School of Public Policy Working Papers   School of Public Policy

8-25-2011

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James Prieger
Pepperdine University, james.prieger@pepperdine.edu

Nicholas J. Sanders

Recommended Citation

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James E. Prieger

School of Public Policy

Pepperdine University

24255 Pacific Coast Highway

Malibu, CA 90263

james.prieger@pepperdine.edu

Nicholas J. Sanders[†]

Stanford Institute for Economic Policy Research

Stanford University

366 Galvez Street

Stanford, CA 94305

sandersn@stanford.edu

August 25, 2011

Verifiable and Non-Verifiable Anonymous Mechanisms for Regulating a Polluting Monopolist

Abstract

Optimal regulation of a polluting natural monopolist must correct for both external damages and market power to achieve a social optimum. Existing non-Bayesian regulatory methods require knowledge of the demand function, while Bayesian schemes require knowledge of the underlying cost distribution. We introduce mechanisms adapted to use less information. Our *P*rice-based *S*ubsidy (*PS*) mechanisms give the firm a transfer that matches or approximates the incremental surplus generated each period. The regulator need not observe the abatement activity or know the demand, cost, or damage functions of the firm. All of the mechanisms induce the firm to price at marginal social cost, either immediately or asymptotically.

1   Introduction

Economic regulation often aims at correcting inefficiency due to firms' market power and the presence of externalities from production. These features of a market pull in different directions: while market power leads to underproduction, negative externalities lead to overproduction. In many cases market outcomes are shaped by both concentration and externalities such as pollution, and optimal regulation of a polluting firm that has market power is an important question. For example, energy is the largest industry in the utility sector that is most likely to still be regulated, and the production of electricity necessarily involves environmental externalities. [1] Using regulatory schemes designed to check market power for firms that do not produce externalities, the case most often considered in the economic regulation literature, is often inefficient.

An optimal regulatory mechanism must account for both market power and externalities, often in the presence of asymmetric information, a third inescapable feature of many markets. The regulator is likely to have less information than the firm about production and externality generation. For example, firms are likely to be more informed about demand in the output market. In the presence of negative environmental externalities, firms are also likely to have more knowledge of their pollution abatement costs and actions, and therefore the environmental damage their production causes. We focus on such environmental externalities, and will often refer to the externality as "pollution", though our mechanisms can be applied in other situations where marginal private and social costs differ.

The importance and relevance of environmental aspects of regulation has led to examination of the performance of traditional interventions in concentrated industries. Concern over the social efficiency of Pigovian taxes began when Buchanan (1969) showed the classic tax fails to achieve optimality in an imperfectly competitive market. A sub-

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[1]Even the telecommunications industry, still usually partially regulated, is not free from externalities, whether positive (e.g., network externalities) or negative (e.g., junk faxes, spam email, and network congestion).

stantial literature since then has focused on how to best design environmental regulatory policy when facing markets in various stages of concentration. Further research on regulating polluting monopolists through taxation includes Pigovian (Barnett, 1980) and other (Oates and Strassmann, 1984) approaches.[2]

Consideration has also been given to oligopolistic settings, beginning with examination of second-best Pigovian taxes in a Cournot duopoly (Levin, 1985). Later work tackles the design of incentive compatible regulation of oligopolistic firms when there is asymmetric information. Regulatory instruments proposed include two-part taxes (Shaffer, 1989), ad valorem taxes (Shaffer, 1995), and surplus subsidy schemes (Kim and Chang, 1993; McKitrick, 1999). Alternative mechanisms include those designed to induce firms to monitor each other (Duggan and Roberts, 2002) and others that work with imperfectly rational firms (Quérou, 2008). All of these mechanisms require at least some knowledge of the demand and external cost functions.[3]

We contribute to the environmental regulation literature by proposing four anonymous (i.e., non-Bayesian) dynamic mechanisms for regulating a polluting natural monopolist with unknown costs of production and pollution reduction.[4] We introduce mechanisms for all combinations of cases of known or unknown demand and social damage functions. Based on the prices a polluting monopolist sets and its actions and outcomes from previous periods, our *PS* (for *P*rice-based *S*ubsidy) mechanisms offer the firm a transfer that matches or approximates the incremental surplus generated each period. Prices and abatement effort converge to optimal levels immediately or asymptotically, depending on how much information the regulator has at hand. Some of the *PS* mecha-

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[2]As an alternative to taxation in the closely related problem where the externalities arise from consumption instead of production, de Villemeur and Gui (2007) propose a price cap scheme. Cavaliere (2000) examines voluntary agreements between a monopolist and consumers concerning environmental product quality (i.e., self-regulation) as an alternative to governmental regulation.

[3]Asymmetric information about abatement costs in non-monopolistic markets has been addressed with non-Bayesian mechanisms by Kwerel (1977) and Dasgupta, Hammond, and Maskin (1980). For a further review of environmental regulation under imperfectly competitive markets, see Requate (2005).

[4]The first use of the term *anonymous* for non-Bayesian regulatory mechanisms seems to be in Baron (1989), although the term has an earlier history in the general mechanism design literature.

nisms are verifiable: they make use only of realized cost, demand, and damage data. As Vogelsang (1988) points out, verifiability is desirable for regulatory mechanisms because it allows third parties (e.g., the courts) to ascertain whether regulation has been properly applied. Other of our mechanisms are non-verifiable, because they assume knowledge of the complete demand function. The anonymity of the mechanisms leads to simple rules for regulators that are practical to implement and do not depend on characterizing unknown parameters with probability distribution functions. We focus solely on the case of monopoly, to make it clearer how the *PS* mechanisms correct for market power and externalities, without the additional complications of strategic interactions among firms. Extending the *PS* mechanisms to oligopoly is explored in the final section.

The *PS* mechanisms stem from the literature on the regulation of natural monopoly with unknown costs, which can be separated into Bayesian and anonymous branches. Bayesian economic regulation, devised for monopolists without externalities by Baron and Myerson (1982) and furthered by Lewis and Sappington (1988b,a) and Laffont and Tirole (1986), assumes the regulator knows the underlying cost distribution of regulated firms. The regulator requests a cost report and gives a transfer to the firm that induces the highest social welfare (subject to individual rationality and incentive compatibility constraints). Bayesian approaches to regulating a polluting monopolist are proposed by Baron (1985a,b) and re-addressed in Laffont (1994), Lewis (1996), Swierzbinski (1994), and Bontems and Bourgeon (2005).[5]

In anonymous methods, such as those proposed in early work by Loeb and Magat (1979), Finsinger and Vogelsang (1981), and Sappington and Sibley (1988), the beliefs of the regulator play no role. Instead, the regulator uses observations on past market quantities and profits to design subsidies which, when offered to the firm, result in marginal cost pricing (there are no externalities in these models). The *PS* mechanisms embody a similar approach, and do not require the regulator to have any knowledge of the under-

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[5]Van Egteren (1996) develops a Bayesian common agency model to discusses the difficulties that arise when the firm's market power and the externalities it produces are regulated by different entities.

lying cost structure of the firm.[6] Despite the recent prominence of Bayesian approaches in the theoretical literature, interest in practical uses of surplus subsidies continues, particularly as options for regulating electricity markets. In a discussion of electricity pricing and performance-based regulation, Vogelsang (2006) considers both Bayesian and non-Bayesian approaches. Gans and King (2000; 2003; 2004) consider applications to the Australian electricity market, and provide a numerical example in which a non-Bayesian surplus subsidy scheme can be used to induce electricity producers to engage in the socially optimal level of capacity investment and transmission under nodal pricing. Tanaka (2005) provides a numerical analysis of regulating an electricity transmission utility using a traditional non-Bayesian surplus subsidy scheme, with a direct implementation of the surplus scheme proposed by Sappington and Sibley (1988) on which our model is loosely based. To our knowledge, such schemes have not yet been implemented in practice. However, the PS mechanisms are members of the general class of anonymous mechanisms incentivizing the regulated firm to improve social welfare. This class includes "incentive regulation" (also known as "performance-based regulation") such as price caps used for public utilities. Our mechanisms thus follow in the tradition of a flourishing applied regulatory practice. As Lyon (1994) says, "[i]ncentive regulation schemes have been applied quite extensively throughout the US in the electric utility, telecommunications, and health care sectors. . . ."

Of mechanisms in the environmental regulation literature, the PS mechanisms are perhaps most closely related to one proposed by Kim and Chang (1993), who modify the Loeb and Magat (1979) (LM) surplus subsidy scheme. Whereas the LM mechanism was designed to induce a natural monopolist to set price equal to marginal cost, Kim and Chang's (1993) extension allows for pollution (and oligopoly), but requires that the regulator know the slopes of the demand and social damage functions. A subsidy is given to each firm corresponding to its overall contribution to social welfare, which is

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[6]In Bayesian models, the regulator typically knows the cost function up to a single parameter. Our mechanisms have no such requirement.

a combination of the gain from additional production and a loss due to pollution from production.

There are two drawbacks to subsidy schemes of the LM form. Regulatory knowledge of the slope of the market demand and social damage functions may be unrealistic. In the *PS* mechanism with the fewest informational assumptions, the regulator instead bases subsidies only on observable past outcomes for both output and environmental damage, similar to the dynamic scheme for a non-polluting monopolist proposed by Finsinger and Vogelsang (1981).[7] When there are no externalities, two of the *PS* mechanisms reduce to Finsinger and Vogelsang's (1981) scheme. [8] Other *PS* mechanisms are designed to use information on either the demand or social damage function if it is available.

Another drawback of the LM mechanism (which is essentially static) is its large subsidy, which appropriates all consumer surplus each period, making funding onerous (and implementation unpopular). The dynamic formulation of the *PS* mechanisms allows the regulator to award much less information rent to the firm by removing excess profit the period after it is earned. While our mechanisms require a subsidy (which may be negative—a tax—if externalities are large), in the long run the per-period subsidy moves to the minimum necessary to induce the firm to produce and is strictly smaller than the LM subsidy in every period. In addition, the consideration of environmental externalities will, in the case of a negative externality, result in an overall *smaller* necessary total subsidy for reaching convergence than a similar incremental subsidy model without such considerations. The socially optimal quantity will be lower than it would be in the absence of externalities, and the optimal quantity produced will be lower as well. As the subsidy cost comes from enticing the firm to increase production, a strictly smaller subsidy will be required to reach the smaller quantity optimum.

An obvious challenge of our mechanism is that it requires the regulator to identify

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[7]Benchekroun and Long (1998) propose a dynamic pollution tax of a different form.

[8]In the working paper version of the present work (available upon request), we extend the literature and provide a formal analysis of the phase plane, stability, and speed of convergence of the dynamical system induced by the regulation.

amount of pollution produced by the firm. There exist, however, various mechanisms through which the amount of pollution created can be measured or estimated. The Toxic Release Inventory (TRI), for example, provides annual records of toxics production for a variety of industries at the firm level, and is made publically available by the Environmental Protection Agency (EPA). For another example, in the case of electricity generation the Department of Energy records the amount of carbon dioxide (CO_2) produced per kilowatt/hour by state. Such information could be used to base damage estimates on the production quantities that are already used for subsidy calculations.

We note that even when measuring pollution is feasible, quantifying the value of damage it creates may be more difficult. Even in the model with the fewest assumptions regarding the marginal damage function, our model assumes that, at minimum, the regulator is able to accurately evaluate the cost of environmental damages, either immediately or *ex post*. This is an admittedly strong assumption, as the true social cost of environmental externalities from some industries remains a constant source of debate. A practical option in such cases would be to impose the preferences of the regulator as the "true" social cost and have the regulator set a subsidy level accordingly. This approach fails to achieve socially optimality if the regulator is not a benevolent social planner possessing the necessary information. However, as long as the regulator places a reasonably accurate valuation on environmental damage, pricing the subsidy accordingly will still move the production outcome closer to the true social optimum.[9]

The rest of the paper is organized as follows. Section 2 presents the four *PS* mechanisms. Section 3 gives numerical examples. Section 4 provides further discussion of the model and its outcomes. Section 5 concludes.

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[9]We thank a referee for noting this more generalized view of achieving a type of second-best outcome.

2 The Mechanisms

Consider an economy with monopolistic and numeraire sectors, where each consumer has quasi-linear utility separable and linear in the numeraire good. Let prices in period t for the goods produced by a monopolist be denoted by $P_t \in \mathbb{R}_+^N$. The firm faces a demand system $Q_t = Q(P_t)$, assumed to be twice differentiable with invertible Jacobian, and lacking income effects. With no income effects on the monopolistic sector, a partial equilibrium analysis is justified. Changes in consumer welfare when prices move from P_{t-1} to P_t are then measured by changes in consumer surplus, defined as the line integral

$$\Delta CS_t = - \int_{P_{t-1}}^{P_t} Q \cdot d\alpha \tag{1}$$

for any continuous path of integration $\alpha \in \mathbb{R}^N$ from P_{t-1} to P_t.[10] Production creates external damages with social cost D_t. When some of the damage from the firm's actions in period t is incurred in the future, D_t represents the present value of the external cost stream. We allow the firm to decrease marginal damages via a scalar abatement effort denoted $a_t \geq 0$, so that the total damage function $D : \mathbb{R}_+^{N+1} \to \mathbb{R}_+$ is $D(Q_t, a_t)$, assumed to be twice differentiable with $\nabla_Q D > 0$ and $\nabla_a D < 0$.[11] Abatement is costly for the firm, so the cost function $C : \mathbb{R}_+^{N+1} \to \mathbb{R}_+$ takes the form $C_t = C(Q_t, a_t)$, assumed to be twice differentiable with $\nabla_Q C > 0$ and $\nabla_a C > 0$. Cost and demand are stable and have no intertemporal effects. The firm is required to supply all demand at a stated price, and operating profits are therefore

$$OP_t = P_t \cdot Q(P_t) - C_t(Q(P_t), a_t) \tag{2}$$

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[10]For equation (1), based on aggregate demand for the monopolist's products, to be used as if from a normative representative consumer, and for the line integral to be path independent, it must be that in addition to each consumer having quasi-linear utility, income must be high enough that there are no income effects in the relevant range of prices. Under more general assumptions on preferences, consumer surplus only approximates actual changes in welfare.

[11]Our notation for derivatives $\nabla_x f$ means the elements of the gradient of f pertaining to vector x.

The firm desires to maximize the present value of future profits, given discount factor $0 < \beta < 1$.[12] Thus, absent regulation, the firm would not abate at all and would set a monopoly price in each period.

The regulator observes prices and last period's production quantities and profit. At a minimum, the regulator can verify changes in damages *ex post*. The regulator is not required to know the marginal damage or cost functions in any of the mechanisms. The firm knows the cost, demand, and marginal damage functions, at least within the relevant range of production. The discount factor β is common knowledge. Regulation begins in period 1, and we assume that in period 0 the firm is unaware of upcoming regulation. This assumption ensures that P_0 and a_0 are exogenous to the regulatory mechanism. When the firm knows regulation is about to be imposed, it may strategically raise prices one period prior to regulation to increase the overall subsidy provided. However, the *PS* mechanisms still converge, though the process will be more costly for the regulator in terms of total subsidy provided.

Social welfare is measured as the sum of consumer welfare (net of damage) and the profit of the firm, and is assumed to be differentiable and to have a unique maximum. We begin with the optimal prices and abatement levels, (P^*, a^*). Given the assumptions of the model, optimality requires pricing at marginal social cost and equating the marginal abatement cost with the marginal social benefits of reducing pollution in each period:[13]

$$P^* = \nabla_Q(C + D)(Q^*, a^*) \tag{3}$$

$$\nabla_a C(Q^*, a^*) = -\nabla_a D(Q^*, a^*) \tag{4}$$

We seek mechanisms that induce the optimal quantities implicitly defined by (3) and (4), immediately if possible or over time if not.

Four variations of such mechanisms follow. In each variation the mechanism consists

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[12]In our discrete time model, with period interest rate r the discount factor is $\beta = (1 + r)^{-1}$.

[13]We follow the conventional abuse of notation in denoting $\nabla_x f|_{x=x^*}$ with $\nabla_x f(x^*)$.

of paying the firm a price-based subsidy. The prices the firm sets (and abatement, in some variants) are the only current actions of the firm that affect the subsidy it receives. We designate the mechanism PS_j^i, where i denotes whether the demand function is known (1) or not (0), and j denotes whether damage is observed in the current period (1) or not (0).

When the regulator knows the demand function and can observe damage in the same period in which it occurs, the PS_1^1 mechanism induces the firm to price and abate optimally in the first period the mechanism is imposed. If instead the regulator knows the demand function but only observes damage with a lag, then as long as the regulator knows the firm's discount factor the PS_1^0 mechanism has the same outcome. If the regulator does not know the demand function, but can observe past production quantities (realized demand) and current damage, then the PS_0^1 mechanism leads to convergence to optimal outcomes over time. The PS_0^0 variant, for when damage is observed only with a lag, also converges to optimal outcomes. Table 1 summarizes the mechanisms.

2.1   Demand is known, damage is immediately observed

Consider first the case where the regulator knows the demand function and immediately observes damage. With the PS_1^1 mechanism, the regulator allows the firm to choose whatever price and abatement level it wishes and then pays the firm a subsidy S_t, where

$$S_t = \Delta CS_t - OP_{t-1} - \Delta D_t \tag{5}$$

The firm's profit each period is then $\pi_t = OP_t + S_t$. The regulator calculates ΔCS_t from (1), which requires knowledge of the demand function in the region of at least one price path between P_{t-1} and P_t.[14] The final term in the subsidy from the change in total damages

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[14]ΔCS_t is often most conveniently calculated from the price path along the edges of the hypercube in $\mathbb{R}^N$ with opposite vertices P_{t-1} and P_t, for then the line integral can be split into regular integrals. Let $P_{i,t}$ be the ith element of P_t and let $g_i(r) = (P_{1,t}, \cdots, P_{i-1,t}, r, P_{i+1,t-1}, \cdots, P_{N,t-1})$. Then ΔCS_t can be found from the

(which reduces the subsidy if output has expanded) requires that realized damage be observed the same period it is incurred.[15] The firm is thus awarded the gain in consumer surplus, net of last period's operating profits, and penalized for the incremental damage caused by increased production.

Figure 1 shows how the subsidy modifies the firm's profit in the single-good case and gives it incentive to price at marginal social cost.[16] The case where the firm's initial price P_0 is too high (i.e., when the distortion due to market power is greater than that due to negative externalities) is depicted, although the mechanism also works in the other case. Given P_0, the firm's operating profit in period 1 from setting price P_1 is area $C + D + E + F$ on the graph, less any fixed costs. Awarding the firm ΔCS_1 adds $A + B$ to its profit. Netting OP_0 from the subsidy removes $A + C + E$ from the firm's profit, and also reimburses its fixed costs. Finally, penalizing the firm for incremental damage is equivalent to charging it the sum of all marginal damages of production for units Q_1 to Q_0, the area between the marginal social cost (MSC) curve and the MC curve (area F). Thus the firm's profit in period 1 after subsidy is area $B + D$, which is also the incremental addition to social welfare (consumer welfare plus profit) caused by the firm lowering its price. The figure highlights the kinship of the $PS_1^{\bullet}$ mechanisms to Sappington and Sibley's (1988) Incremental Surplus Subsidy (ISS) mechanism. The $PS_1^{\bullet}$ mechanisms reduce to the ISS mechanism when there is a single good and no externalities or abatement.

The firm will maximize profit by choosing a price to maximize the welfare gains, which leads to the first-best outcome immediately:

Proposition 1. *If the informational requirements for the regulator to apply the PS_1^1 mechanism are satisfied, then optimal prices and abatement are achieved the first period in which the mechanism is imposed.*

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sum $\sum_{i=1}^{N} \int_{P_{i,t-1}}^{P_{i,t}} Q(g_i(r)) \, dr$.

[15]Alternatively, the regulator could also calculate ΔD_t if it knows the damage function (locally) and can observe abatement effort.

[16]The figure is drawn abstracting away from the abatement dimension. Alternatively, the MSC curve can be thought of as conditional on optimal abatement at each production level.

The proof (see appendix) shows that as long as $\beta < 1$ the first-order conditions from the firm's dynamic programming problem for profit maximization match those for the social optimum in (3) and (4). Thus welfare jumps immediately up to the first-best level under the PS_1^1 mechanism. The mechanism is relatively low cost, in that the firm earns no residual profit after the first period, since the subsidy reduces to a reimbursement of last period's losses (if any) in periods 2 on.[17] In period 1 the firm earns informational rent, the price the regulator pays for knowing less than the firm.

2.2 Demand is unknown, damage is immediately observed

Now assume the regulator does not know the demand function, but can observe lagged quantities sold. With the PS_0^1 mechanism, the regulator observes the firm's price P_t and resulting damage from production, then pays the firm the subsidy

$$S_t = Q_{t-1} \cdot (P_{t-1} - P_t) - OP_{t-1} - \Delta D_t \tag{6}$$

Without knowing the demand function, the regulator cannot grant the firm the exact gain in consumer surplus resulting from price changes. The PS_0^1 mechanism differs from the PS_1^1 in that the change in consumer surplus is approximated with $-Q_{t-1}\Delta P_t$. This is just the difference between the ISS mechanism and Finsinger and Vogelsang's (1981) Approximate Incremental Surplus Subsidy (AISS). When there is no abatement or damage in the model, the $PS_0^\bullet$ mechanisms are equivalent to the AISS mechanism.[18]

Figure 2 shows how the subsidy transforms the firm's profit.[19] The firm's operating profits and incremental damage penalty are as in Figure 1. Instead of awarding area

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[17]The firm may lose money pricing at marginal social cost price if it is a natural monopolist. However, it may not; unlike in traditional natural monopoly regulation, the PS mechanisms do not force price down to marginal *private* cost. In this case the subsidy after period 1 reduces to a tax equal to last period's operational gains.

[18]Vogelsang (1988) proposes (and Lantz (2007) refines) a surplus subsidy mechanism that is a combination of the AISS and ISS schemes. The PS mechanisms could be similarly hybridized, although we do not pursue that approach here.

[19]The figure again assumes the initial price is too high and ignores the abatement dimension.

$A + B$ to the firm, the PS_0^1 mechanism approximates ΔCS_t with area A, which can be calculated based on verifiable prices and quantities. The firm's profit after subsidy is area D, an approximation to the incremental addition to social welfare caused by the firm lowering its price. The firm's total profit after several periods under the mechanism is shown in Figure 3. The firm now faces a trade-off when it decides how much to drop its price. By taking very small steps, the firm captures more of the total surplus available in period 1 ($B + D$ in Figure 1). In the limit, Lemma 2.2 in the appendix shows that the firm can capture all available surplus. However, small steps delay the accrual of surplus to the firm, which is costly when the firm discounts the future. As with the AISS, optimal outcomes are achieved only in the limit:

Proposition 2. *If the informational requirements for the regulator to apply the PS_0^1 mechanism are satisfied, then optimal prices and abatement are achieved in the limit.*

The proof in the appendix shows that the firm can find a sequence of prices and abatement that increase its profit any time welfare is not at the maximum. Furthermore, it is not profitable for the firm to make choices such that welfare endlessly cycles below the maximum. If the firm chooses prices and abatement that lowers welfare, it has negative profit that period. Thus, the firm would do so only if it would create enough additional surplus to be gained in the future to offset the current loss. When there is a global maximum to surplus, however, there is always a price path to the optimum that strictly increases welfare, and the firm gains nothing by ever reducing surplus. Thus the firm's actions cannot cause welfare to cycle or converge to a level below the maximum without the firm forgoing additional profit.

The appendix shows that the solution of the firm's dynamic programming problem leads to an optimal policy function expressed as a difference equation:

$$\Delta P_{t+1} = \frac{1}{\beta} \left[\nabla_P Q_t \right]^{-1} \Delta Q_t + \frac{1 - \beta}{\beta} [P_t - \nabla_Q (C_t + D_t)] \qquad \forall t \geq 2, \beta \neq 0 \qquad (7)$$

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The difference equation is of the second order, since P_{t-1} appears on the left side through the term $\Delta Q_t = Q_t - Q_{t-1}$. Given the quantity produced, abatement is at the optimal level each period. That is, condition (4) holds each period $t \geq 1$ with Q_t replacing Q^*. Thus optimal actions (P^*, a^*) are a fixed point of the system. Once at (P^*, a^*) for two periods, so that $\Delta Q_t = 0$ and that (3) makes the second term on the right side of (7) vanish, then $\Delta P_{t+1} = 0$ for all future periods.

Proof of Proposition 2 (see appendix) follows Finsinger and Vogelsang (1985). Given an initial price P_0, (7) defines the profit-maximizing prices for periods 2 and on, given P_0 and P_1. The determination of P_1 remains. The dynamical system is saddle-path stable as long as the damage function exhibits enough convexity.[20] Therefore P_1 is chosen to put the firm on the saddle path leading toward the fixed point (P^*, a^*).

2.3 Demand is known, damage is observed with a lag

The other two mechanisms PS can be used when the regulator does not observe current damage, but can observe it the next period. Delay in observation of damage may be due to the time needed for emissions to diffuse in the environment or for monitoring data to be analyzed. Since the regulator cannot observe damage in the period in which it occurs, the firm cannot be penalized for damages incurred in period t until period $t + 1$. The subsidies must therefore be adjusted by marking up the penalty so that the firm effectively pays for past damage with interest. The forward-looking firm then treats the upcoming penalty exactly as if it were levied in the current period. If damage costs were not adjusted, the firm would find it profitable to converge to a price lower than MSC and overproduce, because it would not have to bear the full present value of the damages

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[20]We show in an earlier version of this paper that the system defined by (7) is saddle-path stable if demand slopes down and social cost (the sum of private cost and damage) is linear or convex in Q. Note that the convexity of social cost does not require convexity of private cost (as may be lacking due to economies of scale and natural monopoly), as long as the damage function is sufficiently convex. This condition limits the curvature of the marginal social cost when it is concave in Q. Close to the steady state, is also d by the assumed concavity of the welfare function at its maximum guarantees convergence.

caused. When the regulator knows the demand function, but can only observe lagged damage, it can use the PS_1^0 mechanism, with subsidy

$$S_t = \Delta CS_t - OP_{t-1} - \Delta D_{t-1}/\beta$$

The regulator must know the discount factor to calculate the subsidy, and must have observed damage for two periods. With this minor adjustment to the subsidy, the PS_1^0 mechanism works exactly as the PS_1^1 mechanism, and thus leads to the same outcome.

Proposition 3. *If the informational requirements for the regulator to apply the PS_1^0 mechanism are satisfied, then optimal prices and abatement are achieved the first period the mechanism is imposed.*

The proof (in the appendix) shows that the first-order conditions for profit maximization are identical to those for the PS_1^1 mechanism.

2.4 Demand is unknown, damage is observed with a lag

If the regulator can observe past period quantities, prices, and operating profits as well as current prices, knows the firm's discount factor, and can observe total damages of past periods, it can use the PS_0^0 mechanism, with subsidy

$$S_t = Q_{t-1} \cdot (P_{t-1} - P_t) - OP_{t-1} - \Delta D_{t-1}/\beta \tag{8}$$

With this minor adjustment to the subsidy, the PS_0^0 mechanism works exactly as the PS_0^1 mechanism, and thus leads to the same outcome.

Proposition 4. *If the informational requirements for the regulator to apply the PS_0^0 mechanism are satisfied, then social welfare increases each period and optimal prices and abatement are achieved in the limit.*

The proof (see appendix) shows that the first-order conditions for profit maximization are identical to those for the PS_0^1 mechanism.

3   Numerical Examples

In this section, we provide numerical examples of the implementation of the *PS* mechanisms. The market parameters for the examples are chosen to approximate the retail electricity market of Pacific Gas & Electric (PG&E), a regulated electricity utility in California. In all examples, demand is linear and is calibrated so that at the actual average price and total quantity of electricity sold in 2010, demand elasticity is -0.8.[21] To keep the examples simple and on price as the choice variable of interest, we assume that the current level of abatement is optimal, and that optimal abatement is independent of quantity.[22] Under this assumption, if current abatement were not optimal, it would jump to the optimal level immediately in period 1 under any of the mechanisms. Long-run marginal cost is assumed to be constant at current abatement levels at $0.178/kWh.[23] Marginal damage from CO_2, an air pollutant created when generating electricity from fossil fuels, is approximated at $0.01/kWh based on PG&E pollution generation estimates and US government estimates of the marginal damage of CO_2.[24]

For each mechanism, four scenarios are considered. In each, the socially optimal price

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[21]In particular, at a price of $0.183/kWh (PG&E's actual average retail price for the second half of 2010), quantity sold is 44.837 petaWh and the point elasticity of demand is -0.8. The elasticity estimate is the median long-run elasticity found from the metastudy of Espey and Espey (2004).

[22]For example, define abatement to be the fraction of generation sources that are from renewable energy. If we assume that renewable generation sources have a marginal cost one-third higher than non-renewable sources in California, and that marginal cost is the weighted average of the marginal costs of renewable and non-renewable sources, then PG&E's current generation mix of about 70% renewable is nearly optimal.

[23] Given that the long-run marginal cost estimate includes the cost of capital, and that the utility's rate of return on capital is regulated, fixed cost is chosen to be large enough to give the utility very little profit at the initial price. The exact level of fixed cost is not important for demonstrating how the mechanisms work.

[24]PG&E states that each kWh of residential electricity consumption leads to emission of 0.52 pounds of CO_2 (http://www.pge.com/about/environment/calculator/assumptions.shtml). Estimates of the social marginal cost of carbon varies widely. Picking a value of $42 per metric ton, well within the range of estimates from the US government (Interagency Working Group on Social Cost of Carbon, 2010) and other sources, leads to a marginal damage of one cent per kWh.

is \$0.188, the sum of marginal cost and marginal damage. The price in period 0 is either the "low" value of \$0.183/kWh, corresponding to PG&E's actual average retail price in 2010, or the "high" value of \$0.295/kWh, corresponding to the monopoly price. The former represents the case in which the environmental damage problem outweighs the market power problem, in that prices need to rise to the social optimum. The latter represents the opposite, in that prices need to fall to attain the social optimum. For each initial price, the interest rate can also be low (1.8%, from the annual yield on PG&E's corporate bonds as of mid 2011) or high (10%).

3.1 Examples when demand is known

Consider first the situation of $PS_1^\bullet$, in which demand is known. Since the PS_1^1 and PS_1^0 mechanisms provide identical incentives to the firm, the results for the example (shown in Table 2) are the same regardless of which is used. Upon imposition of the mechanism, in all cases the firm immediately sets its price to the first best level of 18.8 cents to maximize its profit.

With a low initial price and low interest rate (the first column in Table 2), the firm earns \$12 billion in profit in present value from periods 1 on, which is \$28 million less than it would have made without the imposition of the mechanism. Since the firm was earning little profit to begin with (see footnote 23), the profit change is relatively small. The subsidy each period is negative (i.e., it is a tax), but relatively small, at around \$−22 million per period. In the steady state, the subsidy covers the fixed cost of the firm and reclaims the operating profits earned each period. The firm earns operating profits in this example because the optimal price is above both average variable and average total cost. The present value of the subsidy is \$−12 billion, large enough (in this case) to reclaim nearly all the profit for consumers.

Since quantity sold falls, the gross consumer surplus (excluding damage and tax receipts) falls compared to before the mechanism, and even when the proceeds of the tax

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are given to consumers, net consumer surplus falls by \$38 million. However, social damage cost falls by \$55 million, and so consumers are better off. Total surplus (the sum of gross profit and consumer surplus, less damage) improves by \$14 million in present value. Since transfers to and from the firm are assumed to be costless, the improvement in total surplus matches what a social planner with full information could gain. The additional surplus the mechanism creates is relatively small because in this scenario, neither the market power problem nor the externality problem is very large. In the next example, still with the low initial price but with a high interest rate (the second column in Table 2), results are similar but the magnitudes are smaller in present value.

With a high (i.e., monopoly) initial price (the third and fourth columns in Table 2), the firm acts the same and earns the same gross profit as with the low initial price. However, the firm is awarded a subsidy of \$90 million in period 1 when it reduces its price from 29.5 to 18.8 cents. The subsidy in period 2 reverts to its steady state negative level, the same as in the previous examples. Net profit is \$135 billion (assuming the low interest rate) or \$24 billion (assuming the high interest rate) less with the mechanism than it is in the assumed counterfactual of continual monopoly pricing. Since quantity sold rises, there are large gains in consumer surplus and smaller increases in damage. Total surplus increases, in this case by much larger amounts than with the low initial price, given the large market power problem that the mechanism corrects. The high initial price scenarios are the only ones in which the subsidy is positive in any period. Although such transfers are assumed to be costless, it is worth noting that with any reasonable social cost of transferring a dollar, the improvement in total surplus of \$62 billion with the low interest rate or \$11 billion with the high interest rate would still be positive.

3.2 Examples when demand is unknown

Now consider the situation with the $PS_0^{\bullet}$ mechanisms, which are appropriate to use when demand is unknown. As in the previous section, the PS_0^1 and PS_0^0 mechanisms provide

identical incentives to the firm, so the results for the example (shown in Table 3) are the same for both.

Comparison with the $PS_1^\bullet$ mechanisms reveals the following. First, the firm takes time to adjust its prices toward the optimum. The higher the interest rate (i.e., the lower the discount factor), the quicker the profit-maximizing price path converges toward the optimum. Second, despite the slower convergence than with the $PS_0^\bullet$ mechanisms, the changes in the present value of net profit, consumer surplus, damage, and total surplus are not far from the results in Table 2. In particular, the total surplus with the $PS_0^\bullet$ mechanisms is only about 6–12% less than with the $PS_1^\bullet$ mechanisms, depending on the scenario. This result highlights a strong point of the non-Bayesian approach of our mechanism. Even with much less information to work with than a typical Bayesian approach requires, the social welfare gains with the $PS_0^\bullet$ mechanisms are not far from the gains available under full information.

4 Discussion

In this section, we discuss the informational requirements for the mechanisms, strategic behavior in response to the mechanisms, budgetary considerations, adverse selection issues, and application of the mechanisms to dynamic markets.

4.1 Information

The informational requirements for the mechanisms are modest. The regulator need not know the cost function facing the firm or the environmental damage function, or be able to observe the firm's abatement efforts (which also stand in for emissions). The $PS_0^\bullet$ variants also remove the need for the regulator to know the demand function. We require that the regulator be able to observe and value realized environmental damage, either contemporaneously or with a lag. This is a weaker assumption than that of many schemes that

require the regulator to know the marginal or total damage function and observe emissions (Shaffer, 1989; Kim and Chang, 1993; Shaffer, 1995; McKitrick, 1999; Duggan and Roberts, 2002; Quérou, 2008).

In most of the tax schemes cited above, firms do not need independent knowledge of the damage function because the taxes incorporate the marginal external damage. In our model, the firm knows the damage function (as in, for example, Duggan and Roberts (2002)). If it does not, then our mechanisms still apply to cases where the regulator knows the marginal damage function but cannot observe abatement or emissions, for the regulator could provide the firm with information on the relationship between pollution and incremental welfare damage. Federal agencies in the US have long attempted to quantify the damage functions from various specific pollutants (see chieh Lui and hun Yu (1976) for an early example of computing parametric damage functions for sulfur dioxide). The *PS* mechanisms give the firm the incentive to learn these damage functions.

The *PS* mechanisms apply to production processes and pollutants for which the links between production, environmental damage, and social welfare are known (or at least reasonably well estimated). If the regulator would not know the welfare cost of the pollution even if it fully observed amounts and locations (e.g., air pollution potentially contributing to global warming), then the mechanisms could not be applied. We do not see this as a limitation specific to the *PS* mechanisms, since if harm cannot be assessed, then the first best cannot be determined.[25]

The $PS_\bullet^0$ versions of the mechanism require the regulator to know the firm's discount factor. Common knowledge of the discount factor is a standard assumption in the literature on dynamic games. Practically speaking, the regulator may be able to assess the firm's discount factor by looking at the yield on outstanding bonds the firm has on the commercial paper market or by considering the credit rating of the firm.

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[25]While cap-and-trade policies allow for regulation without specific knowledge of social damages, they are designed to minimize the cost of achieving a certain level of abatement only and not to reveal the optimal amount of pollution. As such, cap-and-trade fails to reach first-best outcomes.

4.2 Strategic behavior

The *PS* mechanisms are "strategy proof" (using the language of Finsinger and Vogelsang (1985)). In other words, when prices are above the optimum, the firm cannot profit by raising prices just to increase the amount of future subsidies. Similarly, when prices are too low, the firm does not profit by lowering prices to increase subsidies to come. Given the use of dynamic programming in the solution of the firm's problem, we have already assumed that the firm is fully forward looking. Moving prices in the (socially) wrong direction one period to collect larger subsidies as the prices resume converging to the optimum is not profitable when future payments are discounted appropriately.

Furthermore, as with the ISS and AISS mechanisms, the firm's managers will not find it profitable to waste—that is, to produce (given a) at higher than minimum cost. We have not formally included waste in the analysis, to avoid cluttering the model and since the result is nearly obvious. Since cost overruns in any period are not refunded until the next period, waste does not pay when the firm discounts the future. However, as with the ISS and AISS, the mechanisms we propose are open to abuse—waste that provides direct benefits to the managers or owners of the firm.[26] If abuse is likely to be a problem, Lee's (1997) suggestion to require the firm to offer an appropriately designed menu of self-selecting tariffs to eliminate abuse can be adapted to our mechanisms.

4.3 Application to dynamic markets and investment

To this point, the parameters of the market such as costs, demand, and abatement technology have been assumed to be stable. If the market is dynamic, with evolving fundamentals, then the fixed point of marginal social cost pricing becomes a moving target and convergence of the mechanisms will be slowed. Armstrong and Sappington (2007) point out in this regard that surplus subsidy mechanisms can impose financial hardship

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[26]Sappington and Sibley (1993) show that the owners of the firm will limit abuse by managers to its optimal level under the ISS. However, the owners of the firm themselves may engage in abuse if possible.

on the firm if costs rise over time. In particular, expected, increasing costs can make the subsidies in the initial periods insufficient to satisfy the firm's individual rationality constraint. In this case, the PS mechanisms can be modified by subtracting from the subsidy a tax of $1/\beta - \epsilon$ (where $\epsilon > 0$ is arbitrarily small) per dollar of operating profit earned in the previous period. Sappington and Sibley (1988) show that so doing extracts (nearly) all rent from the firm and (more importantly for present purposes) ensures that marginal cost pricing satisfies the individual rationality constraint of the firm. Note, however, that the regulator must know the discount factor (which requires no additional assumption in the cases of the $PS_{\bullet}^0$ mechanisms).

When process innovation in the industry makes lower cost production or abatement technology freely available, the firm always chooses the lowest cost technology. By lowering its costs, the firm increases the amount of incremental surplus it can capture through the subsidies. The mechanisms provide no incentive to delay adopting the new technology. Even when access to (or innovation of) lower-cost technology requires investment on the part of the firm, the mechanisms result in socially efficient investment levels. As Sappington and Sibley (1988) explain, mechanisms of the $PS_1^{\bullet}$ type require the firm to bear and allow the firm to reap the social benefit of investment for exactly one period. Thus the firm bears portion $1 - \beta$ of each and the private and social incentives are aligned.

However, underinvestment is possible with the $PS_0^{\bullet}$ mechanisms when demand is unknown. The firm bears fraction $1 - \beta$ of the investment cost, but can capture strictly less than $1 - \beta$ of the social benefit over time, due to the discrete approximation of changes in welfare used. Underinvestment is greater with lower discount factors, for which less of the social benefit is captured. Of course, underinvestment is not an outcome unique to our mechanisms, since neither unregulated monopoly nor perfectly competitive markets have optimal incentives to innovate in general (Arrow, 1962).

Regarding product innovation, the incentives provided by the mechanisms depend on the treatment of the lagged price for the new good. Following Hicks (1940), it is natural to

treat the price of the new good in the period before it is introduced (since a P_{t-1} is required to compute the subsidy) as the lowest price that would have resulted in no sales.[27] In this case, the $PS_1^{\bullet}$ versions of the mechanism are equivalent to Loeb and Magat's (1979) mechanism for the new good market, and award the firm with full rent extraction (and thus optimal incentives to innovate). The $PS_0^{\bullet}$ mechanisms do not provide full incentive, for the usual reason that the entire amount of surplus could only be obtained if there were no discounting and the firm lowered price by infinitesimal steps. Again, however, by awarding more of the surplus to the firm than an unregulated monopolist charging a uniform price would be able to capture, these mechanisms outperform the unregulated market with regard to product innovation.[28]

4.4 Other issues

The rest of this section covers a few remaining issues, including whether the mechanisms are self-financing, whether they efficiently select firms for production, the optimality of the mechanisms, and regulatory commitment.

4.4.1 Budget balance

The PS mechanisms share the feature with the AISS and ISS that more money may be required to subsidize the firm than the transfer brings in to the regulator in periods in which it is negative (that is, is a tax on the firm). Of course, as noted above our mechanisms will require much less subsidy than a mechanism in the style of Loeb and Magat (1979) since only increments to welfare are subsidized rather than awarding all surplus

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[27]That is, for purposes of computing (5), for period $t-1$ use the lowest price for the new good $N+1$ such that at the actual prices for the existing goods, demand for good $N+1$ is zero.

[28]An additional complication when the regulator does not know the demand function is that the counterfactual choke price to use for good $N+1$ in period $t-1$ to calculate subsidy (6) will not be known. Good $N+1$ can be left out of the subsidy calculation for the period of its introduction, in the same spirit that new services subject to U.S. federal telecommunications price regulation in the 1990s were added to the price cap formulas only after their first year. However, then it is as if the initial price is endogenous for the new good and the problem noted in the text following equation (2) regarding the firm's strategic choice of P_0 applies.

to the firm each period. Furthermore, the inclusion of the adjustment in the subsidy for increases in environmental damage also lowers the transfer from the amount it would be under a pure AISS or ISS scheme. However, when applying the mechanisms to a natural monopoly in cases where marginal social cost pricing is not profitable, at least near the steady state positive transfers will be required, since the subsidy converges to a refund of last period's operating losses. We do not explicitly consider the social cost of the transfers, as does much of the Bayesian mechanism design literature, because in the U.S. (and increasingly also elsewhere) subsidies are typically paid by consumers of the firm's services rather than from general tax revenue. Thus we follow Sappington and Sibley (1988) in assuming that subsidies can be funded through two-part tariffs with a fixed-fee component that is presumed not to affect demand.[29]

4.4.2   Selection of firms

Do the mechanisms efficiently select firms for production? In other words, do the mechanisms ensure that the firm remains in business if and only if it is socially beneficial for it to do so? In general, as with other incremental surplus subsidy schemes, the answer is no, because there is no way to compare the size of the subsidy with the total amount of surplus obtained for society through production. However, when the transfers are funded through two-part tariffs as suggested above, the maximum amount that could be collected through the fixed fees is the net social surplus (assuming that the consumers or ratepayers are also the population harmed by the pollution). Thus socially inefficient production is unfundable under two-part tariffs, which signals to the regulator that the firm should not continue to be subsidized.[30]

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[29]This accords with our assumption that there are no income effects in demand.

[30]This argument presumes that consumers have homogeneous demand. If not, then uniform fixed fees will cause those consumers with the lowest surplus from consumption to cease purchasing before the total amount of revenue collected exhausts total net surplus. In this case, the ability to fund the mechanisms through uniform fixed fees becomes a sufficient but not necessary condition for efficient selection. Efficient selection with heterogeneous demand can be restored by treating the determination of the fixed fee for each consumer, given the usage price, as an exercise in contributing toward the provision of an indivisible public good (in this case, the existence of the firm). Then we could apply anonymous mechanisms such as that

4.4.3 Optimality of the mechanisms

The PS_1^1 mechanism also has the property that (in the absence of knowledge of the discount factor and the cost function) no other Markovian mechanism that guarantees marginal cost pricing and optimal abatement in all periods awards lower rents to the firm in periods after the first.[31] The proof is as in Sappington and Sibley (1988), whose ISS mechanism shares a similar property when there is no external damage. To ensure optimal prices and abatement, the subsidy can differ from consumer surplus (net of external damage) only by a constant. That constant can be no lower than what the PS_1^1 mechanism awards in periods 2 on.[32]

4.4.4 Regulatory commitment

Given that the mechanisms are dynamic, the regulator must commit to the regulatory policy over time. There are two departures from commitment that one can anticipate. The first regards dynamic inconsistency. A complication arising under Bayesian regulation is that the regulator, once knowing the firm reveals its true characteristics, has incentive to renege on the regulatory contract and not allow the firm to collect information rent (Crew and Kleindorfer, 2002). But without regulatory commitment, the firm lacks the incentive to reveal its true type, and the Bayesian approach collapses. The PS mechanisms do not share this challenge. When they are funded under the two-part tariffs discussed above, the regulator cannot improve social welfare by reneging on the promised subsidy.[33] Furthermore, under the informational constraints assumed, the regulator has no incentive to

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proposed by Jackson and Moulin (1992) to elicit from consumers their willingness to pay for access to the good, to ensure that the project is undertaken if and only if the collective benefit outweighs the total cost, and to collect only enough fees to cover the costs of the project.

[31]The mechanism is Markovian since the subsidy depends only on the firm's action and the state in the present period, where the latter is determined by the firm's actions in the previous period.

[32]Consider subsidies of the form $CS_t - D_t - k$ that induces optimal outcomes, and let $\hat{S}$ be the lowest subsidy S for which $OP^* + S \geq 0$. For the firm to stay in business requires $CS_t - D_t - k \geq \hat{S}$, or $k \leq CS^* - D^* + OP^*$ at the optimal outcomes. Inspection of the PS_1^1 subsidy (5), evaluated at optimal prices and abatement, shows that the subsidy is exactly $CS^* - D^* + OP^*$ in periods 2 on.

[33]If there is a social cost of transferring money to the firm, then the rent-extraction proposal discussed in section 4.3 can be used to remove the incentive for *ex post* regulatory opportunism.

change the proffered subsidies for future periods. In the case of the PS_1^1 mechanism, this is because no better Markovian mechanism exists, as discussed in the previous subsection. The other mechanisms (for which optimality is not proven) are dynamically consistent in the sense that if any mechanism would perform better than the PS mechanism, there is no reason the regulator would not have adopted it in the previous period.

Another reason regulatory commitment may fail is that the regulator's view of marginal social cost may change due to varying political pressure, changes in other ambient environmental conditions, or newly gained information about the impacts of the externality. In such situations, the regulator would adjust the part of the subsidy stemming from damages, and push the firm toward the "new" equilibrium. This is the situation discussed in section 4.3. However, such changes can be costly, because sufficient uncertainty about future subsidies may lead to unpredictable actions on the part of the firm. The difficulties of long-term commitment in both Bayesian and non-Bayesian regulatory mechanisms are discussed in greater detail in Vogelsang (2006).

5 Conclusion

If a monopolist's production results in externalities, application of either regulatory methods for competitive polluting firms or mechanisms for non-polluting natural monopoly will yield socially suboptimal output levels. We introduce four mechanisms to regulate an externality-producing monopolist that are easier to implement than prior mechanisms. Unlike Bayesian regulatory methods, the PS mechanisms do not require that the regulator have any knowledge of the firm's cost function or underlying cost distribution. Our mechanisms also advance the literature on anonymous regulation for polluting firms by reducing the size of the subsidy given to the firm and not using information about the entire demand curve. In the mechanism requiring the least information, the regulator need know only fully observable and verifiable outcomes from prior periods and the discount

factor of the firm.

The proposed mechanisms should not be applied blindly without careful investigation of the particular market to ascertain whether the required assumptions hold. Just as with cap-and-trade plans or price cap regulation of public utilities, the simple, elegant theoretical mechanisms may need to be expanded in practice. What can be expressed with a few equations in the sterile laboratory of the theorists' model often ends up as hundreds of pages in the bureaucrat's regulations, and no doubt more research will be needed to bridge the gap between theory and practice.

A natural extension of the mechanisms would be to oligopoly. The $PS_1^\bullet$ mechanisms can be extended as in Schwermer (1994) or Kim and Lee (1995), who modify the ISS mechanism for Cournot oligopoly without environmental externalities. If only total damage is observed each firm must be assessed the entire incremental damage, which may lead to inefficient exit decisions. If the damage function is known, the information can be used to penalize each firm for only its contribution to incremental damage, as in Kim and Chang's (1993) nonlinear pollution tax. The extension of the $PS_0^\bullet$ mechanisms to oligopoly are more difficult to analyze, since each firm must consider not only how its actions today affect its future subsidy, but also how other firms will respond to its actions. As an additional consideration, Benchekroun and Ray Chaudhuri (2008) show that dynamic pollution taxes can help sustain collusion. Moving from monopoly to oligopoly takes the firm's problem from decision theory to game theory, making it beyond the scope of this article. We intend to pursue such exploration in future work.

Appendix

A-1.1 Proof of Proposition 1

The firm's profit stream, discounted with factor β and expressed as a function of choice variable sequences $\mathbf{x} = (\mathbf{P}, \mathbf{a})$, where $\mathbf{P} = \{P_t\}_{t=0}^{\infty}$ and $\mathbf{a} = \{a_t\}_{t=0}^{\infty}$, has present value

$$\Pi(\mathbf{x}) = OP_0 + \sum_{t=1}^{\infty} \beta^t \pi_t \tag{A-1}$$

where $x_0 = (P_0, a_0)$ is assumed to be exogenous. Define period profit to be

$$\pi(x_t, x_{t-1}) = OP_t + S_t = OP_t + (\Delta CS_t - OP_{t-1} - \Delta D_t) \tag{A-2}$$

$$= P_t \cdot Q(P_t) - C(Q(P_t), a_t) + \Delta CS(P_t, P_{t-1})$$

$$- [P_{t-1} \cdot Q(P_{t-1}) - C(Q(P_{t-1}), a_{t-1})]$$

$$- [D(Q(P_t), a_t) - D(Q(P_{t-1}), a_{t-1})]$$

Note that profit is a function of the previous period's prices and abatement through the subsidy. Thus the firm's choice of x_t affects that period's operating profit and subsidy, along with next period's subsidy. The terms in $\Pi(\mathbf{x})$ relevant to the firm's decision at t are

$$\mathcal{L}_t = \pi(x_t, x_{t-1}) + \beta \pi(x_{t+1}, x_t) \tag{A-3}$$

The first order conditions to maximize (A-3) with respect to x_t (where C, D, P, and Q are treated as column vectors), after noting that $\nabla_{P_t} \Delta CS = -Q_t$,[34] are

$$
\begin{aligned}
\nabla_{P_t} \mathcal{L}_t &= Q_t + \nabla_P Q_t \left(P_t - \nabla_Q C_t \right) - \left(Q_t + \nabla_P Q_t \nabla_Q D_t \right) \\
&\quad + \beta \left\{ Q_t - \left[Q_t + \nabla_P Q_t \left(P_t - \nabla_Q C_t \right) \right] + \nabla_P Q_t \nabla_Q D_t \right\} \\
&= (1 - \beta) \left[P_t - \nabla_Q \left(C_t + D_t \right) \right] = 0 \\
&\Rightarrow P_t = \nabla_Q \left(C_t + D_t \right) \qquad \forall t > 0, \beta < 1
\end{aligned} \tag{A-4}
$$

We use the notation $\nabla_x y$ to mean the matrix with element $\partial y_j / \partial x_i$ in row i and column j. Equation (A-4) implies that at P_t^* price is set equal to marginal social cost, the sum of marginal private cost and marginal damages. The first order condition with respect to a_t is

$$
\begin{aligned}
\nabla_{a_t} \mathcal{L}_t &= -(1 - \beta) \nabla_a \left(C_t + D_t \right) = 0 \\
&\Rightarrow \nabla_a C_t = -\nabla_a D_t \qquad \forall t > 0, \beta < 1
\end{aligned} \tag{A-5}
$$

At a_t^*, the marginal private cost of abatement is equal to marginal social benefits of abatement.

A-1.2 Proof of Proposition 2

The working paper version of the present work (available upon request) contains a proof that the dynamical system defined by (7) is saddle-path stable and converges to the social optimum as long as the damage function exhibits enough convexity. In particular, it can be shown that saddle-path stability obtains if (in addition to assumptions already noted in the text) demand slopes down and social cost (the sum of private cost and damage) is linear or convex in Q.

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[34]The equality follows from the lack of income effects and Roy's identity, or directly from a fundamental theorem of line integrals (Apostol, 1969, p.338)

However, instead of following a purely abstract approach relying on the theory of difference equations, here we adapt Finsinger and Vogelsang's (1985) proof of convergence of their mechanism to our setting. This alternative proof of convergence provides intuition regarding some important features of the mechanism, as discussed in the text. Given our focus on the intuition behind the mechanism, in a few places below we refer the reader to various other work by Finsinger and Vogelsang for proofs of some technical details. Given a reference price P_0, define consumer surplus (not including damage) in a period as

$$V(P) = \int_P^{P_0} Q \cdot d\alpha$$

for any continuous path of integration $\alpha \in \mathbb{R}^N$ from P to P_0. Using $V(P)$ as if from a normative representative consumer is the same as assuming that each actual consumer has quasi-linear utility[35] and there must be no income effects in the relevant range of prices. Define social welfare as $W(P, a) = V(P) + OP(P, a)) - D(Q(P), a)$, where OP is as defined in (2) and D is the damage function as in the text. First, note that the welfare gains from a series of the firm's choices are always higher than the undiscounted sum of the subsidy payments:

Lemma 1. *With no discounting, the welfare gains always outweigh the subsidies and operating profits. For any $j, k \in \mathbb{N}, k \geq j$, and any sequence $\{a_i\}_{i=j-1}^k$, we have*

$$W(P_k, a_k) - W(P_{j-1}, a_{j-1}) \geq \sum_{t=j}^k \pi_t. \tag{A-6}$$

Proof: The lack of income effects implies that V is convex (Vogelsang and Finsinger, 1979, see). Since $\partial V / \partial P = -Q$, $V(P_i) - V(P_{i-1}) \geq Q(P_{i-1})(P_{i-1} - P_i)$. Adding $\Delta OP_i -$

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[35]Utility need not be identical among consumers, but the marginal utility of income must be constant and identical for all (see Mas-Colell, Whinston, and Green, 1995, p. 119).

ΔD_i to both sides gives $\Delta W_i \geq \pi_i$ (see equation (A-12) below to explain the right side of the inequality), which holds for any (a_{i-1}, a_i). So

$$W(P_k, a_k) - W(P_{j-1}, a_{j-1}) = \sum_{t=j}^{k} \Delta W_t \geq \sum_{t=j}^{k} \pi_t \qquad \blacksquare \qquad \text{(A-7)}$$

Lemma 2. *The firm can:*

1. *increase its profit whenever it is possible to increase welfare; for any differentiable path $x(t) = (P(t), a(t))$, $t \in [0, 1]$, such that $W(x(t))$ is strictly increasing in t, there exists a sequence $\{x_i\}_{i=0}^{k}$, with $x_0 = x(0)$, $x_k = x(1)$, such that $\Pi_k(\beta) = \sum_{i=1}^{k} \beta^{i-1} \pi(x_i) > 0$; and*

2. *appropriate all welfare gains in the limit by changing prices by ever-smaller amountsthe discretization of the path $x(t)$ becomes finer; $\lim_{k \to \infty} \Pi_k(0) = W(x(1)) - W(x(0))$.*

Proof: Because $\partial V / \partial P = -Q$, we have

$$V(p(1)) - V(p(0)) = -\int_{P(0)}^{P(1)} Q \cdot dP = -\int_0^1 Q(P(t)) \cdot \dot{p}\, dt$$

$$= \lim_{k \to \infty} \sum_{i=1}^{k} -Q(P(t_i)) \cdot [P(t_i) - P(t_{i-1})]$$

where the first equality follows from the second fundamental theorem of calculus for line integrals (Apostol, 1969, p. 334), the second from the definition of the line integral, and the third from the definition of integrals, assuming that $\{t_i\}_{i=1}^{k}$ is a partition of $[0, 1]$. Thus for every $\varepsilon > 0$ there exists an m such that for all $k \geq m$,

$$\left| [V(p(1)) - V(p(0))] + \left[\sum_{i=1}^{k} Q(P_i) \cdot (P_i - P_{i-1}) \right] \right| < \varepsilon \qquad \text{(A-8)}$$

Adding $(OP_k - D_k) - (OP_0 - D_0)$ to both expressions in brackets implies

$$\left| W(x(1)) - W(x(0)) - \sum_{i=1}^{k} \pi_i \right| < \varepsilon \qquad \text{(A-9)}$$

Lemma 1 implies that the expression inside the absolute value bars on the left side of (A-9) is non-negative, and so

$$\sum_{i=1}^{k} \pi_i \leq W(x(1)) - W(x(0)) < \sum_{i=1}^{k} \pi_i + \varepsilon \qquad \text{(A-10)}$$

thus proving part 2 of the lemma. Furthermore, note that (A-10) along with the monotonicity of W on $x(t)$ implies that for a sufficiently fine partition, $\sum_{i=1}^{k} \pi_i > 0$. This implies that there is a partition for which π_i is positive for all i. If not, then there is always a $\pi_j < 0$ no matter how large k is. Since $\pi_j = \Delta OP_j - Q_{j-1} \cdot \Delta P_j - \Delta D_j]$ (see equation (A-12) below), (A-8) implies that π_j can be made arbitrarily close to ΔW_j. However, recall that x_j is constructed from partition $\{t_i\}_{i=1}^{k}$, and that W is monotonic on $x(t)$. It cannot be that π_j is negative but W is monotonic. Since k can be found such that $\pi_i > 0 \ \forall i$, part 1 of the lemma follows. ∎

Lemma 3. *Welfare converges to its maximum.* $\{W(x_t)\}_{t=1}^{\infty} \to W^* = \max_x W(x)$.

Proof: If not, then define $\bar{W} = \limsup\{W(x_t)\}_{t=1}^{\infty}$. Then unless W_t cycles endlessly below $\bar{W}$, which lemmas III and IV of Finsinger and Vogelsang (1982) show cannot be profitable, $W_t \to \bar{W}$ and the firm eventually can gain only an arbitrarily small amount of additional profit, per Lemma 1. Since W is assumed to have a unique maximum, there is a differentiable path $x(t)$ such that $W(x(t))$ is strictly increasing in t (starting near $\bar{W}$ and ending at W^*), and so part 1 of Lemma 2 implies that the firm is leaving additional profit unearned, which cannot be optimal. ∎

To complete the proof of Proposition 2, note that with a unique maximum, there is a unique $x^* = \arg\max_x W(x)$. The absence of non-global maxima in welfare simplifies

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the proof but is not essential. See Finsinger and Vogelsang (1982) for the additional steps needed to prove convergence in the absence of concavity.

A-1.3 Derivation of the Policy Function for the PS_0^1 Mechanism

Under mechanism PS_0^1 profit in (A-2) becomes

$$\pi(x_t, x_{t-1}) = OP_t + S_t = OP_t + [Q_{t-1} \cdot (P_{t-1} - P_t) - OP_{t-1} - \Delta D_t] \tag{A-11}$$

$$= \Delta OP_t - [Q_{t-1} \cdot \Delta P_t + \Delta D_t] \tag{A-12}$$

which can be written as

$$\pi(x_t, x_{t-1}) = P_t \cdot Q(P_t) - C(Q(P_t), a_t) - [P_t \cdot Q(P_{t-1}) - C(Q(P_{t-1}), a_{t-1})]$$
$$- [D(Q(P_t), a_t) - D(Q(P_{t-1}), a_{t-1})]$$

The first order conditions to maximize (A-3) are now

$$\nabla_{P_t}\mathcal{L}_t = Q_t + \nabla_P Q_t (P_t - \nabla_Q C_t) - (Q_t + \nabla_P Q_t \nabla_Q D_t)$$
$$+ \beta \{Q_t - \nabla_P Q_t \Delta P_{t+1} - [Q_t + \nabla_P Q_t (P_t - \nabla_Q C_t)] + \nabla_P Q_t \nabla_Q D_t\}$$
$$= \Delta Q_t + (1 - \beta) \nabla_P Q_t [P_t - \nabla_Q (C_t + D_t)] - \beta \nabla_P Q_t \Delta P_{t+1} = 0$$
$$\Rightarrow \Delta P_{t+1} = \frac{1}{\beta} (\nabla_P Q_t)^{-1} \Delta Q_t + \frac{1-\beta}{\beta} [P_t - \nabla_Q (C_t + D_t)]$$

$$\forall t \geq 2, \beta \neq 0 \tag{A-13}$$

which is the law of motion (7) in the text. Note that the first term on the right side uses the assumption that the Jacobian of the demand system is invertible (or, equivalently, that the Hessian of consumer welfare is nonsingular). The first order condition for abatement is the same as in (A-5).

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A-1.4   Proof of Proposition 3

Under mechanism PS_1^0 profit in (A-2) becomes

$$\pi(x_t, x_{t-1}, x_{t-2}) = OP_t + S_t = OP_t + (\Delta CS_t - OP_{t-1} - \Delta D_{t-1}/\beta)$$

$$= P_t \cdot Q(P_t) - C(Q(P_t), a_t) + \Delta CS(P_t, P_{t-1})$$

$$- [P_{t-1} \cdot Q(P_{t-1}) - C(Q(P_{t-1}), a_{t-1})]$$

$$- [D(Q(P_{t-1}), a_{t-1}) - D(Q(P_{t-2}), a_{t-2})]/\beta$$

Since the subsidy two periods ahead is affected by actions in t, an extra term is added to (A-3):

$$\mathcal{L}_t = \pi(x_t, x_{t-1}, x_{t-2}) + \beta\pi(x_{t+1}, x_t, x_{t-1}) + ?^2\pi(x_{t+2}, x_{t+1}, x_t) \qquad \text{(A-14)}$$

The first order conditions to maximize (A-14) are

$$\nabla_{P_t}\mathcal{L}_t = Q_t + \nabla_P Q_t \left(P_t - \nabla_Q C_t \right) - Q_t$$

$$+ \beta \left\{ Q_t - \nabla_P Q_t \Delta P_{t+1} - \left[Q_t + \nabla_P Q_t \left(P_t - \nabla_Q C_t \right) \right] - \nabla_P Q_t \nabla_Q D_t/\beta \right\}$$

$$+ \beta^2 \nabla_P Q_t \nabla_Q D_t/\beta$$

which match those in (A-4) exactly. Thus the mechanism induces exactly the same behavior from the firm as does the PS_1^1 version.

A-1.5 Proof of Proposition 4

Under mechanism PS_0^0 profit in (A-2) becomes

$$\pi(x_t, x_{t-1}, x_{t-2}) = OP_t + S_t = OP_t + [Q_{t-1} \cdot (P_{t-1} - P_t) - OP_{t-1} - \Delta D_{t-1}/\beta]$$
$$= P_t \cdot Q(P_t) - C(Q(P_t), a_t) - [P_t \cdot Q(P_{t-1}) - C(Q(P_{t-1}), a_{t-1})]$$
$$- [D(Q(P_{t-1}), a_{t-1}) - D(Q(P_{t-2}), a_{t-2})]/\beta$$

and the period-specific maximand is as in (A-14).The first order conditions to maximize (A-14) are now

$$\nabla_{P_t} \mathcal{L}_t = Q_t + \nabla_P Q_t (P_t - \nabla_Q C_t) - (Q_t + \nabla_P Q_t \nabla_Q D_t)$$
$$+ \beta \{ Q_t - \nabla_P Q_t \Delta P_{t+1} - [Q_t + \nabla_P Q_t (P_t - \nabla_Q C_t)] - \nabla_P Q_t \nabla_Q D_t/\beta \}$$
$$+ \beta^2 \nabla_P Q_t \nabla_Q D_t/\beta$$
$$= \Delta Q_t + (1 - \beta) \nabla_P Q_t [P_t - \nabla_Q (C_t + D_t)] - \beta \nabla_P Q_t \Delta P_{t+1} = 0$$

which match those in (A-13) exactly. Thus the mechanism induces exactly the same behavior from the firm as does the PS_0^1 version.

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Table 1: The price-based subsidy mechanisms

	Demand function known	Output observed with lag
Damage observed immediately	PS_1^1 (immediate convergence)	PS_0^1 (asymptotic convergence)
Damage observed with lag	PS_1^0 (immediate convergence)	PS_0^0 (asymptotic convergence)

Table 2: Numerical examples of the PS_1^1 and PS_0^1 mechanisms

	Low initial price				High initial price			
	low interest rate (r =1.8%)		high interest rate (r =10%)		low interest rate (r =1.8%)		high interest rate (r =10%)	
period	price (cents)	subsidy ($Billion)	price (cents)	subsidy ($Billion)	price (cents)	subsidy ($Billion)	price (cents)	subsidy ($Billion)
0	18.30	-	18.30	-	29.49	-	29.49	-
1	18.80	-0.21	18.80	-0.21	18.80	0.90	18.80	0.90
2,3,	18.80	-0.22	18.80	-0.22	18.80	-0.22	18.80	-0.22
Optimal price	18.80		18.80		18.80		18.80	
Present value								
Gross profit	12.03		2.16		12.03		2.16	
Subsidy	-12.03		-2.16		-10.93		-1.15	
Gross CS	272.49		49.05		272.49		49.05	
Damage	24.36		4.38		24.36		4.38	
Total surplus	*260.16*		*46.83*		*260.16*		*46.83*	
Δ Present value								
Net profit	-0.028		-0.003		-135.27		-23.53	
Net CS	-0.383		-0.071		209.07		36.81	
Damage	-0.548		-0.099		11.64		2.09	
Total surplus	*0.138*		*0.025*		*62.17*		*11.19*	

Notes: Price and subsidies at top part of table are not discounted. Present values include periods 1 on. Gross (net) profit excludes (includes) the subsidy. Gross (net) consumer surplus (CS) excludes (includes) the subsidy, and excludes damage. Total surplus includes gross profit, gross CS, and damage. Δ present value is the change in present value when moving from a no-mechanism market to the market with the mechanism imposed. For the PS_0^1 mechanism, it is assumed that prices are the same in period 0 and the one before.

Table 3: Numerical examples of the PS_1^0 and PS_0^0 mechanisms

	Low initial price				High initial price			
	low interest rate (r =1.8%)		high interest rate (r =10%)		low interest rate (r =1.8%)		high interest rate (r =10%)	
period	price (cents)	subsidy ($Billion)	price (cents)	subsidy ($Billion)	price (cents)	subsidy ($Billion)	price (cents)	subsidy ($Billion)
0	18.30	-	18.3	-	29.49	-	29.49	-
1	18.36	-0.03	18.42	-0.05	28.24	-2.19	27.01	-1.94
2	18.41	-0.05	18.51	-0.09	27.13	-2.16	25.11	-1.84
3	18.46	-0.07	18.58	-0.12	26.15	-2.10	23.65	-1.65
4	18.50	-0.09	18.63	-0.14	25.29	-2.00	22.53	-1.42
5	18.53	-0.10	18.67	-0.16	24.53	-1.89	21.66	-1.21
10	18.66	-0.16	18.77	-0.20	21.87	-1.29	19.57	-0.52
20	18.76	-0.20	18.80	-0.22	19.68	-0.57	18.86	-0.24
30	18.79	-0.21	18.80	-0.22	19.06	-0.32	18.81	-0.22
Optimal price	18.80		18.80		18.80		18.80	
Present value								
Gross profit		10.64		1.67		33.92		9.94
Subsidy		-10.63		-1.67		-32.95		-9.16
Gross CS		273.94		49.56		245.59		39.49
Damage		-24.43		-4.41		-23.00		-3.90
Total surplus		*260.15*		*46.83*		*256.51*		*45.53*
Δ Present value								
Net profit		-0.028		-0.004		-135.40		-23.76
Net CS		-0.326		-0.050		204.19		35.27
Damage		-0.484		-0.076		10.27		1.61
Total surplus		*0.130*		*0.022*		*58.52*		*9.89*

Notes: See notes to Table 2. For the PS_0^0 mechanism, it is assumed that prices are the same in period 0 and the one before.
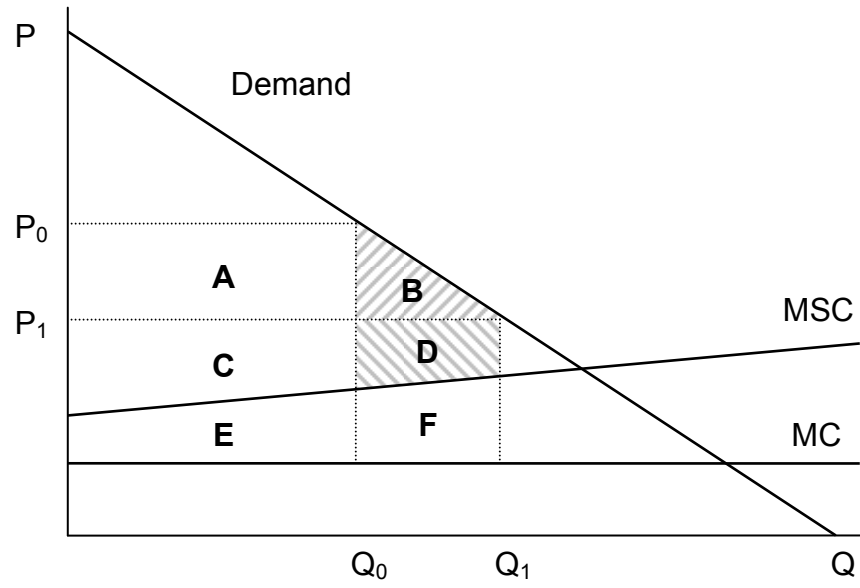
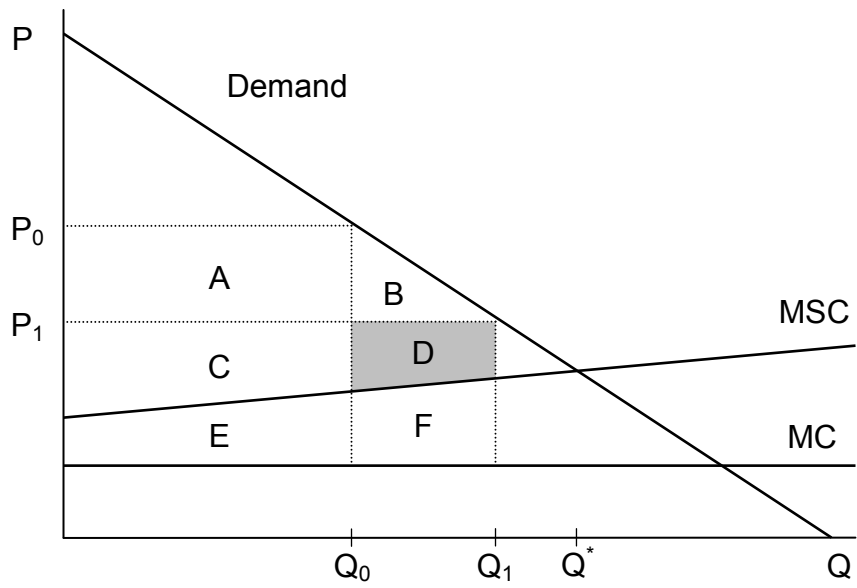Figure 1: The firm's profit under the PS_1^1 mechanism

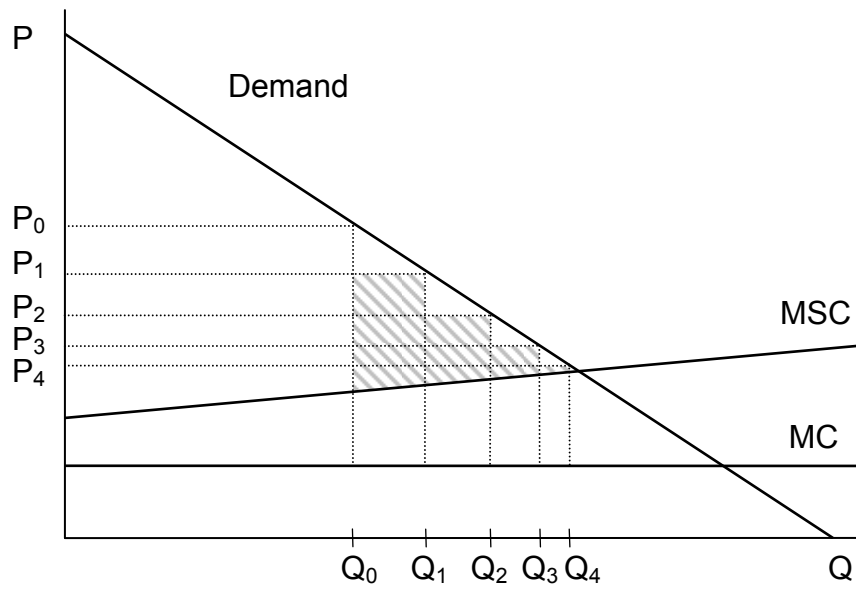Figure 2: The firm's profit in one period under the PS_0^1 mechanism

Figure 3: The firm's profit over time under the PS_0^1 mechanism