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# **On Valuing Employee Stock Option Plans With The Requisite Service Period Requirement**

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## **Introduction**

This paper examines issues involving employee stock option plans as a part of employee compensation. In particular, employee stock options requiring a requisite service period are viewed as an option on options and hence, a compound option, as the employee's future market wage rate is unknown. Non-transferability of employee stock options is overcome by the well known put-call parity theorem. Difficulties arise, however, if a firm is privately held. The paper derives a simple formula to value equities for privately held firms. The formula will then be used in valuing the employee stock options.

Many U.S. corporations, in addition to paying regular wages and salaries, offer employees stock options as a part of an employee compensation package. As such, employee stock options are treated as still another form of employee incomes, as they provide an employee with a substantial economic gain. The employee stock options are call options giving employees privilege to be able to purchase stocks at a price substantially below the prevailing market price by the time when the options are exercised. The difference between ordinary wages and salaries, and the employee stock options is that with the employee stock options, although accrued, the actual compensation is deferred while ordinary wages and salaries are paid now. Normally, the employee stock options are subject to certain restrictive conditions.

Unfortunately, the standard accounting practice, however, has not allowed these costs to be a part of an employee compensation package, and hence, true wages and salaries have been largely underreported. In order to recognize these costs explicitly as a matter of GAAP, the

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recent FASB Rule 123R requires a business entity to impute these costs of the employee stock options.

The purpose of this paper is to value employee stock options especially with required requisite service periods, both when the firm's shares are publicly traded and when the firm is privately held. Section A discusses some practical valuation issues. Section B introduces valuation formulas followed by some numerical examples in Section C. Section D discusses the option valuation when the underlying stock value is difficult to compute when a firm is privately held. The value of the underlying is, however, shown to depend on the firm's asset turnover ratio and the cost of goods sold as well as the industry's correlation with the economy. Section E concludes the paper.

### **A. Some Valuation Issues**

The majority of employee stock options are compensatory and as such, they are a part of the employee wages and salaries. Some employee stock options, however, are non-compensatory. Therefore, they should be reported under a possible heading, "extra incomes or bonuses," or "the disposition of capital surplus," and not under the heading, "wages and salaries." In this paper, we do not distinguish between compensatory and non-compensatory employee stock options. However, the following will impact the value of employee stock options so we discuss them here.

#### **1. Valuation Date and Financial Reporting Standard**

Employee stock options can be valued either at the grant date or at any date thereafter, more meaningfully, at the financial statement date. In this paper, we are not concerned about how changes in the market value of the options will impact the company's financial statement, until the options are exercised, or expire unexercised. The employee stock options will be valued at the grant date.

Example: Assume that on a given date in 2004, John Doe received an employee stock option to buy 10,000 shares of his employer's common stocks at a price of \$10 a share on December 24, 2005. If the employee stock option was valued at \$2 a share at the grant date, the company will have debited the entire \$20,000 as an expense for the year, if the option is fully vested. Or the company will have amortized it. A subsequent rise or fall in the value of the employee stock option is not accounted for in the financial statement, until the options are actually exercised, or simply expire unexercised on December 24, 2005.

#### **2. Life of Employee Stock Options**

In one extreme end of the spectrum, employee stock options may be exercisable immediately after the options are granted but for a specified time period. This is often known as the American option. In the other extreme end, options can be written open ended and continue on for a long time, while options can be exercisable immediately. The life of this option may be infinite.

The simplest option is of European type where an option is sold with the timing of the option's exercise made certain. That is, the life of the European option is finite.

The major difference between the American and European call options is that with the American options, options can be exercised at any time leading to the option's contractual expiry, while with the European options, options can be exercisable only at the option's expiry. Consequently, the American options are more valuable than the European options in principle.

In this paper, we present varied situations where the life of employee stock options differs in valuing the employee stock options.

### **3. Vesting and Vested Options**

Some stock options are forfeited due to premature cessation of employment prior to an agreed-upon compensatory service period. Hence, we do not rule out a possibility that the options may be forfeited. That is, although writer of an option may not be able to cancel the options contract, employees can, and this event must be discounted in pricing the options. Obviously, if in actuality, such an event occurs, we simply make reverse or adjusting entries.

Staying on the job long enough at least until after the requisite service period ensures that the employee stock option is fully vested. Whether to stay with the job to reap the profits from the fully vested option requires an economic decision.

However, the decision to stay at the current employment depends on the value of the employee stock options and the total compensation both at the end of the requisite service period relative to the employee's anticipated market wages. Therefore, if an employer offers a stock option to an employee, the employer has effectively sold a call option for an employment at no cost to the employees with an exercise price equal to the employee's anticipated market wages, which is unknown.<sup>1</sup> Consequently, a compensatory employee stock option, which requires a certain requisite service for some period, can be viewed as an option on an option, or a compound option. In this paper, we will show how such an option is valued.

Generally, the economic value of employee stock options at the grant date will be amortized on a straight-line basis over the requisite service (or the vesting) period. If, however, the option is fully vested, no such amortization would be necessary.

### **4. Valuing Options When the Firm is Privately Held**

If a firm is privately held, difficulties exist in valuing employee stock options, as the value of these options depend on the firm's market value of equity. There are at least three different approaches discussed in finance literature regarding valuing stocks, i.e. (1) the Capital Asset Pricing Model (CAPM) approach, (2) the constant dividend growth model, and (3) such multiples approaches as earnings, cash flows or sales multiples. More fundamentally, however, one can assess the firm value by examining the firm's asset turnover and the labor productivity directly in conjunction with the usual industry's relations with the economy and the aggregate market volatility. We examine some of these issues in this paper.

#### **B. Valuation Models in General**

As discussed, the value of employee stock options can differ widely depending on whether or not the options are fully vested. In case where the options are fully vested, there are at least two valuation models available. They are the famous Black-Scholes Option Pricing Model and the lattice model, which refers to binomial or trinomial models. The Black-Scholes model offers a closed form solution for European options, but if the employee stock options are exercisable at any time on or before the option's expiry, i.e. the American options, unfortunately, there is no closed form solution for the American options. So from a theoretical

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<sup>1</sup> For an analysis on options contracts with uncertain exercise price, see, for example, S. Fisher (1978) and W. Margrabe (1978).

standpoint, people talk about the lattice model as an alternative to the Black-Scholes' model as it offers at least an "analytic" solution.

Options can be either fully vested, or may not be fully vested requiring some vesting period. We analyze each one of these cases below. As it turns out, employee stock options, which are of compensatory nature, should be treated as an option on an option to stay with the current employment for long enough a period, and hence, a compound option.

### (1) Fully vested options with no dividend

If an option is fully vested, employees can exercise their options anytime on or before the option's expiry. In the most simplistic case where the stock pays no dividend, the general rule is to use the Black-Scholes option-pricing model to value the employee stock options, even if the Black-Scholes model is for the European options.

It is well known that if no dividends are paid on the underlying asset, it never pays to exercise American options early. Consequently, the valuation formula for American call options becomes identical to that for the Black-Scholes formula for European options. Therefore, it is almost senseless to talk about the lattice model in this case, as the Black-Scholes model offers the exact solution.

In order to see why no early exercise is optimal with the American options, assume that the option can be sold for money at any time. Consider a fully vested employee stock option, which was issued a year ago, but will expire in two years. Assume that the exercise price is \$10 a share. The option's intrinsic value is the amount of money one makes if the option is exercised immediately, which is the difference between the price of the underlying and the exercise price. Obviously, if the stock price is \$15 today, the intrinsic value would be \$5 a share. Is it possible that the call option is valued lower than \$5?

The answer is clearly not. In fact, the call value is always higher than the intrinsic value at any time, as long as there are still time remaining till the option expires. In this example, we still have two years remaining. Having an extra time to exercise is valuable. So, the time value of an option must always be positive. Therefore, if the option can be sold, it will sell for more than the intrinsic value, that is, at the price of the intrinsic value of \$5 plus the extra time value. It never pays to exercise the American options early. What if the stock price is below the exercise price, e.g. \$8? Clearly, the option will not be exercised and hence, the intrinsic value is zero.

Conclusion: Use the Black-Scholes option pricing equation to value the fully vested employee stock options.

Some authors<sup>2</sup> erroneously argue that employee stock options are not transferable and hence, have no market value. Consequently, it is argued that many people will simply exercise their options in that a bird on hand is better than two birds in the bush. With non-transferability, the true value of the employee stock option will be considerably smaller than one would expect. For one thing, making an argument that risks are avoided through exercise is questionable, as the entire option-pricing model is based on the idea of probability-free, risk neutral pricing.

Clearly, the employee stock options are not directly transferable and have no established secondary markets for them. However, selling back a call option can be synthetically created through a "do-it-yourself" alternative. The alternative to selling a call

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<sup>2</sup> For example, see Les Barenbaum, *et al.*, (2004).

option is to write a put, sell short stocks and invest in a risk-free asset.<sup>3</sup> Although the relationship among calls, puts, stocks and risk-free assets does not hold precisely in the case of American options, these alternatives are real and close enough in reality. Therefore, the fact that an option is not directly transferable does not mean that the stock option cannot be valued, especially when the secondary market exists for stocks and options. For example, the famous put-call parity theorem states that buying a put and a stock is equivalent to buying a call and investing in a risk-free asset that guarantees a future payoff. In terms of present values, the theorem suggests that:

$$p + S_0 = c + X \cdot e^{-rT} \quad (1)$$

The symbols,  $p$  and  $c$ , represent the current put and call premium, while  $S_0$  and  $X$  are the current stock price and the exercise price of options. The exercise price also represents the guaranteed future cash flows from a riskless investment. The risk-free interest rate is  $r$  and the time remaining till the option expires is  $T$ . Rearranging Eq. (1) to solve for  $-c$  gives  $-c = -p - S_0 + X \cdot e^{-rT}$ , which states that writing or selling a call is equivalent to writing a put, selling short stocks and investing in a risk-free asset. That is, if an employee stock option cannot be sold, an employee can follow her or his “do-it-yourself” alternatives in the open capital market. Consequently, there is absolutely no need to rely on using the lattice model, when a firm pays no dividends. The Black-Scholes model suffices. The following comments are in order, however.

First, in reality, such “do-it-yourself” alternatives may not exist, if a firm is a private entity whose shares are not traded in the public’s capital market. How do we model the employee stock option when the option is truly not transferable? We will address this question later. Second, obviously, if the employee stock options have identical strike prices and identical maturities, we may not even need to go through tedious computations, as the market call premium would fully reflect the Black-Scholes option pricing objectively. In reality, however, the majority of employee stock options are custom tailored meaning that the exercise prices and maturities are all different for different plans.

## (2) Fully vested options with dividend

It is also well known that the Black-Scholes option-pricing model can price European options on stocks with known discrete dividends by subtracting the present value of the dividends from the stock price. In addition, the Black-Scholes model can also price European options on stocks that pay dividends at a known continuous rate by discounting the stock price by that rate. Remember, however, that with dividends the stock price falls by the amount of dividends as the stock goes ex-dividend. This will reduce incentives to exercise the option, during ex-dividend days, while increasing the incentives to exercise the option before the stock goes ex-dividend. Whether an employee will actually exercise her or his options depends on the amount of dividends.

However, when an option is exercisable anytime on or before the option’s expiry, it is generally known that the binomial model will produce an analytical (not a closed form) solution. The solution is analytical in that one needs to evaluate at every node of trees if the option should be exercised depending on whether the option premium then would be greater

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<sup>3</sup> This is known as the put-call parity in the options literature.

than the intrinsic value then. That is, the value of an employee stock option now is the present value,  $c$ , of either the intrinsic value, i.e.  $(S_t - X)$ , or the expected call premium, i.e.  $c_t$ , whichever is greater, i.e.

$$c = \max(c_t, S_t - X) \cdot e^{-rt} \quad (2)$$

Again, whether the option is transferable is irrelevant for reasons already cited, unless no secondary markets exist for stocks and options. Consequently, computing for the call option value now requires computing for the future call premium at time  $t$ , i.e.,  $c_t$ , and predicting the future stock price,  $S_t$ . In the case of the binomial lattice model, the future call premium is solved for through recursive backward iterations and the future stock price is predicted in a series of up and down stochastic jump process with a given volatility. Differencing time intervals can be made infinitely smaller leading to a solution in continuous time.

### (3) Options with the requisite service period

Staying on the job long enough at least until after the requisite service period ensures that the employee stock option is vested. But this requires an economic decision. Assume that the option is of European type and hence, can be exercised only at the end of requisite service period,  $T$ , which is often the case. Then, the value of an employee stock option at the end of the requisite service period is not only the difference between the stock and the exercise price, i.e.  $S_T - X$ , but also the amount by which the employee's total compensation package,  $S_T - X + W_T$ , exceeds the employee's anticipated market wages,  $M_T$ . The symbol,  $W_T$ , is the employee wages and salaries. It is noted that  $S_T - X + W_T \geq 0$  and that the exercise price,  $M_T$ , is uncertain. The value of the option at time  $T$  can now be stated as follows.

$$c_T = \begin{cases} 0, & \text{if } S_T - X + W_T \leq M_T \\ S_T - X + W_T - M_T, & \text{otherwise} \end{cases} \quad (3)$$

The total compensation package,  $S_T - X + W_T$ , is a random variable, because  $S_T$  is not known, even though the values of  $X$  and  $W_T$  are known a priori.

It is well known<sup>4</sup> that the exact valuation depends on the standard deviations of the stock price,  $S_T$ , and the market competitive wages,  $M_T$ ; and the correlation between them,  $\rho_{sm}$ . The valuation formula for this particular option,  $\zeta$ , is:

$$\zeta = (S - X + W) \cdot N(d_1) - M \cdot N(d_2) \quad (4)$$

<sup>4</sup> See S. Fisher, "Call Option Pricing When the Exercise Price is Uncertain and the Valuation of Index Bonds," *Journal of Finance*, March 1978, pp. 169-176; and W. Margrabe, "The Value of an Option to Exchange One Asset for Another," *Journal of Finance*, March 1978, pp. 177-186.

$$\text{where } d_1 = \frac{\ln\left(\frac{S - X + W}{M}\right) + \sigma^2 T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}; \quad \text{and } \sigma^2 = \sigma_s^2 - 2 \cdot \rho_{sm} \sigma_s \sigma_m + \sigma_m^2.$$

The pricing formula presented here departs from the usual Black-Scholes option pricing model. Essentially, the employee stock option is an option on an option to stay with the job, and hence, a compound option. That is, when the employer first sold an employee stock option to an employee at no cost, it has also effectively sold her or him a call option to “buy” the employee’s current job at an exercise price equal to the employee’s future market wages,  $M_T$ , which of course is not known today.

### C. Numerical Example

The unknowns in Eq. (4) are the standard deviations of the stock returns and the market wage rates, and the correlation between them. The rest of the variables, i.e.  $S$ ,  $X$ ,  $W$ ,  $M$ , and  $T$  are known. Consider the following data:

Requisite period in years	2
Number of shares outstanding	10,000,000
Stock price per share	\$25.00
Exercise price per share	\$24.00
No of stock options given	50,000
Annual salary	\$125,000
Annual salary per share	\$0.0125
Market salary elsewhere	\$150,000
Market salary per share elsewhere	\$0.0150
Annual volatility for wages and salaries in the company	60%
Annual volatility for market wages and salaries	40%
Correlation coefficient	0.85
Total standard deviation	33.47%
Risk-free rate	5%

Applying the information to Eq. (4) gives the value of the employee stock option, which is \$4.20 a share, or \$209,856 for the whole 50,000 shares given.

One may wish to conduct a series of Monte Carlo simulations to predict changes in the call value by recognizing that the stock price process underlying Eq. (4) follows the Wiener process that

$$dS = \mu(t) \cdot S \cdot dt + \sigma(t) \cdot S \cdot \varepsilon \sqrt{dt}, \quad \varepsilon \sim N(0, 1) \quad (5)$$

The symbols  $\mu(t)$  and  $\sigma(t)$  are the instantaneous drift and volatility.

The standard deviations and the correlation are often assumed to vary with time, especially if the firm is still growing. Stock price volatility is most popularly modeled as the standard autoregressive model of the form<sup>5</sup>

$$\sigma_t^2 = \gamma_0 V + \gamma_1 r_{t-1}^2 + \gamma_2 \sigma_{t-1}^2 \quad (6)$$

The symbol  $V$  is the long-run volatility while the  $r$  represents percent change in the stock price. Typically, we assume that the sum of all  $\gamma$ 's is one, i.e.  $\gamma_0 = 1 - \gamma_1 - \gamma_2$ . The variance rate on day  $t + k$  are:<sup>6</sup>

$$\sigma_{t+k}^2 - V = \gamma_1 (r_{t+k-1}^2 - V) + \gamma_2 (\sigma_{t+k-1}^2 - V) \quad (7)$$

Assuming  $E[r_{t+k-1}] \approx 0$ , the expectation on Eq. (6) is  $E[\sigma_{t+k}^2 - V] = (\gamma_1 + \gamma_2)E[\sigma_{t+k-1}^2 - V]$ . This produces

$$E[\sigma_{t+k}^2] = V + (\gamma_1 + \gamma_2)^k E[\sigma_t^2 - V] \quad (8)$$

Similarly, the covariance between the stock returns and the future market wage rates can be updated through the GARCH model as

$$Cov_t = \omega + \alpha S_{t-1} M_{t-1} + \beta Cov_{t-1} \quad (9)$$

#### D. Employee Stock Options for Closely Held Firms

In the Modigliani-Miller world, value of a firm is a direct function of the firm's asset beta. The asset beta is generally known as unlevered beta, which typically is obtained from levered equity beta. However, if a firm is not a public company, it is difficult to measure equity betas, and consequently, it seems necessary to examine components of asset betas directly. Assuming for simplicity that a firm is financed with 100% pure equity, we find the following factors determine asset betas:

1. The production technology in terms of the firm's capital-output ratio (or asset turnover ratio) and the productivity of labor representing the variable factor of production (or the cost of goods sold or the gross profit margin)
2. The economic conditions facing the industry
3. The volatility of the market as a whole.

To see this, we follow a theory of firm valuation similar to the ones found in Subramanyam and Thomadakis (1980). If a firm's output is produced with one variable and

<sup>5</sup> This formula is generally known as the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) of order (1,1) and is abbreviated as GARCH(1,1). See T. Bollerslev (1986).

<sup>6</sup> Eq (5) is generally estimated by either a regression, or the maximum likelihood technique, or variance targeting technique. See, for example, J. C. Hull (2006), pp. 465-477.

one fixed factor of production, the conventional definition on the return on assets for a competitive firm in an industry  $j$  is  $r_{ji} = \frac{p_j q_{ji} - w_j L_{ji}}{V_{ji}} = p_j k_{ji} - w_j \lambda_{ji}$  where:

$p_j$  = the price of the output  $j$ ,

$k_{ji}$  = the average product of the fixed factor of production, capital, for a firm  $i$  in producing the output  $j$ , i.e.  $k_{ji} = \frac{q_{ji}}{V_{ji}}$ .

$w_j$  = the wage rate in industry  $j$ ,

$L_{ji}$  = the number of variable factors of production, e.g. workers, hired by a firm  $i$  in producing the output  $j$ , and

$\lambda_{ji}$  = the labor capital ratio for a firm  $i$  in producing the output  $j$ , i.e.  $\lambda_{ji} = \frac{L_{ji}}{V_{ji}}$ .

If the factor market is competitive, the wage rate equals the value of the marginal product of labor, i.e.  $w_j = p_j MPL_{ji}$ , where  $MPL$  is the marginal product of labor and given

by  $MPL_{ji} = \frac{\partial q_{ji}}{\partial L_{ji}}$ . Suitably rearranging the expression for the marginal product of labor,

$MPL_{ji}$  can be expressed in the elasticity form as  $MPL_{ji} = \frac{\partial q_{ji}}{\partial L_{ji}} \frac{L_{ji}}{q_{ji}} \frac{q_{ji}}{V_{ji}} \frac{V_{ji}}{L_{ji}} = \eta_{q_{ji}, L_{ji}} \frac{k_{ji}}{\lambda_{ji}}$ , where

$\eta_{q_{ji}, L_{ji}}$  represents the elasticity of output with respect to labor. In this circumstance, Rhee (2005) has shown that the asset beta becomes equal to:

$$\beta_i = \frac{Cov(r_i, r_m)}{\sigma_m^2} = \frac{Cov(p_i, r_m)}{\sigma_m^2} \cdot k_{ji} \cdot [1 - \eta_{q_{ji}, L_{ji}}] \quad (10)$$

The symbol  $\sigma_m^2$  is the variance rate for the return on market portfolio. We will assume that  $0 < \eta_{q_{ji}, L_{ji}} < 1$  so  $\beta_i > 0$ . Eq. (10) states that the firm's asset beta is higher, the greater the

correlation between the industry's demand conditions and the economy, i.e.  $\frac{Cov(p_i, r_m)}{\sigma_m^2}$ , the

greater the average product of capital,  $k_{ji} = \frac{q_{ji}}{V_{ji}}$ , and the lower the output elasticity of labor,

$\eta_{q_{ji}, L_{ji}}$ . As a result, it is possible that a technology firm with a high average product of capital operating on a low output elasticity of labor is seen riskier, at least on surface, as they seem to increase the asset beta.

The significance of Eq. (10) lies in the fact that one can directly estimate the value of equity by looking directly at the components of a firm's asset beta. The industry's covariance

with the market portfolio, i.e.  $Cov(p_t, r_m)$ , the market volatility,  $\sigma_m^2$ , as well as the firm's asset turn over ratio,  $k_{ji}$ , and the output elasticity of labor,  $\eta_{q_{ji}, L_{ji}}$ , are important factors. Consequently, if the asset beta is assumed to vary with time, the expected future estimate of Eq. (10) becomes:

$$E[\beta_{t+k}] = \alpha_k k_{ji} = \frac{E[Cov_{t+k}]}{E[\sigma_{t+k}^2]} \cdot [1 - \eta_{q_{ji}, L_{ji}}] \cdot k_{ji}. \quad (11)$$

Eq. (11) assumes that the industry is competitive.<sup>7</sup> Practicability of Eq. (11) in valuing employee stock options is as follows.

First, notice that the ratio,  $\frac{E[Cov_{t+k}]}{E[\sigma_{t+k}^2]}$ , is same for all firms in a given industry,  $j$ . As a result, one can compute the value from data available for other publicly traded firms in the same industry. The labor elasticity of output differs for different firms, however.

If  $\frac{E[Cov_{t+k}]}{E[\sigma_{t+k}^2]} = 1.20$ , and  $\eta_{q_{ji}, L_{ji}} = 0.2$ , for example, it can be shown that  $E[\beta_{t+k}] = \frac{E[Cov_{t+k}]}{E[\sigma_{t+k}^2]} \cdot [1 - \eta_{q_{ji}, L_{ji}}] \cdot k_{ji} = (1.20) \cdot (1 - 0.2) \cdot k_{ji}$ , i.e.  $E[\beta_{t+k}] \equiv \alpha_k k_{ji} = 0.96 \cdot k_{ji}$ . In practice, the labor elasticity of output can be approximated in terms of gross profit margin from its costs of goods sold, as labor is generally regarded as variable factors of production. Consequently, all that is remaining in computing the future expected asset beta,  $E[\beta_{t+k}]$ , is to give a value for the capital output ratio, which can be measured in fact by the asset turnover ratio. For example, if the capital output ratio, or the asset turnover ratio,  $k_{ji} = 1.5$ , then,  $E[\beta_{t+k}] \equiv \alpha_k k_{ji} = 0.96 \cdot 1.5 = 1.44$ . Therefore, we conclude that, given a firm's gross profit margin and the asset turnover ratio, an overtime change in a firm's asset beta may arise as a result of overtime changes in the industry covariance relative to the market.

One can attempt to write theoretical future values of  $\beta_{t+k}$  solely as a function of the capital output ratio, or the asset turnover ratio, as:

$$\beta_{t+k} = \alpha_k k_{ji} + \varepsilon_{ik} \quad (12)$$

It is assumed that  $E[\varepsilon_{ik}] = 0$ ,  $E[\varepsilon_{ik}^2] = \sigma^2$ , and  $E[\varepsilon_{ik} \varepsilon_{is}] = 0$ . Taking expectations on both sides of Eq. (12) produces Eq. (11). It should be noted, however, that the value of  $\frac{E[Cov_{t+k}]}{E[\sigma_{t+k}^2]}$  varies with a future time differencing interval,  $k$ , so that the coefficient,  $\alpha_k$ , is expected to change for

<sup>7</sup> Subramanyam and Thomadakis (1980) discuss the impact of the industry structure on asset betas, where a firm could face negatively sloping industry demand curve, e.g. monopoly power.

different values of  $k$ . It appears quite natural to use the GARCH model to estimate the values of  $\alpha_k$  from earlier Eqs. (8) and (9).

### **E. Summary and Conclusions**

In this paper, we have identified some of the potential issues in valuing the employee stock options, which may be consistent with the recent FASB rule changes, and offered some valuation procedures. FASB Rule 123R may also have an impact on tax accounting in that annual amortized portion of option expenses is mailed out to taxpayers for their income tax purposes. Those who follow the accrual tax accounting must file same with the Internal Revenue Service as an extra income.

Generally, non-transferability of employee stock options is of no relevance to the valuation, as long as the underlying stock is publicly sold and one can write a put option. When options can be exercisable on a particular date in the future, we have shown that the Black-Scholes option pricing model offers the most straightforward solution to fully vested options. However, some employee stock options are of “American” type such that options can be exercisable at any time starting immediately. In this case and if the underlying pays dividends, the binomial lattice model is the right choice in valuing the fully vested options.

Employee stock options, which require a minimum requisite service period, pose some difficulties. However, they are like a call option’s contract written on the employee’s option to stay with the job, and therefore, it constitutes as a compound option except that the exercise price is unknown a priori. The reason is that the employee has an option to leave the job, depending on the employee’s market salary then, before the option is fully vested after satisfying the requisite service period requirement.

In this paper, we have also shown a method to predict possible changes in option values allowing time-varying volatilities. Further research may be needed especially when the volatility of stock prices embodies fractality.<sup>8</sup> Finally, the paper also showed that when the firm is privately held, changes in firm value are triggered by changes in the firm’s asset beta. Three major determinants to the firm’s asset beta were (1) such production technology parameters as the asset turnover ratio and the gross profit margin, (2) the industry’s correlation with the economy, and (3) the volatility of the economy as a whole.

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<sup>8</sup> For example, please see T. Lux (2004), and L. Calvet and A. Fisher (2001, 2002). However, the work was originally inspired three decades earlier by the work by B. B. Mandelbrot and J. W. van Ness (1968), and B. B. Mandelbrot and H. W. Taylor, (1967). See also B. B. Mandelbrot, A. Fisher, and L. Calvet, (1997).

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