratio on the no-growth branch in Equation (A1), $\frac{Y}{r^*}$, is the value of a permanently non-growing business (per dollar of capital). The second term is the value of the option to begin growth for a firm currently not growing. This option value is a net amount—the value of incremental future profitability when the business commences growth less the cost of growth investment. The first term on the right hand side on the growth branch in Equation (A1), $\frac{Y}{r^*-g}$, is the value of a permanently growing business (per dollar of capital). The second term is the expected profitability loss should the entrepreneur suspend future growth due to low profitability. The third term is the cost of growth that recognizes that the entrepreneur might suspend growth at times in the future.

The return threshold for expansion of the operating business is,

$$\xi^* = r^* \times \left[\frac{r^*-g}{r-g}\right] \times \left[\frac{\alpha}{\alpha-1}\right] \times \left[\frac{\lambda}{\lambda-1}\right] \qquad \text{(A3)}$$

This return maximizes the value of the operating business in Equation (A1). When $Y \geq \xi^*$, growth investment creates net value because the market/book ratio exceeds one. The entrepreneur suspends growth when $Y < \xi^*$ because the market to book ratio is less than one and net value creation from growth investment is negative.

The expansion threshold in Equation (A3) is the product of four terms. The first term, r^*, is the cost of capital for a hypothetical business that permanently commits to no-growth. The second term, $\left[\frac{r^*-g}{r-g}\right]$, represents growth leverage risk from unavoidable growing capital investment costs for a permanently growing business. The third term, $\left[\frac{\alpha}{\alpha-1}\right] \geq 1$, represents the delaying force of irreversible investment on the entrepreneur's expansion decision. The final term, $0 \leq \left[\frac{\lambda}{\lambda-1}\right] \leq 1$, represents the accelerating force of follow-on investment options for the entrepreneur's expansion decision.

Appendix B

To find the market to book ratio, $\pi(Y)$, for the new venture prior to start of the operating business, we use Goldstein and Zapatero's (1996) valuation methodology as applied by Goldstein et al. (2001). Because Goldstein and Zapatero employ equilibrium analysis, our use of it determines the new venture cost of capital endogenously in section II.

The market to book ratio for the new venture, $\pi(Y)$, satisfies the differential equation:

$$r\pi = -\theta\sigma_{x,c} Y\pi' + Y^2\pi''\sigma^2/2 \tag{A4}$$

which says that the dollar return, at the riskless rate, on the new venture's value (left hand side) equals an expected dollar net gain (right hand side). The expected net gain is a loss due to risk aversion, plus an expected capital gain due to the convexity of the value function. Convexity arises from the upside earnings potential of the business, which the entrepreneur captures with the start of the new business, and downside earnings protection that the entrepreneur achieves by delaying the start of the new business. For a numerical example, Figure1 depicts the convexity of the value function, $\pi(Y)$, that solves Equation (A4) with the conditions in Equation (A5) and Equation (A6) below. Equation (A4) recognizes that the venture earns no income, *X*, prior to the start of the new business.

Value matching and smooth pasting conditions jointly determine "c," the arbitrary constant in Equation (1), and ψ^*, the value maximizing return threshold for start of the new business,

$$\pi(\psi^*) = m(\psi^*) - 1 \tag{A5}$$

$$\pi'(\psi^*) = m'(\psi^*) \tag{A6}$$

Value matching in Equation (A5) says that the value of the new venture (per dollar of capital) immediately prior to the start of the new business equals the net value of the operating business immediately after the start. The net calculation is the value of the operating business less required capital. Smooth pasting in Equation (A6) says that the two value functions–pre and post new venture start–meet one another at the new venture start boundary, ψ^*, without kinks which ensures that the new venture start boundary, ψ^*, maximizes value.

Appendix C

We prove that the new venture start boundary exceeds the expansion boundary, $\psi^* \geq \xi^*$. Presuming the opposite, $\psi^* < \xi^*$, leads to a contradiction. Under this presumption, solving Equation (A5) and Equation (A6) using the no growth branch of Equation (A1) leads to, $\psi^* = r^* \dfrac{\alpha}{\alpha - 1}$. Because $\alpha > 1$, this result means that $\psi^* > r^*$. At the same time, Blazenko and Pavlov (2009) prove that $r^* \geq \xi^*$. Combining these two results leads to, $\psi^* > \xi^*$, which contradicts our original presumption.

Appendix D

In this Appendix we prove two results. First, we prove $\lim_{\sigma \to 0+} \psi^* = r^*$. Second, we prove that when volatility becomes unbounded, the return threshold for start of the new business also becomes unbounded, $\lim_{\sigma \to \infty} \psi^* = \infty$. First, one can show that $\lim_{\sigma \to 0+} \lambda = -\frac{r-g}{\theta\sigma_{x,c}}$ and $\lim_{\sigma \to 0+} \frac{\alpha}{\alpha-1} = 1$. Substitute these results into Equation (A3), to reveal, $\lim_{\sigma \to 0+} \xi^* = r^*$. Substitute all of these results into Equation (2) to reveal that this equation is satisfied only when $\psi^* = r^*$, and therefore, $\lim_{\sigma \to 0+} \psi^* = r^*$. Second, one can show that, $\lim_{\sigma \to \infty} \frac{\alpha}{\alpha-1} = \infty$, $\lim_{\sigma \to \infty} \lambda = 0$, and $\lim_{\sigma \to \infty} \xi^* = r^* - g$. From Equation (2), because $\lim_{\sigma \to \infty} \frac{\alpha}{\alpha-1} = \infty$, then either $\lim_{\sigma \to \infty} \psi^* = \infty$, $\lim_{\sigma \to \infty} \psi^* = -\infty$, or $\lim_{\sigma \to \infty} \psi^* = 0$. The latter two possibilities contradict the result we proved in Appendix C, $\psi^* > \xi^*$ because $\lim_{\sigma \to \infty} \xi^* = r^* - g > 0$. Therefore, it must be the case that $\lim_{\sigma \to \infty} \psi^* = \infty$.

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Figure I
Value of the New Venture Prior to Business Start
and the Value of the Operating Business
Versus the Return on Capital

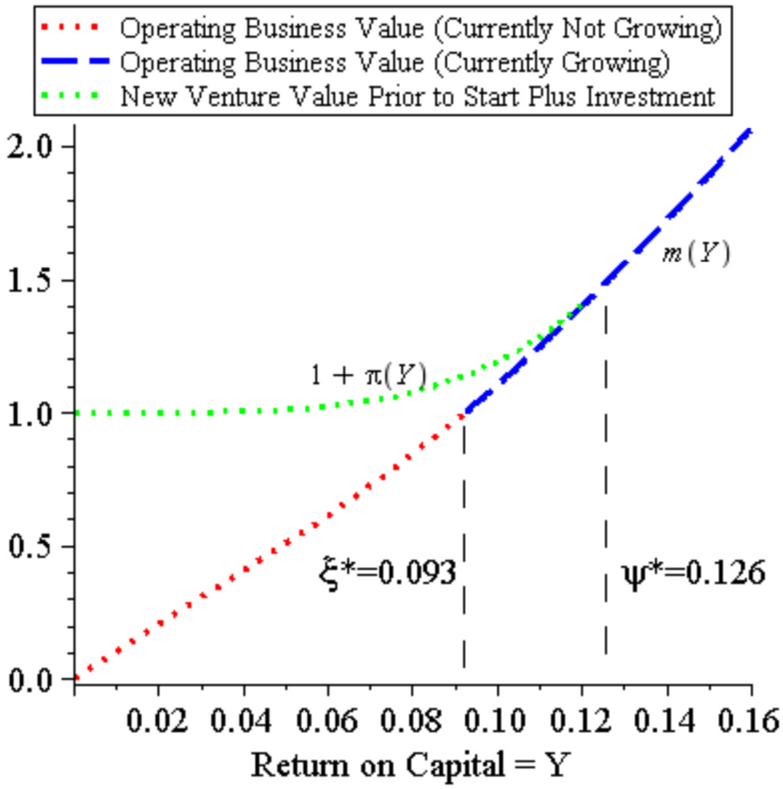

Figure I plots one plus the new venture market to book ratio prior to start, $1+\pi(Y)$, and the operating business market to book ratio, $m(Y)$. Parameters equal $g = 0.06$, $r = 0.05$, $\theta\sigma_{x,c} = 0.05$, $\sigma = 0.2$. As the return on capital Y increases from the left, convergence of the first function to the second illustrates that the return threshold for new venture start, $\psi^{*}=0.126$, exceeds the return threshold for expansion of the operating business, $\xi^{*}=0.0927$, by over 300 basis points.

Figure II
Return Thresholds and the New Venture's Cost of Capital Versus Earnings Volatility

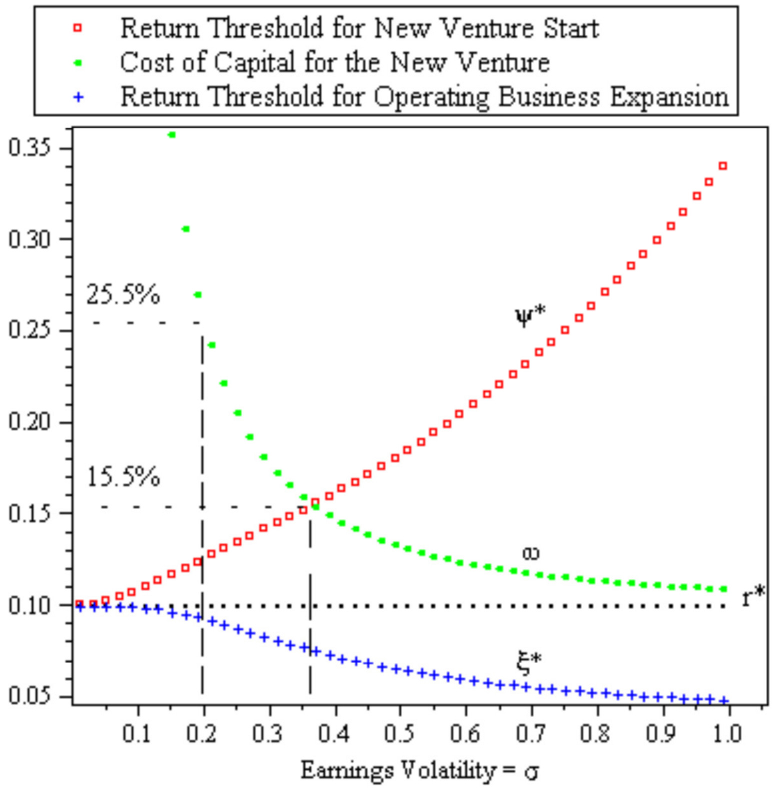

Figure II plots the new venture's cost of capital, ω, the entrepreneur's return threshold for start of the business, ψ^*, and the manager's return threshold for expansion of the operating business, ξ^*, against earnings volatility, σ. Parameters of the model equal r= 0.05, g = 0.06, $\theta\sigma_{x,c}$ =0.05.

Figure III
Return Thresholds and the New Venture's Cost of Capital Versus Business Risk

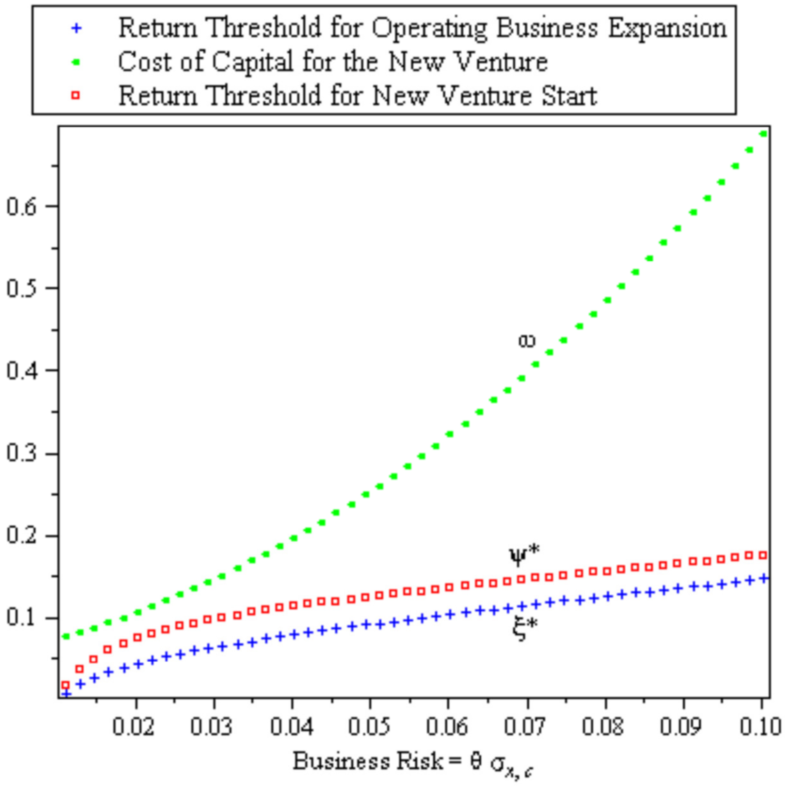

Figure III plots the new venture's cost of capital, ω, the entrepreneur's return threshold for start of the business, ψ^*, and the manager's return threshold for expansion of the operating business, ξ^*, against business risk, $\theta\sigma_{x,c}$. Parameters of the model equal r= 0.05, g = 0.06, $\sigma = 0.2$.

Figure IV
Return Thresholds for Start of the New Venture
and for Expansion of the Operating Business Versus Growth

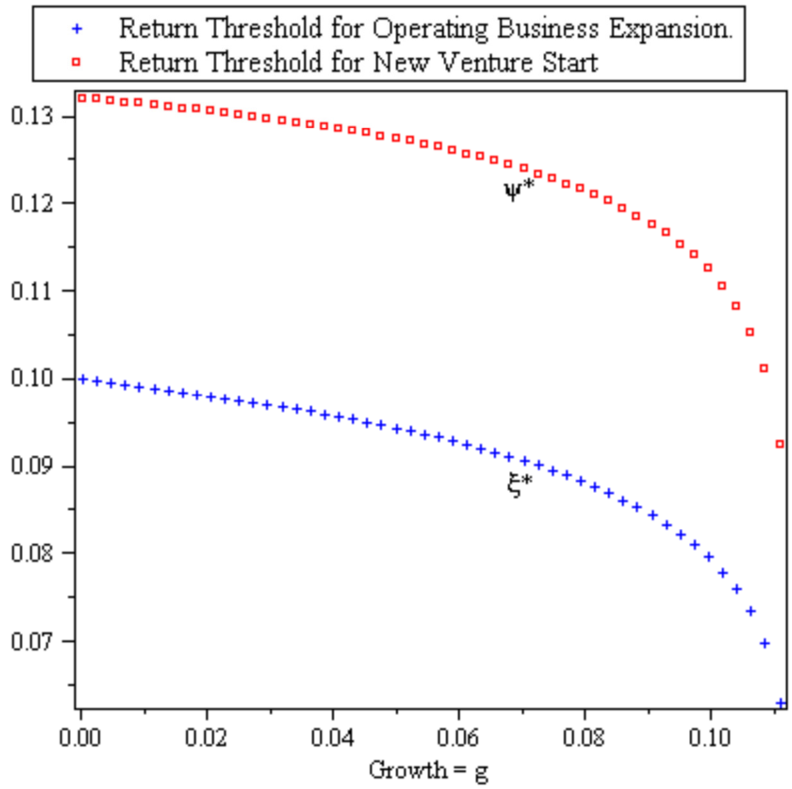

Figure IV plots the entrepreneur's return threshold for start of the business, ψ^*, and the manager's return threshold for expansion of the operating business, ξ^*, against the maximum rate of growth, g, when growth is at the discretion of the manager. Parameters of the model equal r= 0.05, $\theta\sigma_{x,c}$ =0.05, $\sigma = 0.2$.

Figure V
Return Thresholds for Start of the New Venture
and for Expansion of the Operating Business Versus the Riskless Interest Rate

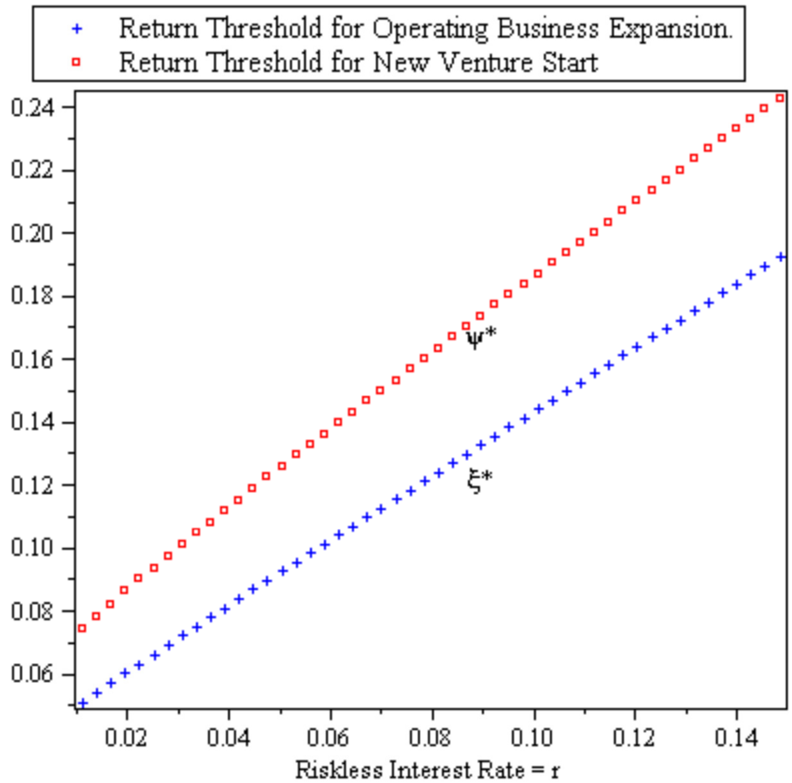

Figure V plots the entrepreneur's return threshold for start of the business, ψ^*, and the manager's return threshold for expansion of the operating business, ξ^*, when growth is at the discretion of the manager versus the riskless interest rate, *r*. Parameters of the model equal $\sigma = 0.2$ $\theta\sigma_{x,c}$ =0.05 g=0.06.